Estabilización de fluctuaciones en dinámica de poblaciones

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DDays, Badajoz, 14-11-2014

Why the size of certain populations fluctuate?

Intrinsic to the species. Science (1974), Nature (1976).

review article

Simple mathematical models with very complicated dynamics

Robert M. May*

Is there chaos in real ecology?

Hassel and May (1976).

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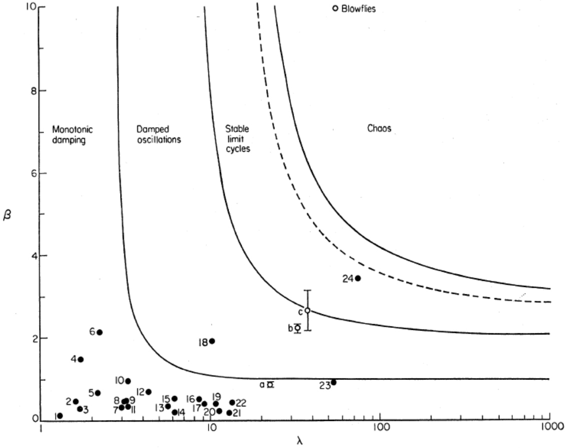
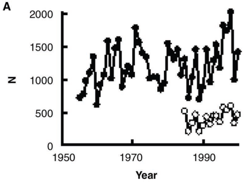


More examples

Costantino et al. Flour beetle (1997) Science.

Cousin et al. Soay Sheep (2001) Science.



And even more...

Becks et al. Microbial food web (2005) Nature.

Beninca et al. Plankton community (2008) Nature.

Why control chaotic fluctuations?


PLAN AHEAD

Outline

Evolution of population size

Discrete population model

$$x_t = f(x_{t-1})$$

- x_t population size
- f describes the population production

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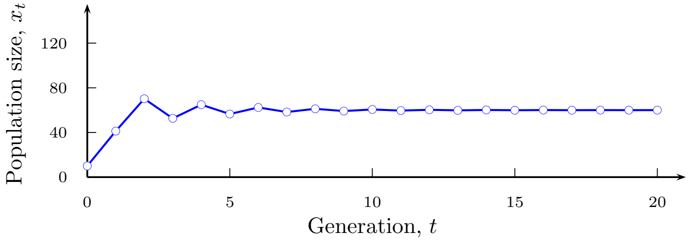

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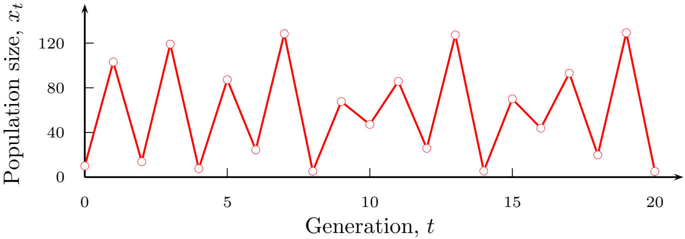
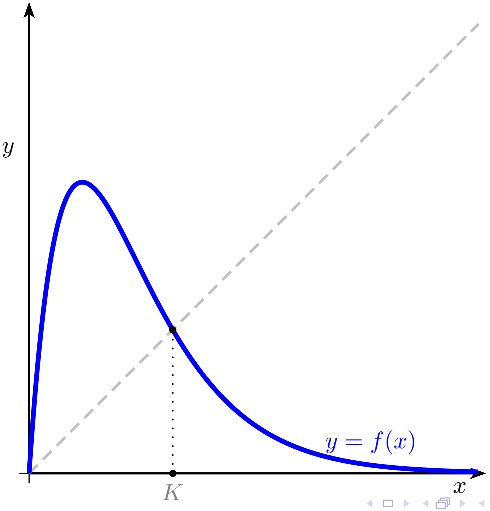


How is f in overcompensatory models?

1. $f\colon \mathbb{R}_+ \to \mathbb{R}_+$ continuous, $f(0) = 0$ and $f(x) > 0$ for $x > 0$.
2. f has a unique positive equilibrium K, $f(x) > x$ for $x \in (0, K)$, and $f(x) < x$ for $x > K$.
3. There exists $d < K$ such that f is increasing in $(0, d)$ and decreasing elsewhere.

How is *f* in overcompensatory models?



Chaos control problem (stabilization)

- Fluctuating population
- Transform it in a (more) stable population
- How?
- Small perturbations of f are not considered suitable, e.g. OGY method

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Adding / Stocking

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Removing / Harvesting

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Mixing both

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Constant harvesting, Constant enrichment, Constant effort harvesting, Constant effort enrichment, Limiter controls, Target oriented controls, Pulse strategies,...

Stabilization, Global stability, Hydra effects, Bubbles, Generated risks (Allee effects), Cost, Initial transient,...

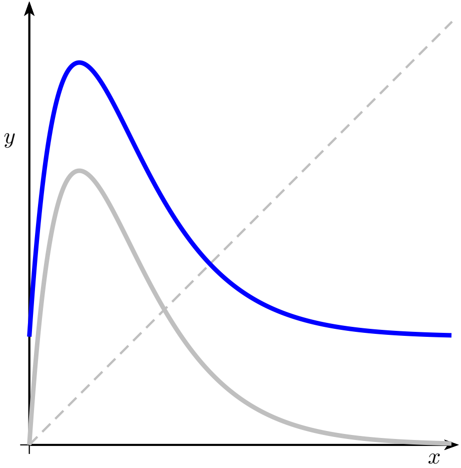
Constant control

Constant immigration

$$x_{n+1} = f(x_n) + C, \qquad C > 0$$

McCallum (1992) J. Theor. Biol.; Parthasarathy & Sinha (1995) Phys. Rev. E; Stone & Hart (1999) Theor. Pop. Biol; Wieland (2002) Phys. Rev. E.

Constant control



Population maps and GAS

Global stability?

Population maps and GAS

Local dynamics drive global dynamics for *S*-unimodal maps!
D. Singer (1978) *SIAM J. Appl. Math.*

4. f satisfies $Sf < 0$ with

$$(Sf)(x) := \frac{f'''(x)}{f'(x)} - \frac{3}{2}\left(\frac{f''(x)}{f'(x)}\right)^2.$$

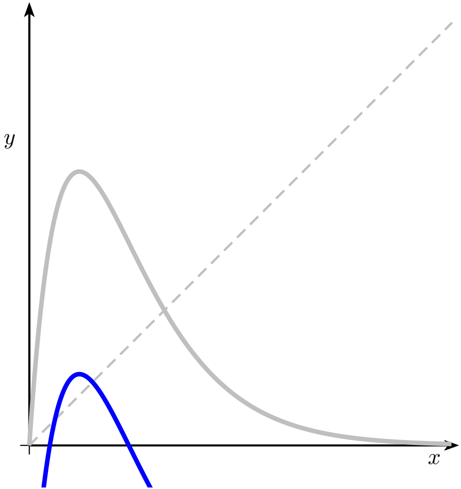
Constant control

Harvesting a constant quota

$$x_{n+1} = f(x_n) - C, \qquad C > 0$$

Gueron (1998) Phys. Rev. E ; Schreiber (2001) J. Math. Biol.; Liz (2010) Theor. Ecol.

Constant control



Constant control

$$x_{n+1} = \max\{0, f(x_n) - C\}, \qquad C > 0$$

- Extinction
- Essential extinction
- Bistability

Schreiber (2001) J. Math. Biol.

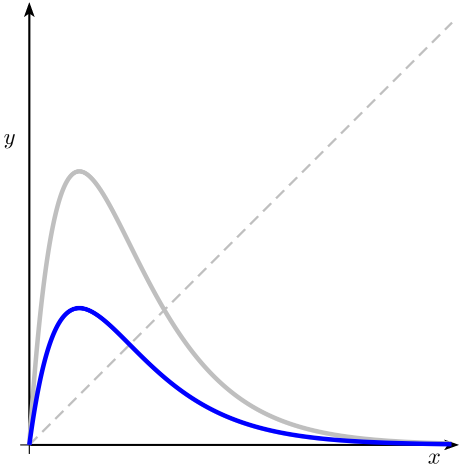
Proportional Control

Harvesting with constant effort

$$x_{n+1} = (1-\gamma) f(x_n), \qquad \gamma \in (0,1).$$

Güémez & Matías (1993) Phys. Lett. A; Liz (2010) Phys. Lett. A

Proportional Control



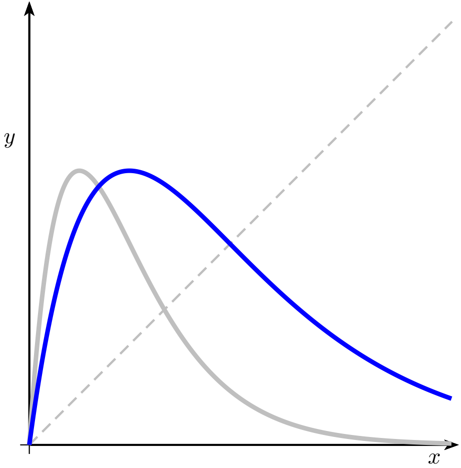
Proportional Control

Harvesting before reproduction,

$$x_{n+1} = f((1-\gamma)x_n), \qquad \gamma \in (0,1),$$

discloses an interesting effect.

Proportional Control



Proportional Control

Hydra effect

Proportional Control

Proportional immigration

$$x_{n+1} = f((1+\gamma)x_n), \qquad \gamma \in (0,\infty).$$

Carmona & Franco. (2011) Nonlinear Anal. R.W.A.

Target oriented control (focusing on the size)

$$x_{n+1} = f(\gamma T + (1-\gamma)x_n), \qquad \gamma \in (0,1).$$

Dattani et al. (2011) Phys. Lett. A; Franco & Liz (2013) Int. J. Bifurcations and Chaos

- There is not danger of extinction.
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- LS implies GAS.

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Description

- Defines certain threshold (the limiter)
- Acts when the population surpasses the limiter
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Limiter constant over time

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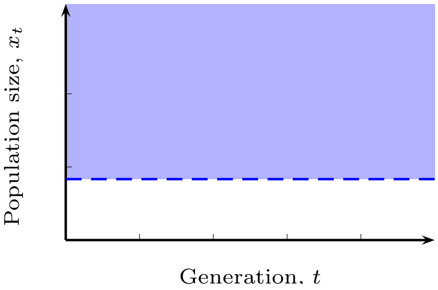

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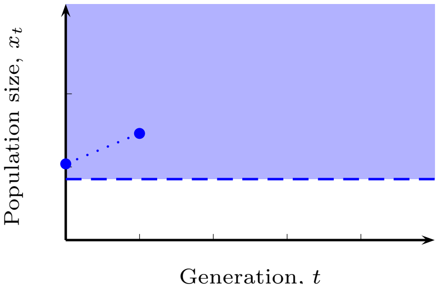

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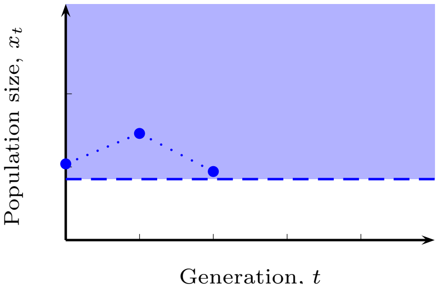

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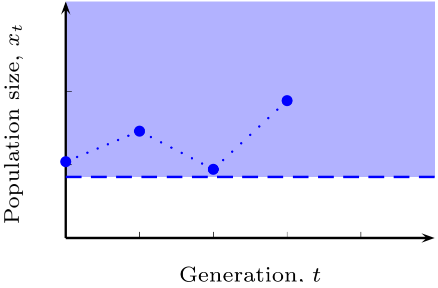

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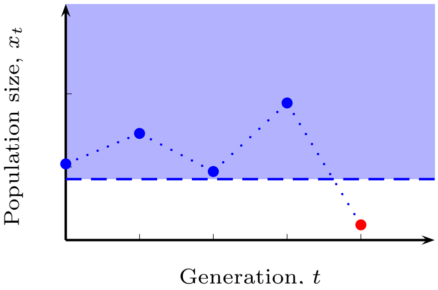

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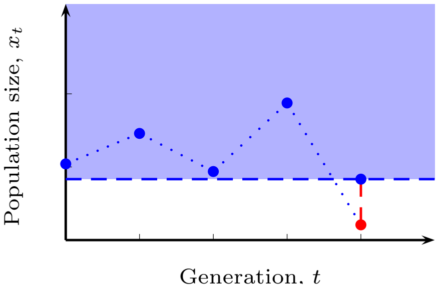

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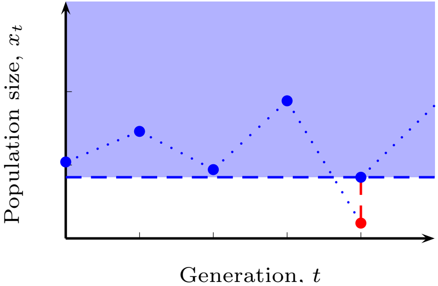

Avoiding crashes
Adaptive limiter control (ALC)

Limiter variable over time

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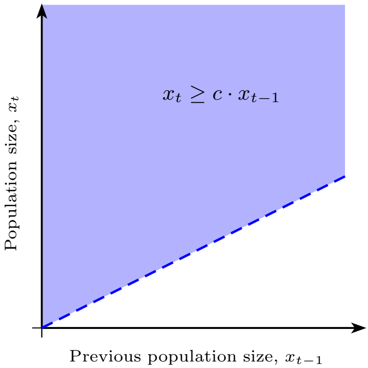

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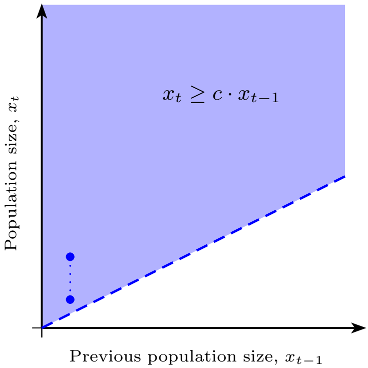

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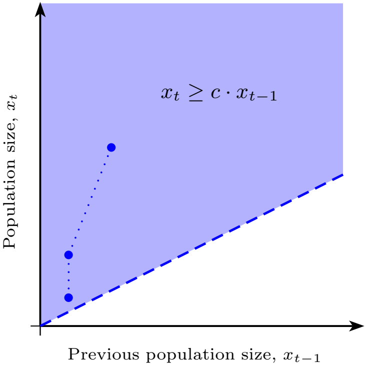

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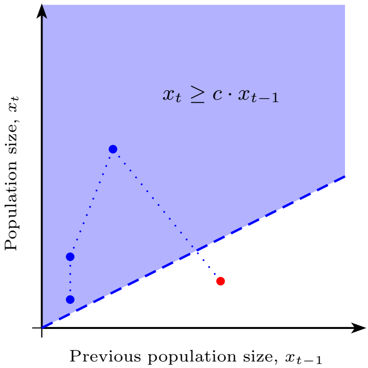

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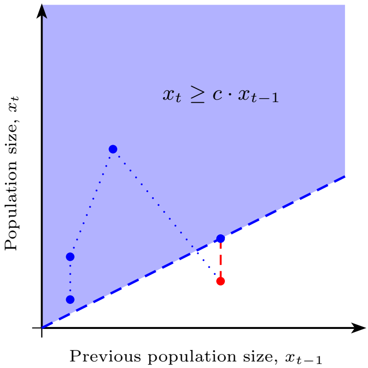

Atypical control

Adaptive limiter control (ALC)

Sah, Salve and Dey (J. Theor. Biol. 2013)

Mathematical model

Numerical simulations

Empirically tested

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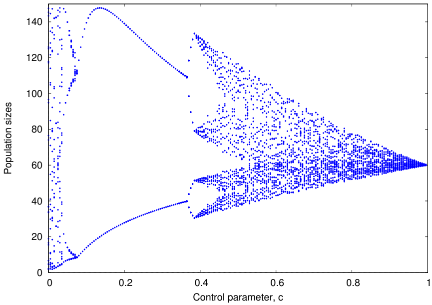

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Analytical results

Effect of increasing the ALC intensity

For a general family population maps (Franco & Hilker (2013) *J. Theor. Biol.*):

1. ALC is not able to stabilise an equilibrium
2. ALC globally confines the population in a trapping region with size tending to 0 as c increases
3. The trapping region is sharp

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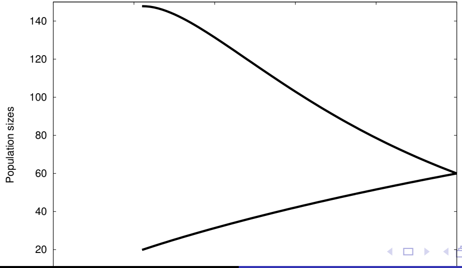

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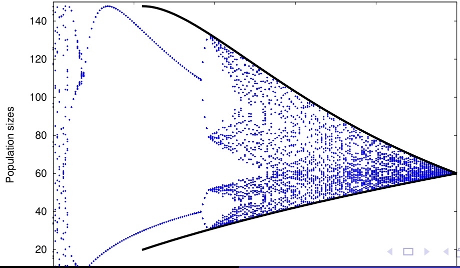

Application

How to choose the ALC intensity

Recipes for:

1. Avoid dropping below a minimum value.
2. Avoid surpassing a maximum value.
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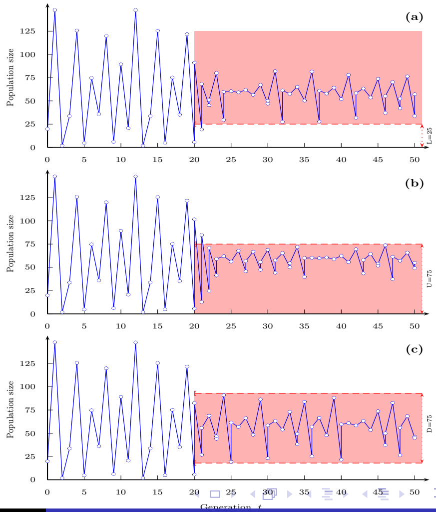

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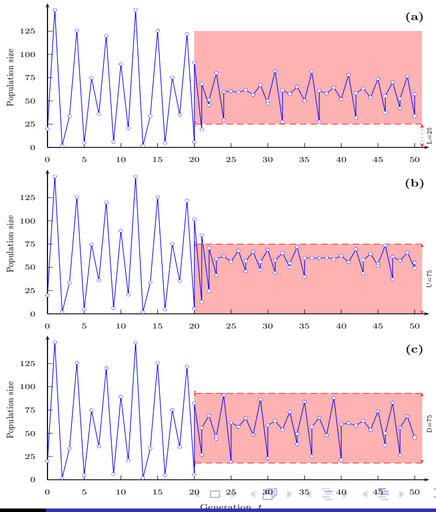

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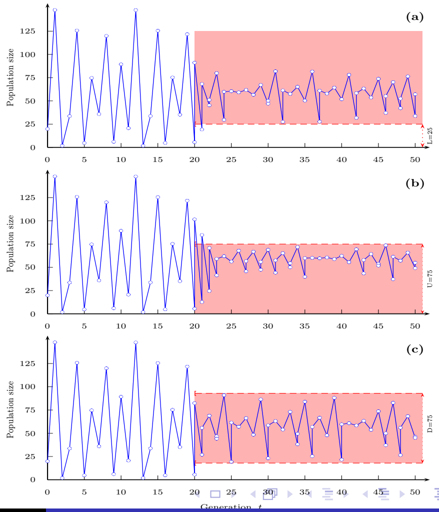

Main tool

Importance of intra-generation variations

- Correct mathematical model $a_{t+1} = \max\{f(a_t), ca_t\}$
- When ALC acts we have two population sizes b_t and a_t
- We must choose one to define the limiter in $t+1$
- In experiments and numeric simulations a_t was selected
- $$b_{t+1} = f(a_t) \qquad \text{and} \qquad a_{t+1} = \begin{cases} b_{t+1}, & b_{t+1} \geq c \cdot a_t \\ c \cdot a_t, & b_{t+1} < c \cdot a_t \end{cases}$$
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Activation threshold

There exists a unique A_T such that $a_{t+1} = \begin{cases} f(a_t), & a_t \leq A_T, \\ c \cdot a_t, & a_t > A_T, \end{cases}$

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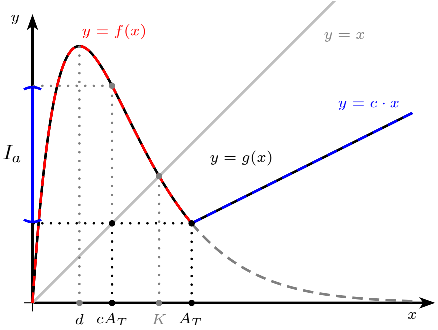
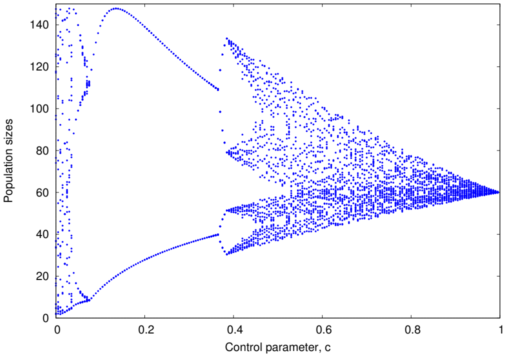


Effort

Measured as the number of individuals added



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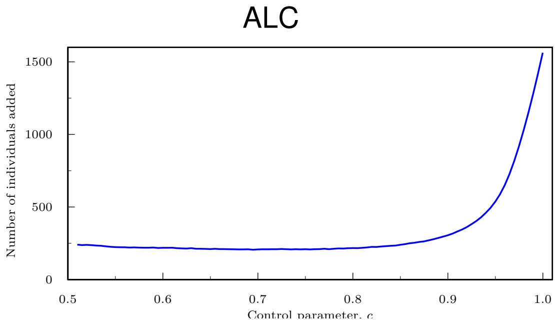

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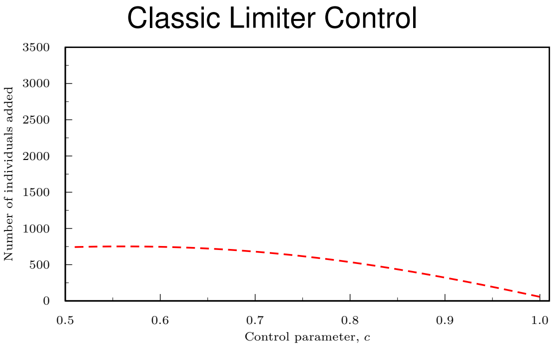

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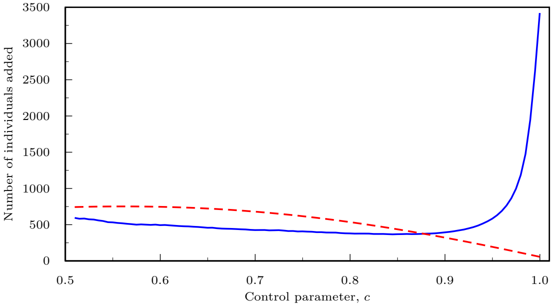
Measured as the number of individuals added

LC vs ALC



Importance of initial transients

Effort

Measured as the number of interventions

Number or consecutive interventions

ALC never acts consecutively inside the trapping region.

What happens if we use b_t?

ALCa

$$a_{t+1} = \begin{cases} b_{t+1}, & b_{t+1} \geq c \cdot a_t, \\ c \cdot a_t, & b_{t+1} < c \cdot a_t, \end{cases}$$

Franco & Hilker (2014) *SIAM J. Appl. Dyn. Syst.*

What happens if we use b_t?

ALCb

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Together with $b_{t+2} = f(a_{t+1})$

Franco & Hilker (2014) *SIAM J. Appl. Dyn. Syst.*

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Model suggested for ALCa!

Franco & Hilker (2014) *SIAM J. Appl. Dyn. Syst.*

Properties of ALCb

Similarities with ALCa

Proposition equilibria

ALCb it is not able to stabilize equilibria.

Proposition activating threshold

ALCb adds individuals in generation t with $t \geq 1$ if and only if $b_{t-1} > A_T$.

Proposition frequency

ALCb never acts in two consecutive generations.

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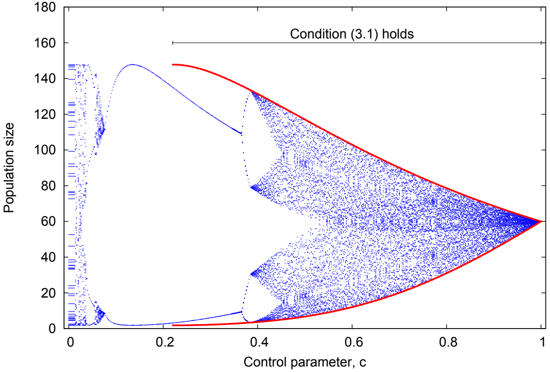
ALCb confines the population sizes before intervention in an interval around K which shrinks as c grows.

Properties of ALCb
Similarities with ALCa



Properties of ALCb

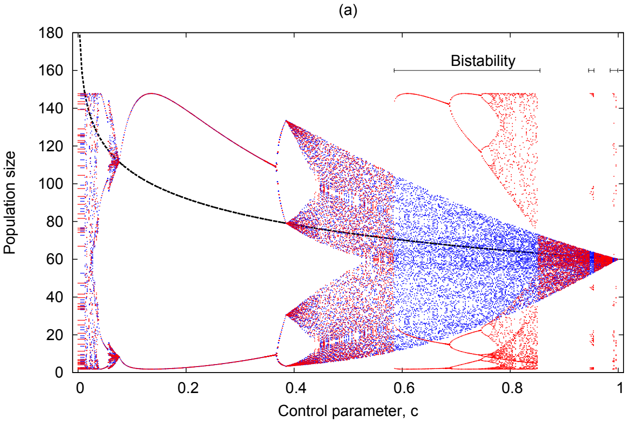
Difference with ALCa

Proposition trapping region

ALCb confines the population sizes before intervention in an interval around K which shrinks as c grows, provided that the initial population size b_0 belongs to certain subinterval of the trapping region.

Properties of ALCb

Difference with ALCa



Global stability in higher order systems

1-D Ricker $x_{n+1} = x_n e^{r - x_n}$ (40 years ago)

2-D Ricker $x_{n+1} = x_n e^{r - x_{n-1}}$ (1 year ago, computer aided proof)

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Global stability in higher order systems

Juveniles and adults

$$\begin{cases} x_{t+1,1} = f((1-h_2)x_{t,2}), \\ x_{t+1,2} = (1-h_1)s_1 x_{t,1} + (1-h_2)s_2 x_{t,2}, \end{cases} \tag{1}$$

$x_{t,1}$ juveniles; $x_{t,2}$ adults; $h_1, h_2 \in [0,1)$ harvest rates; $s_1, s_2 \in (0,1]$ survivorship rates; $f(y) = \alpha y e^{-\beta y}$ Ricker map with $\alpha > 1$, $\beta > 0$.

E. Liz & P. Pilarczyk. *J. Theoret. Biol.* (2012)

Using a result of I. Győri & S. Trofimchuk, system (1) has a positive global attractor for

$$\ln \alpha \in (r_0, r_0+1], \quad \text{where} \;\; r_0 := \ln\Big(\frac{1-(1-h_2)s_2}{(1-h_1)(1-h_2)s_1}\Big).$$

General setting

Lure system

$$x_{t+1} = Ax_t + bf(c^T x_t), \qquad (2)$$

with A a non-negative matrix in $\mathbb{R}^{n \times n}$; $b, c \in \mathbb{R}^n_+ \setminus \{0\}$; and $f \colon \mathbb{R}_+ \to \mathbb{R}_+$ continuous, $f(0) = 0$, and $f(y) > 0, \forall y \in \mathbb{R}_+ \setminus \{0\}$.

Absolute stability

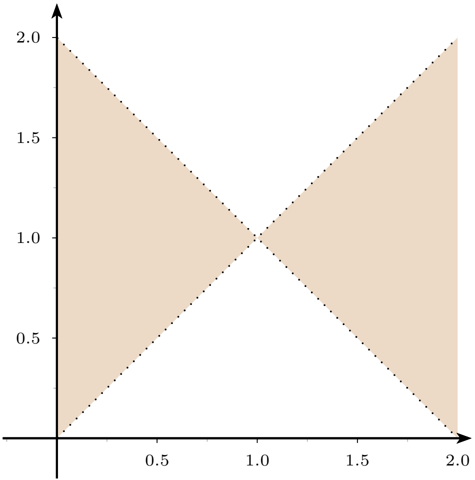
Sector bound condition (Townley et. al. *Systems & Control Letters* 2012)

There exists a unique $y^* > 0$ so that $f(y^*) = py^*$ and

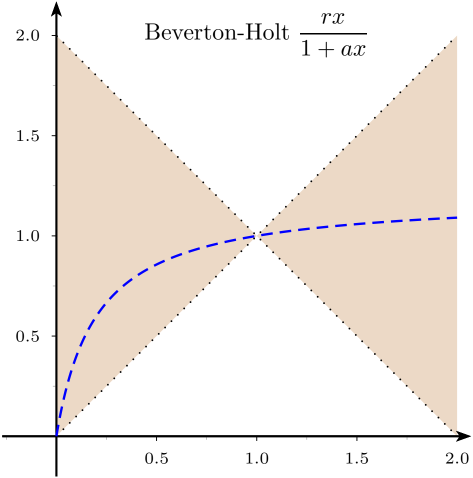
$$|f(y) - py^*| < p|y - y^*|, \quad y \in \mathbb{R}_+ \setminus \{0, y^*\}$$

with $p := \frac{1}{c^T(I-A)^{-1}b}$.

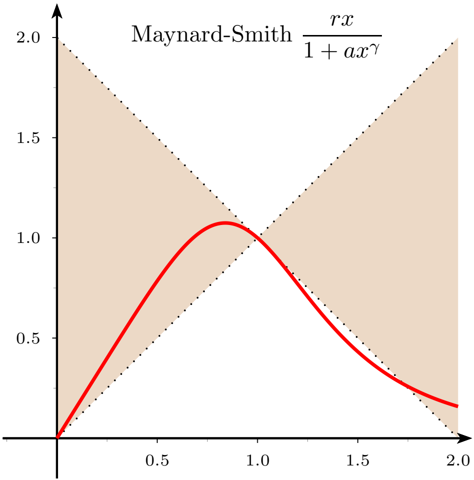
Absolute stability



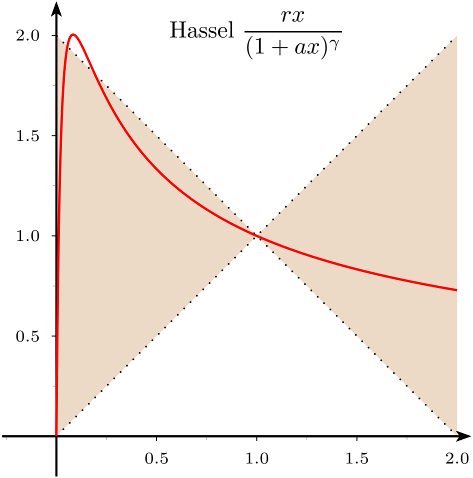
Absolute stability



Absolute stability



Absolute stability



Particular case

$$A = \begin{pmatrix} 1-\delta & 0 & 0 & \cdots & 0 \\ a_1 & 0 & 0 & \cdots & 0 \\ 0 & a_2 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \cdots & 0 & a_{n-1} & 0 \end{pmatrix}, \quad b = \begin{pmatrix} b_1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix},$$

where $0 < \delta \leq 1$, and $a_i, b_1 > 0$,

Results from El-Morshedy and Jiménez-López (2008) can be adapted to show that the GAS of the map implies GAS for the system.

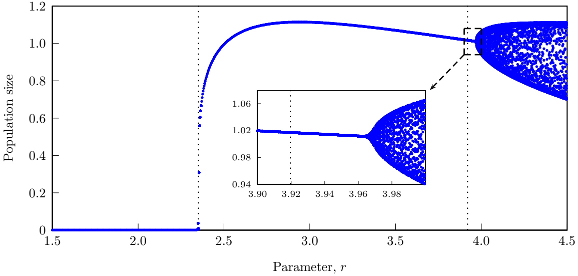
Consequence

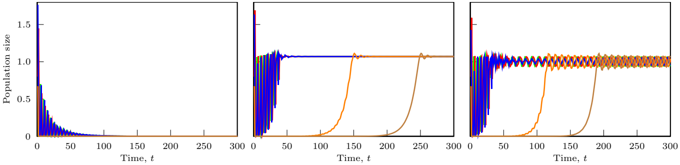
System (1) has a positive global attractor for

$$\ln \alpha \in (r_0, r_0 + 2], \quad \text{where} \;\; r_0 := \ln\Big(\frac{1-(1-h_2)s_2}{(1-h_1)(1-h_2)s_1}\Big).$$

Franco, Logemann & Perán (2014) *System & Control Letters*

Sharp?


1.2
1.0
0.8
0.6
0.4
0.2
0
Population size
1.5
2.0
2.5
3.0
3.5
4.0
4.5
Parameter, r
1.06
1.02
0.98
0.94
3.90
3.92
3.94
3.96
3.98


Population size
1.5
1.0
0.5
0
0
50
100
150
200
250
300
Time, t

Summary

1. Analytical support for ALC and TOC.
2. New interesting properties of ALC and TOC.
3. Recent global stability results improved.

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Liz

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