Satellite constellations: properties and applications

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DANCE Network. Ddays 2016

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OUTLINE

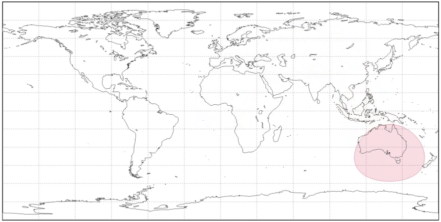
- Introduction
- Satellite Constellation Design
- Flower Constellations
- Station-keeping
- Time distribution methodology
- Applications
- Conclusion and Future work

Motivation

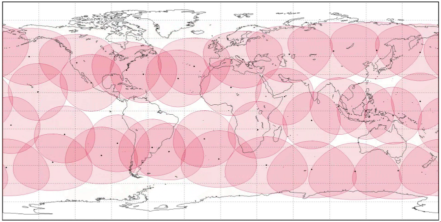
Artificial satellite VS. Satellite constellations

The general trend of proposing the substitution of a single large satellite with a set of smaller satellites working in cooperation has spread through almost all aspects of space research and applications.

A single satellite can cover only a limited portion of the Earth for some particular time intervals.



Satellite constellations can provide continuous global or regional coverage for low revisit interval.



Current situation of satellites orbiting the Earth

Current data

According to NASA, the total number of launched satellites is **7526**. (November 07, 2016) `http://nssdc.gsfc.nasa.gov/nmc/spacecraftSearch.do`

Discipline

The number of satellites (s/c) can be cataloged in different disciplines:

- Astronomy 319 s/c.
- Earth Science 969 s/c.
- Planetary Science 316 s/c.
- Solar and Space Physics 857 s/c.
- Human Crew 332 s/c.
- Life Science 97 s/c.
- Micro-gravity 72 s/c.
- **Communications** 2146 s/c.
- Engineering 419 s/c.
- Navigation and GPS 475 s/c.
- Resupply-Repair 218 s/c.
- **Surveillance and Military** 2302 s/c.
- Technology Applications 281 s/c.

Altitude classifications for geocentric orbits

Altitude classifications

Another way to classify the satellites is according to the altitude of the satellite with respect to the Earth surface.

- Low Earth Orbits **(LEO)**: altitudes up to $2,000$ km.
- Medium Earth Orbits **(MEO)**: altitudes from $2,000$ km. up to $35,786$ km.
- Geostationary Orbits **(GEO)**: altitudes of $35,786$ km. (circular and planar)

- International Space Station (ISS) is in **LEO** region.
  The altitude is about 415 km.
  The velocity is 7.7 km/s.
  Period: 92.65 m.
- Other missions: Earth observation satellites, spy satellites...

Altitude classifications for geocentric orbits

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- Global Positioning System (GPS) is in **MEO** region. The altitude is about $20,200$ km. The velocity of the satellites is 3.8 km/s. Period: 12 h.
- Other missions: Navigation (GPS, Galileo), communication...

Altitude classifications for geocentric orbits

Altitude classifications

Another way to classify the satellites is according to the altitude of the satellite with respect to the Earth surface.

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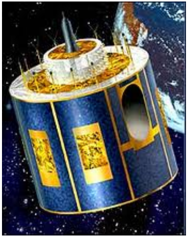


- Meteosat is in **GEO** region.
  The altitude is about $35,786$ km.
  The velocity is 3.07 km/s.
  Period: 24 h.
- Other missions: Weather forecast, meteorology, communications...

Satellite Constellation: Definition and examples

Definition

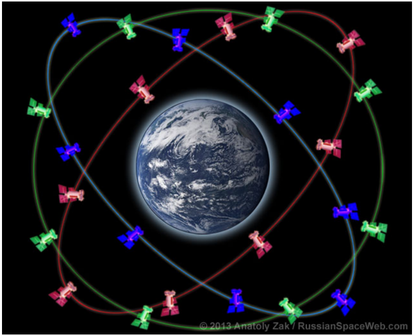
A satellite constellation is a group of artificial satellites working together and following the same goal.

- Global Positioning System (GPS).
- Nationality: USA.
- Number of satellites: 32.
- First launch: February 1978.

Satellite Constellation: Definition and examples

Definition

A satellite constellation is a group of artificial satellites working together and following the same goal.

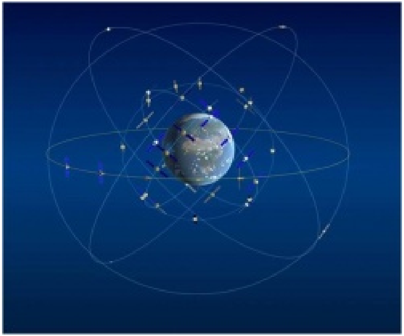


- GLObal NAvigation Satellite System (GLONASS).
- Nationality: Russia.
- Number of satellites: 27. (24 in orbit)
- First launch: October 1982.
- Last launch: May 28, 2016.

Satellite Constellation: Definition and examples

Definition

A satellite constellation is a group of artificial satellites working together and following the same goal.

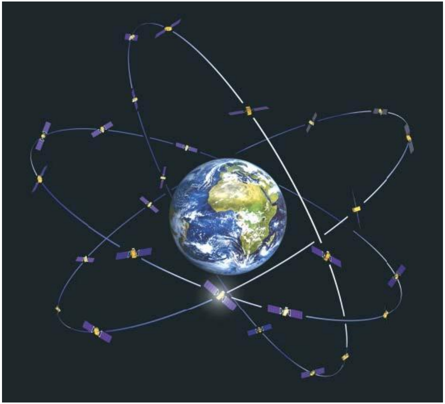


- The BeiDou Navigation Satellite System
- Nationality: China.
- Number of satellites: 35. (21 in orbit)
- First launch: 30 October 2000.
- Last launch: 12 June 2016.

Satellite Constellation: Definition and examples

Definition

A satellite constellation is a group of artificial satellites working together and following the same goal.

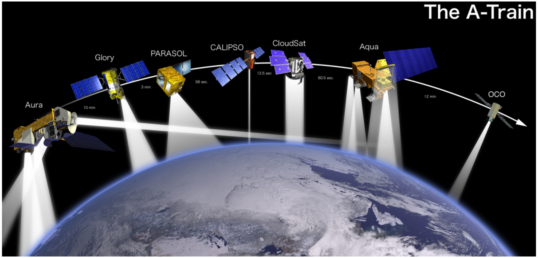


- Galileo
- Nationality: Europe.
- Number of satellites: 30 (12 in orbit)
- First launch: 21 October 2011
- Last launch: 17 December 2015.

Satellite Constellation: Definition and examples

Definition

A satellite constellation is a group of artificial satellites working together and following the same goal.



- A-train constellation
- Nationality: Multiple
- Number of satellites: 6
- Aura, Glory, PARASOL, CALIPSO, CloudSat...

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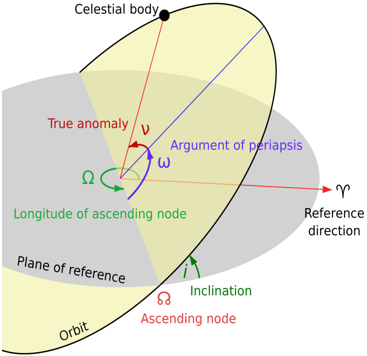
Determination of a satellite orbit

Classical Orbital Elements

The **Classical Orbital Elements** are the parameters required to uniquely identify a specific orbit. The traditional orbital elements are the six Keplerian elements.

- Semi-major axis (a).
- Eccentricity (e).
- Inclination (i).
- Argument of perigee (ω).
- Longitude of the ascending node (Ω).
- Mean anomaly (M).



Satellite Constellation History

Constellation design is generally a very difficult problem because each orbit has an infinite number of choices for the six orbital parameters.

$$\boxed{\text{Parameters } = 6 \times \text{ Number of satellites}}$$

Designers adopt other ideas:

Streets-of-coverage design **1961**

Satellites in circular polar orbits separated such that the ground coverage overlaps to provide full coverage.

Walker Constellations **1970**

Uniformly distributed satellites in equally spaced planes in circular (e = 0) orbits. (i: t/p/f)

Draim constellations **1987**

Highly eccentric orbits may be better than circular ones.

Original Flower Constellations Theory **2004**

Not constrained to circular orbits. Satellites belong to the same repeating space track.

Background of Flower Constellations

The most relevant feature of this model consists of the visualization and study of the constellations using a **rotating reference frame** instead of an inertial frame. That way, a relative orbit whose geometry reminds the shape of the petals of a flower is obtained. Furthermore, **symmetries** of satellites (in space and/or time) appear playing a key role.

Evolution of Flower Constellation

Original Flower Constellations. They make easy to design constellations for continuous or persistent observations of Earth sites and regions.
Difficult parametrization.

Lattice Flower Constellations. They provide all symmetric solutions with a minimum parameterization.
Method: Hermite normal form.

Necklaces on Flower Constellations. They provide all symmetric solutions using subsets of the admisible locations.
Method: number theory.

Original Flower Constellation Theory

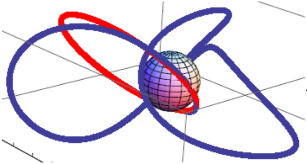
Flower Constellations (FCs) are satellite constellations whose satellites share the same closed and repetitive ground track with respect to a rotating reference frame.

A **FC** is defined by a set of 9 parameters. The main characteristics are:

- The orbital parameters a, e, i and ω are the same for all satellites.
- The RAAN and the Mean anomalies are given by

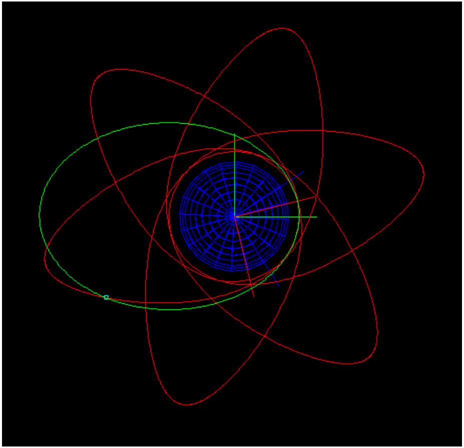
$$\Omega_{k+1} = \Omega_k + 2\pi \frac{F_n}{F_d}, \qquad M_{k+1} = M_k - 2\pi \frac{N_p F_n + F_d F_h}{N_d F_d}, \qquad k \in \{0, 1, ..., N_{s-1}\}$$

- where, F_n, F_d, F_h are three phasing (integer) parameters which give the distribution of the satellites into admissible positions along the relative trajectory.
- The orbital period (a) is determined by compatibility equation $N_p T_p = N_d T_d$. Thus, the relative ground track is **closed, repetitive, and contains all the satellites**.



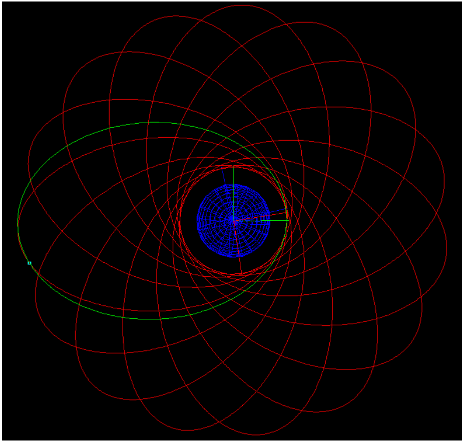
⇐The orbit in the ECI and ECEF frames of reference.

Original Flower Constellation Theory. Example



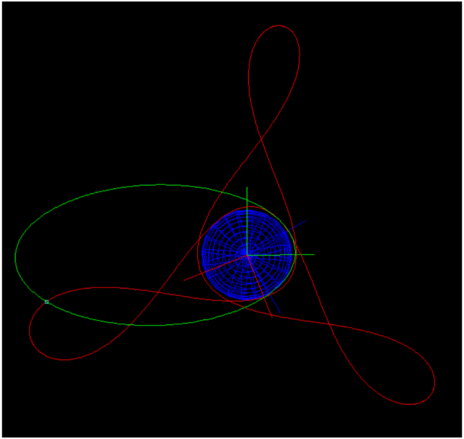
- $N_p = 6$, $N_d = 1$
- $N_p T_p = N_d T_d \quad \rightarrow \quad a.$
- $e = 0.5$, $i = 40^\circ$, $\omega = 0^\circ$.

Original Flower Constellation Theory. Example



- $N_p = 15$, $N_d = 1$
- $N_p T_p = N_d T_d \quad \rightarrow \quad a.$
- $e = 0.6$, $i = 20^\circ$, $\omega = 0^\circ$.

Original Flower Constellation Theory. Example



- $N_p = 3$, $N_d = 1$
- $N_p T_p = N_d T_d \quad \rightarrow \quad a.$
- $e = 0.65$, $i = 45°$, $\omega = 0°$.

Original Flower Constellation Theory

The original theory of FCs presents some problems:

- Is this the minimum parametrization?
- Equivalence problem: different parameters give the same constellation.
- Parameters F_n, F_d, F_h must be coprimes, and they don't have physical meaning.

The Lattice Theory of Flower Constellations (LFCs)

- A Lattice Flower Constellation (LFC) is described by a set of 7 parameters.
- The theory provides all symmetric solutions (space/time).
- In this formulation, the satellites are not required to share the same relative orbit. Consequently, the constellation will have one or multiple relative orbits.
- The ground track(s) is(are) closed and repetitive only if the compatibility equation is satisfied $N_p T_p = N_d T_d$.

2D Lattice Flower Constellation Theory

Continuous parameters:

- Semi-major axis a, eccentricity e, inclination i, argument of perigee ω.
- Ω_{00} and M_{00} are the RAAN and Mean anomaly of the reference satellite.

Integer parameters:

- N_o : Number of orbits.
- N_{so} : Number of satellite per orbit.
- N_c : The configuration number. $N_c \in [0, 1, \ldots, N_o - 1]$.

Equations

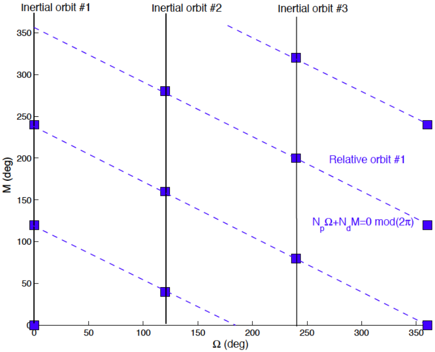
These parameters (that do not need to be coprimes) determine the (Ω, M)-space:

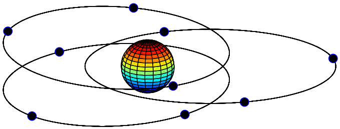
$$\begin{pmatrix} N_o & 0 \\ N_c & N_{so} \end{pmatrix} \begin{pmatrix} \Omega_{ij} - \Omega_{00} \\ M_{ij} - M_{00} \end{pmatrix} = 2\pi \begin{pmatrix} i \\ j \end{pmatrix}.$$

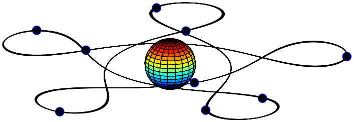
where $i = 0, \ldots, N_o - 1$, $j = 0, \ldots, N_{so} - 1$. Then, the satellite (i, j) is the j^{th} satellite on the i^{th} orbital plane.

2D-LFC example

2D-LFC : $N_o = 3$, $N_{so} = 3$, $N_c = 2$, $N_d = 3$, $N_p = 5$.

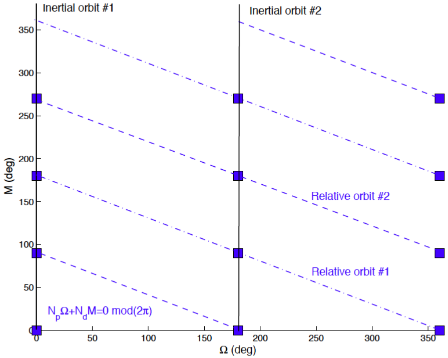


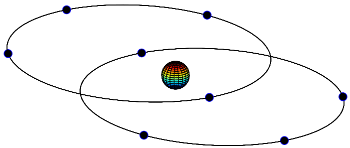


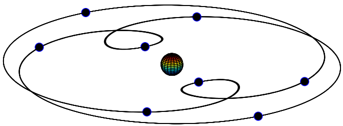


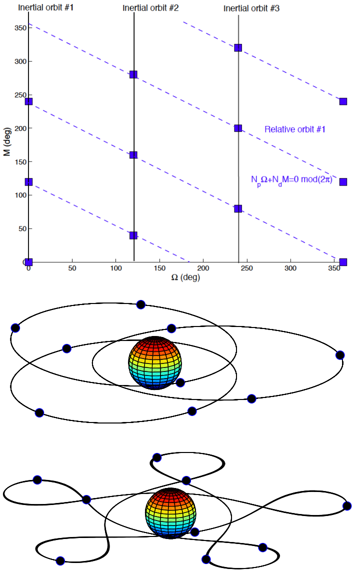
2D-LFC example

LFC : $N_o = 2,\ N_{so} = 4,\ N_c = 0,\ N_d = 4,\ N_p = 2.$

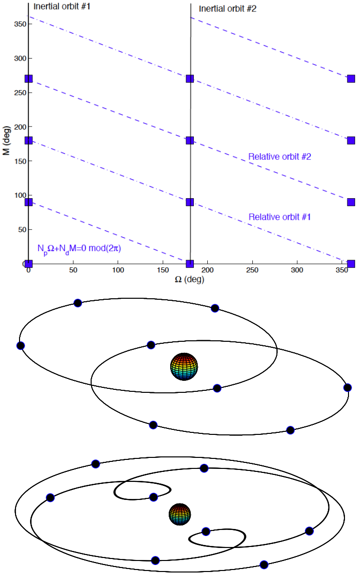








LFC : $N_o = 3$, $N_{so} = 3$, $N_c = 2$, $N_d = 3$, $N_p = 5$.



LFC : $N_o = 2$, $N_{so} = 4$, $N_c = 0$, $N_d = 4$, $N_p = 2$.

3D Lattice Flower Constellation Theory

Continuous parameters:

- Semi-major axis a, eccentricity e, inclination i.

Integer parameters:

- N_o : Number of orbital planes.
- N_ω : Number of orbits in each plane.
- N'_{so} : Number of satellites on each orbit. (Note that: $N_{sat} = N_o N_\omega N'_{so}$)
- $N_{c_1}, N_{c_2}, N_{c_3}$: The phasing parameters.

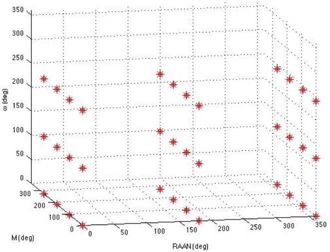
These parameters determine the (ω, Ω, M)-space:

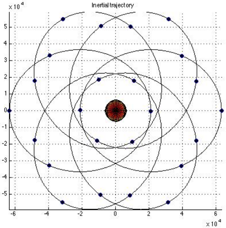
$$\begin{pmatrix} N_o & 0 & 0 \\ N_{c_3} & N_\omega & 0 \\ N_{c_1} & N_{c_2} & N'_{so} \end{pmatrix} \begin{pmatrix} \Omega_{ijk} \\ \omega_{ijk} \\ M_{ijk} \end{pmatrix} = 2\pi \begin{pmatrix} i \\ j \\ k \end{pmatrix}.$$

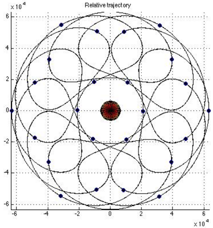
where $i = 0, 2, \ldots, N_o - 1$, $j = 0, 2, \ldots, N'_{so} - 1$, $k = 0, 2, \ldots, N_\omega - 1$, $N_{c_1} \in [0, N_o - 1]$, $N_{c_2} \in [0, N_\omega - 1]$, $N_{c_3} \in [0, N_o - 1]$

3D-LFC example

$N_o = 2$, $N_\omega = 3$, $N'_{so} = 4$, $N_{c_1} = N_{c_2} = N_{c_3} = 0$, $e = 0.5$, $i = 20^\circ$







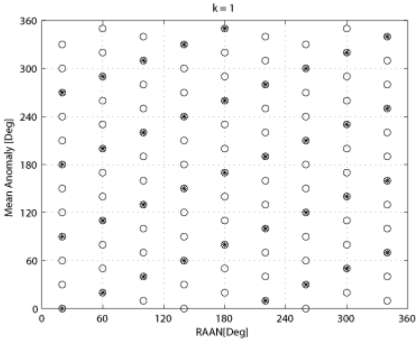
Software: Ikebana 5.0

$\boxed{\text{Let us play!}}$

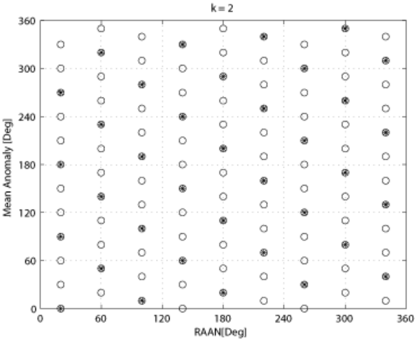
Necklace Flower Constellations (NFC)

Necklace Theory on Flower Constellations

We would like to have a constellation with the same overall dynamics of a Lattice Flower Constellation (keep symmetries) but fewer satellites.



2D-LFC : $N_{so} = 12$,
NFC : $N_{rso} = 4, k = 1.$



2D-LFC : $N_{so} = 12$,
NFC : $N_{rso} = 4, k = 2.$

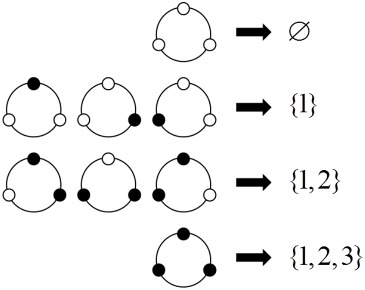
Necklace Flower Constellation. Formulation

We identify the first orbit of the (Ω, M)-space with a subset $\mathcal{G} \subseteq \{1, \cdots, N_{so}\}$.

Definition

Two subsets $\mathcal{G}$ and $\mathcal{G}'$ that differ by an additive constant are considered identical:

$$\mathcal{G} = \mathcal{G}' \iff \exists\, s : \mathcal{G} \equiv \mathcal{G}' + s \mod (n).$$



Unlabeled necklaces with three pearls and two colors.

Necklace Flower Constellation. Formulation

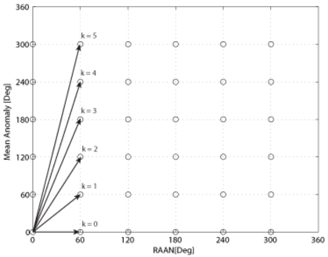
Symmetry number

Let $\mathcal{G}$ be a necklace. $\mathcal{G}$ has a symmetry of length r if $\mathcal{G}$ and $\mathcal{G} + r$ coincide modulo n.

$$\text{Sym}(\mathcal{G}) = \text{mín}\{1 \leq r \leq n : \mathcal{G} + r \equiv \mathcal{G} \mod (n)\}.$$

Shifting parameter

The initial necklace is duplicated for each subsequent orbital plane using a constant **shifting parameter** (an integer $k \in \{0, 1, \cdots, N_{so} - 1\}$).



Shifting problem.

Necklace Flower Constellation. Consistency and minimality

Consistency problem. Due to the modular nature of the Ω parameter, the shifting has to be chosen in such a way that the necklace in the orbit with $\Omega = 0$ coincides with the necklace in the orbit with $\Omega = 2\pi$.

Consistency condition

$$\text{Sym}(\mathcal{G}) | kN_o - N_c$$

Minimality problem. Sometimes, for the same $\mathcal{G}$, there are two values of the shifting parameter which generate the same (Ω, M)-space.

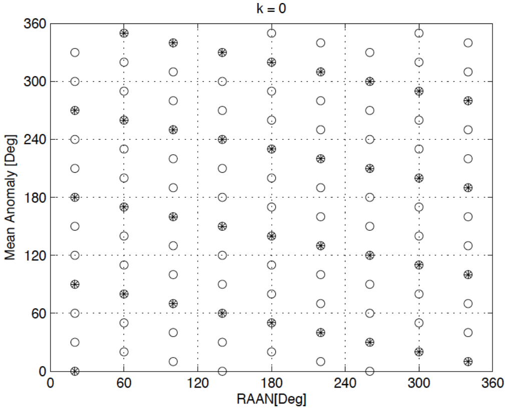
Minimality condition

$$0 \leq k \leq \text{Sym}(\mathcal{G}) - 1$$

Necklace Flower Constellation. Admissible pair

Admissible pair

A pair $(\mathcal{G}, k)$ is an **admissible pair** if the distribution of the satellites is lattice-invariant.

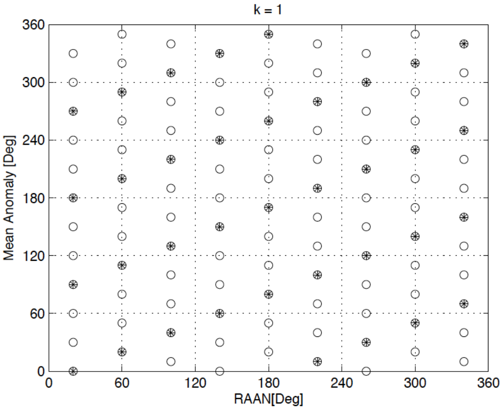


NFC with $\mathcal{G} = \{1, 4, 7, 10\}$, $k = 0$, $N_o = 9$, $N_{so} = 12$, $N_{rso} = 4$, $N_c = 3$.

Necklace Flower Constellation. Admissible pair

Admissible pair

A pair $(\mathcal{G}, k)$ is an **admissible pair** if the distribution of the satellites is lattice-invariant.

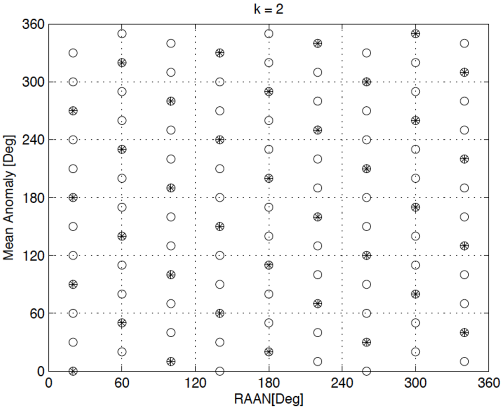


NFC with $\mathcal{G} = \{1, 4, 7, 10\}$, $k = 1$, $N_o = 9$, $N_{so} = 12$, $N_{rso} = 4$, $N_c = 3$.

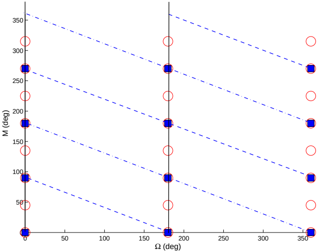
Necklace Flower Constellation. Admissible pair

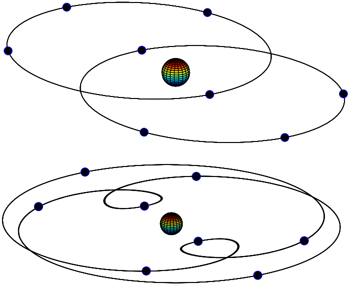
Admissible pair

A pair $(\mathcal{G}, k)$ is an **admissible pair** if the distribution of the satellites is lattice-invariant.

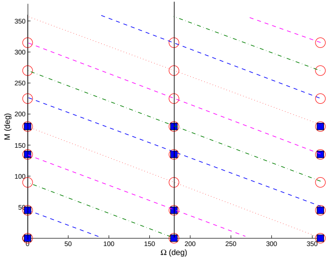


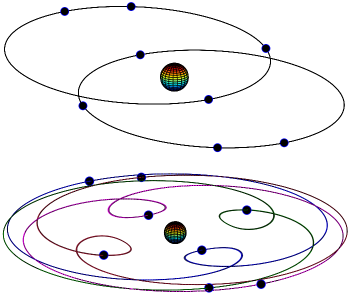
NFC with $\mathcal{G} = \{1, 4, 7, 10\}$, $k = 2$, $N_o = 9$, $N_{so} = 12$, $N_{rso} = 4$, $N_c = 3$.





NFC : $N_o = 2$, $N_{so} = 8$, $N_{rso} = 4$,
$\mathcal{G} = \{1, 3, 5, 7\}$, $k = 0$,
$N_d = 4$, $N_p = 2$





NFC : $N_o = 2$, $N_{so} = 8$, $N_{rso} = 4$,
$\mathcal{G} = \{1, 2, 4, 5\}$, $k = 0$,
$N_d = 4$, $N_p = 2$

Necklace Flower Constellations

It's interesting to compute the total number of **admisible pairs** $(\mathcal{G}, k)$ that can be obtained from a Lattice Flower Constellation with parameters N_{so}, N_o, and N_c.

Counting solutions

The Diophantine equation $d|ak - b$, where a, b, d are positive integers and the unknown k takes integer values in the range $[0, d-1]$ has $Y(d, a, b)$ solutions:

$$Y(d,a,b) = \begin{cases} 0 & \text{if } \gcd(d,a) \nmid b \\ \gcd(d,a) & \text{otherwise.} \end{cases}$$

Work in progress!

A close formula is still required. Or not?

n-D Necklace Flower Constellations

2-D Necklace Flower Constellations

$$\mathrm{Sym}(\mathcal{G})|kN_o - N_c.$$

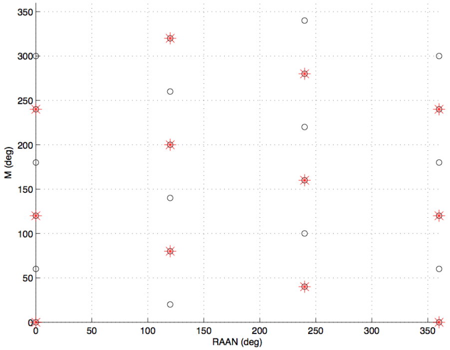
3-D Necklace Flower Constellations

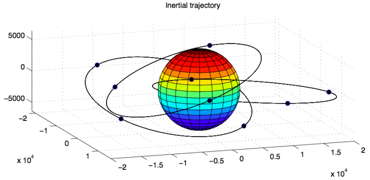
$$\mathrm{Sym}(\mathcal{G}_\omega)\,|\,S_{\omega\Omega}N_o - N_{c_3},$$
$$\mathrm{Sym}(\mathcal{G}_M)\,|\,S_{M\omega}N_{f\omega} - N_{c_2},$$
$$\mathrm{Sym}(\mathcal{G}_\omega)\,|\,S_{M\Omega}N_o - (N_{c_1} - S_{M\omega}N_{c_3}).$$

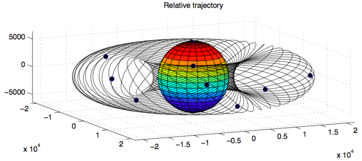
n-D Necklace Flower Constellations

D. Arnas is working on this particular problem.

Parameters: $N_o = 3$, $N_{so} = 6$, $N_{rso} = 3$, $N_c = 2$, $N_p = 5$, $N_d = 1$; $\mathcal{G} = \{1, 3, 5\}$, $k = 2$.







Station-keeping for Lattice Flower Constellations

Problem: Relative distance between the satellites of the constellation is time changing under J_2 perturbation.

Relative station-keeping

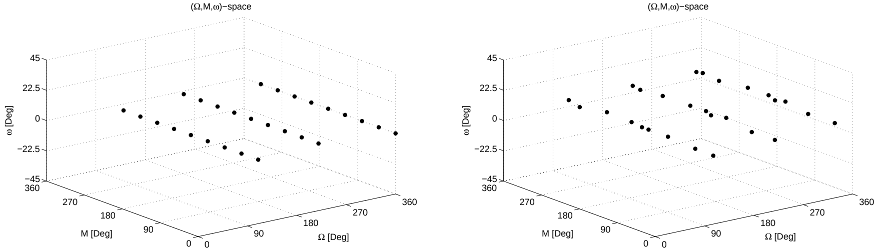
Novel way to design LFCs that preserves the initial distribution of satellites and the initial symmetries over time.

Methodology:

- First, we perform a slight modification of a to get the same secular perturbation for all the satellites.
- Next, the values of e and i are computed so that they minimize the non-secular perturbation of the osculating elements as much as possible.
- Hence, it is possible to obtain a constellation where all the satellites are perturbed in a similar way.

Station-keeping for Lattice Flower Constellations

Example: Galileo constellation (described using the LFC theory: $N_o = 3$, $N_{so} = 9$, $N_c = 2$, $a = 29600.137$km, $e = 0$, $i = 56°$, $\omega = 0°$, $\Omega_{00} = 0°$ and $M_{00} = 0°$).



(a) Initial lattice (b) Lattice after 1 year

Nonetheless, the lattice-preserving property is not valid indefinitely and some orbit-maintenance maneuvers must be planned in order to compensate the ground track shift.

Station-keeping for Lattice Flower Constellations

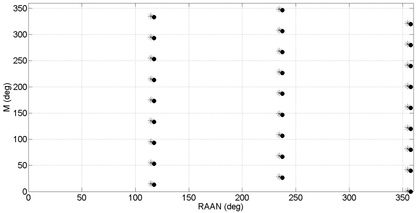
Absolute station-keeping

It is achieved by an impulsive-maneuvering strategy to compensate the shifting in the relative track.

Methodology:

- First, establish a relation between the Ω and the deviation of the ground-track after a repetition cycle.
- Next, plan in-plane maneuvers when the deviation reaches the maximum allowed.
- The new configuration $(\Omega^*_{ij}, M^*_{ij})$ has to follow the LFC design equations.

Example: Galileo constellation



Maximum deviation allowed = 3°
$\Rightarrow$ a maneuver required each 98.5 days.

It results in a $\Delta v = 0.16\ km/s$ per satellite each 3 months, which proves that the absolute station keeping is feasible in this design.

Time Constellations

Objective

Generate satellite constellations that include the effects of orbital perturbations such as the gravitational potential of the Earth, the atmospheric drag, the Sun and the Moon as disturbing third bodies or the solar radiation pressure.

Design possibilities:

- Constellation design with a common relative trajectory.
- Constellation design with multiple relative trajectories.
- Constellation design with minimum number of inertial orbits

The time distribution methodology is able to generate all kinds of satellites configurations including equally spaced time distributions (as the Flower Constellations Theory does) but also formation flying.

Constellation design with a common relative trajectory

Objective

Generate a constellation whose satellites share the same relative trajectory over time in the rotating frame of reference.

Satellite positions can be expressed in the ECI (Earth Centered Inertial) or the ECEF (Earth Centered - Earth Fixed) frame of reference:

$$\begin{aligned} \mathbf{x}|_{ECI} &= \mathcal{R}_3(\Omega)\,\mathcal{R}_1(i)\,\mathcal{R}_3(\omega)\,\mathbf{x}, \\ \mathbf{x}|_{ECEF} &= \mathcal{R}_3(-\psi_{G0} - \omega_{\oplus} t)\,\mathbf{x}|_{ECI}. \end{aligned}$$

where ψ_{G0} is the longitude of Greenwich at the time of reference t_0 and $\omega_{\oplus}$ is the angular velocity of rotation of the Earth.

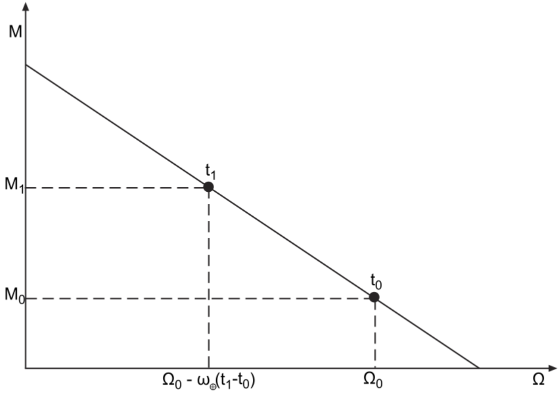
Methodology

- a, e, i and ω must be equal for all the satellites.
- We compute a reference trajectory (named $\mathbf{x_0}$) where t_0 is the reference time of the constellation which also locates ψ_{G0}.
- Locate the remaining satellites in that trajectory

Constellation design with a common relative trajectory

The right ascension of the ascending node Ω_1 and the mean anomaly M_1 of the second satellite can be expressed as a function of the values of the first one:

$$\begin{aligned}\Omega_1 &= \Omega_0 - \omega_\oplus(t_1 - t_0),\\ M_1 &= M_0 + n(t_1 - t_0).\end{aligned}$$



$(\Omega,\ M)$-space representation of a relative trajectory.

Example of constellation defined in a single relative trajectory

- $a = 14420\ km$, $e = 0.4$, $i = 63.435^o$. $\omega = 0^o$.
- $\Omega_0 = 0$, $M_0 = 0$ and $t_0 = 0$ be the parameters of the leading satellite.
- $t_q = 300(q-1)$ where $q \in [1, 5]$ (five satellites), and 300 represents the delay in seconds between satellites.

The Mean anomaly and RAAN for each satellite is computed as follows:

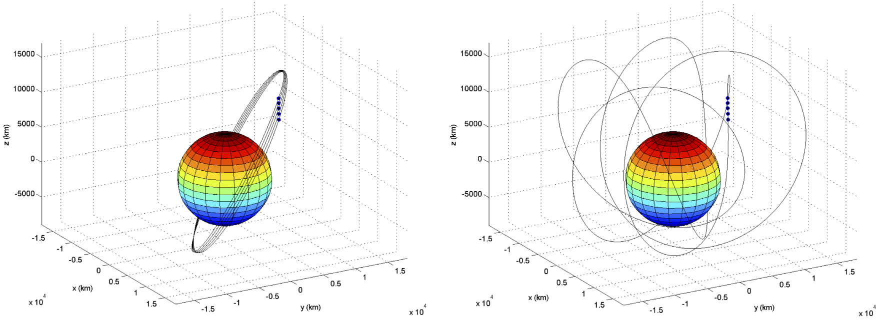
$$\begin{aligned} \Omega_q &= -\omega_{\oplus} t_q; \\ M_q &= n t_q; \end{aligned}$$

which leads to the configuration shown in the following table:

Element	Sat. 1	Sat. 2	Sat. 3	Sat. 4	Sat. 5
$\Omega_q\ (rad)$	0.000	-0.022	-0.044	-0.066	-0.088
$M_q\ (rad)$	0.000	0.109	0.219	0.328	0.438

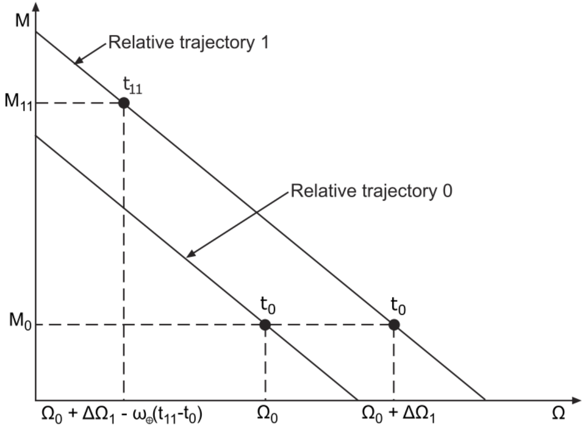
Example of constellation defined in a single relative trajectory

The figure shows the **inertial and relative trajectories** of the constellation. It is very unlikely for the satellites to **collide** because they are moving in the same relative trajectory.



Inertial (left) and relative (right) trajectories of the constellation.

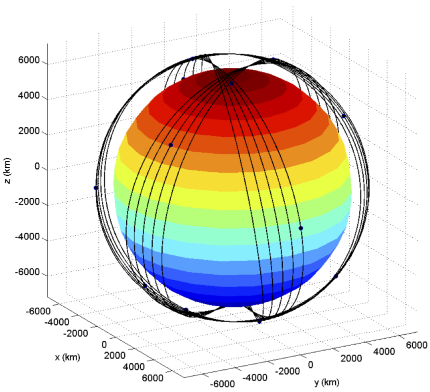
Constellation design with multiple relative trajectories



$(\Omega,\ M)$-space representation of the configuration for multiple relative trajectories.

Example of constellation defined in various relative trajectories

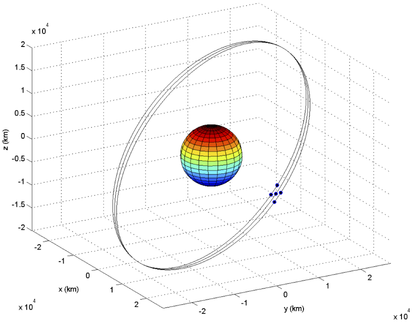
There are 15 different orbits, one for each satellite, however there are only three different relative trajectories (a solid line, a dashed line and a dotted line), which was the objective sought.



Inertial (left) and relative (right) trajectories of the constellation.

Example of constellation defined in various relative trajectories with minimum number of inertial orbits

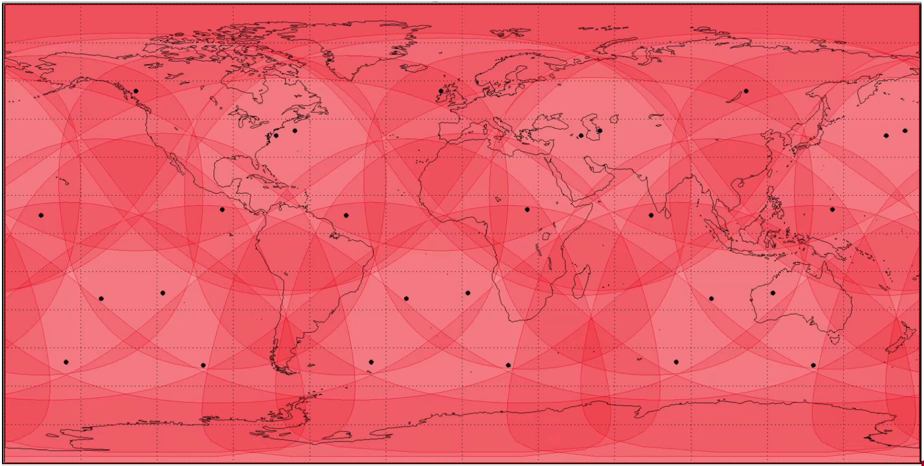
The constellation is built in three different inertial orbits and three relative trajectories generating the "+" shape that we were aiming for.



Inertial (left) and relative (right) trajectories of the constellation.

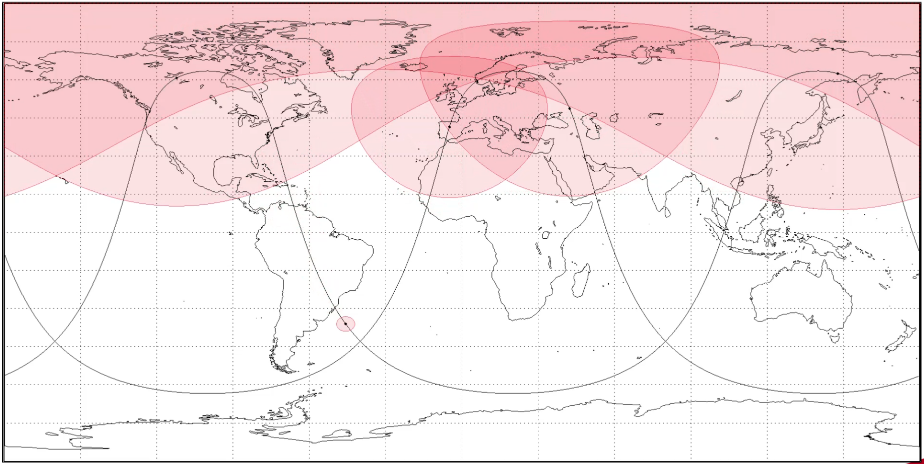
Global Coverage Constellation

Lattice Flower Constellation for Global coverage (Galileo constellation).
Parameters: $N_o = 3$, $N_{so} = 9$, $N_c = 2$, $N_p = 10$, $N_d = 17$.



USA, Europe, Japan connection

Lattice Flower Constellation to connect USA, Europe, Japan.
Parameters: $N_o = 5$, $N_{so} = 1$, $N_c = 3$, $N_p = 3$, $N_d = 1$.



Conclusions

- **Flower Constellations** represent an improvement in the difficult art of satellite constellation design.
- **Necklaces FCs** represent a dramatic step forward with the mission design impact.
- **Station-keeping** for Flower Constellations has been studied. In particular, relative and absolute station-keeping has been applied to Galileo constellation.
- **Time Constellations** allow to include the effect of orbital perturbation in the design procedure.
- **Applications.** The theory has already been applied to design reconnaissance orbits for Earth sites, or GPS-like navigation systems.

Future work

- **Generalization of Necklace FCs** may represent a dramatic step forward with the design methodology.
- **Time Constellations** represent a new procedure to design satellite constellations. It allows to include orbital perturbations to the basic design.
  - **Applications.** It allows to design constellations whose orbits are sun-synchronous, frozen or present a repeating ground-track property.
  - **Formation flying** is a new design procedure where low-cost satellites can be used.

Thanks for your attention!

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Satellite phasing

The phasing of the satellites in the constellation is computed using **three integer parameters**:

- The number of orbits, $N_o = F_d$.
- The number of satellites per orbit,

$$N_{so} = \frac{N_d}{\gcd(N_d, N_p F_n + F_d F_h)}.$$

- The configuration number,

$$N_c = E_n \frac{N_p F_n + F_d F_h}{\gcd(N_d, N_p F_n + F_d F_h)} \pmod{F_d}$$

where E_n is any integer such that $E_n F_n \equiv 1 \pmod{F_d}$.

Back

Satellite phasing

The phasing of the satellites in the constellation is computed using **three integer parameters**:

- Number of orbits, N_o.
- Number of satellites per orbit, N_{so}.
- The configuration number, $N_c \in [0, \ldots, N_o - 1]$.

And it corresponds with all the solutions of the **system of equations**:

$$\begin{bmatrix} N_o & 0 \\ N_c & N_{so} \end{bmatrix} \begin{bmatrix} \Omega \\ M \end{bmatrix} \equiv \begin{bmatrix} 0 \\ 0 \end{bmatrix} \pmod{2\pi} \quad (1)$$

Back