

# Modelos en neurociencia: sistemas dinámicos y más allá

Toni Guillamon

Departament de Matemàtiques, Universitat Politècnica de Catalunya (UPC)  
& Institut de Matemàtiques de la UPC-BarcelonaTech (IMTech)  
& Centre de Recerca Matemàtica (CRM)

Sistemas dinámicos y neurociencia  
Ddays 2021

# Outline

- 1 Models of dynamical systems in neuroscience
  - The role of dynamical systems
- 2 Parameter estimation: At the edge of DS
  - Using the period function in spiking regimes
  - Estimation of synaptic inputs using sliding mode control
- 3 Empiric models: Beyond DS
  - Black-box models
  - Gray-box models: food for DS

# 1. Models of dynamical systems in neuroscience

# Models of neurons/populations activity

- Based on **biophysical principles** (Hodgkin-Huxley type models)

$$\begin{cases} C \frac{dv}{dt} = -g_L(v - V_L) - \sum_{\iota \in \Upsilon} g_{\iota} w_{\iota}^{p_{\iota}}(v - V_{\iota}) + I_{app}, \\ \frac{dw_{\iota}}{dt} = \frac{W_{\iota}(v) - w_{\iota}}{\tau_{\iota}(v)}, \quad |\Upsilon| \approx \# \text{ ionic channels.} \end{cases}$$

$v \in [-100, 60]$  mV approximately is the **membrane potential** of the neuron, and  $w_{\iota} \in [0, 1]$  are the **gating variables**.

# Models of neurons/populations activity

- ▶ Based on **biophysical principles** (Hodgkin-Huxley type models), or (over)simplifications of them (Fitzhugh-Nagumo, integrate and fire, ...).

$$\left\{ \begin{array}{l} \dot{v} = -f(v) + w + I_{app}, \\ \dot{w} = \epsilon(-v - \gamma w), \end{array} \right. \quad \left\{ \begin{array}{l} \dot{v} = -g_L(v - V_L) + I_{app}, \\ v(t^-) = \theta \Rightarrow v(t^+) = v_{reset}. \end{array} \right.$$

## Models of neurons/populations activity

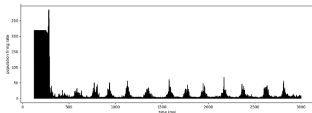
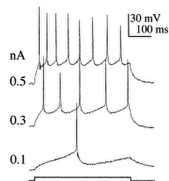
- ▶ Based on **biophysical principles** (Hodgkin-Huxley type models), or (over)simplifications of them (Fitzhugh-Nagumo, integrate and fire, ...).
- ▶ **Mean-field models** derived *analytically* from the biophysical ones (firing rate models, population density models, neural mass models, neural field models, ...).

$$\begin{cases} \tau_E \dot{E} &= -E + (1 - r E) S_E [w_{EE} E - w_{EI} I + k P(t)], \\ \tau_I \dot{I} &= -I + (1 - r' I) S_I [w_{IE} E - w_{II} I + k' Q(t)], \end{cases} \quad [\text{Wilson-Cowan, 1972}]$$

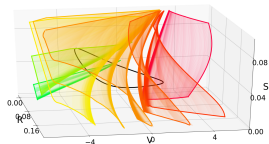
$$\begin{cases} \dot{r} &= \delta/\pi + 2rv, & [\text{Montbrió et al, PhysRevX 2015}] \\ \dot{v} &= v^2 + \bar{\eta} - \pi^2 r^2 - Jr + I_{app}(t). \end{cases}$$

# Models of neurons lead to dynamical systems

- ▶ Thanks to the underlying **(bio)physical analogies**, they are written as **dynamical systems**; so, from a biological point of view, we can benefit from the study of “familiar elements and features” like:
  - **limit cycles** (spikes, oscillations) and their **bifurcations**,
  - **slow-fast** systems,
  - **invariant manifolds**,
  - ...



[N. Gutiérrez, TFM 2021]



[Pérez-Cervera et al, J Nonl Sci 2019]

# Models of neurons lead to dynamical systems

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  - limit cycles (spikes, oscillations) and their bifurcations,
  - slow-fast systems,
  - invariant manifolds,
  - ...
- ▶ So, the **connection** between **dynamical systems** and **neuroscience** is clear.

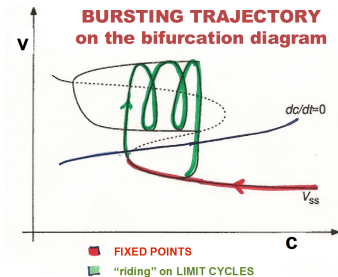


# How do dynamical systems help?

- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems (**bursting**).



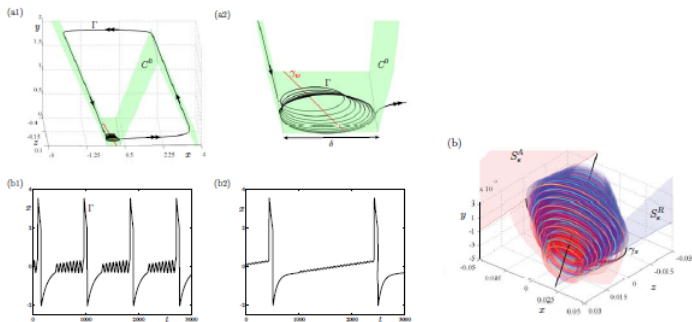
$v, w$  fast,  $c$  slow:



[Rinzel, 1980's]

# How do dynamical systems help?

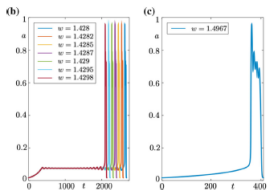
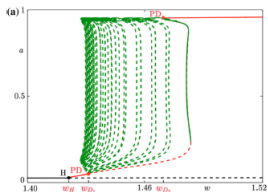
- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems (bursting, **mix-mode oscillations**).



[Desroches et al., SIAM Rev. 2016]

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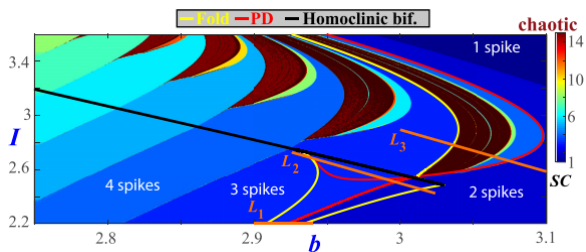
- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems (bursting, mix-mode oscillations, **mix-mode bursting oscillations**).



[Köksal-Ersöz et al., J Math Biol 2020]

# How do dynamical systems help?

- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems (bursting, mix-mode oscillations, mix-mode bursting oscillations, **spike-adding**).

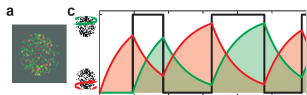
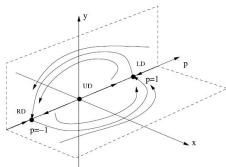


[Barrio et al., CNSNS 2020]

# How do dynamical systems help?

- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems (bursting, mix-mode oscillations, mix-mode bursting oscillations, spike-adding).
- ▶ What is necessary to **rise complexity**? Use of separatrix maps for **quasi-periodic forcing** in **binocular rivalry systems**.

$$\begin{cases} \dot{p} = h(p) + x^2(1-p) + y^2(-1-p), \\ \dot{x} = f(p, x, y) + I_x x + \varepsilon \eta_x(t), \\ \dot{y} = f(-p, y, x) + I_y y + \varepsilon \eta_y(t). \end{cases}$$

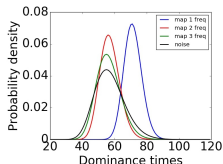
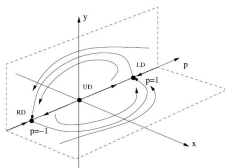


[Pastukhov et al., Front. Comp. Neuro.  
2013]

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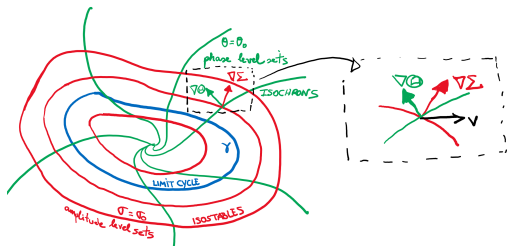


Comparison between fittings to log-normal distribution of the dominance times histograms.

[Delshams, G., Huguet, Chaos 2018]

# How do dynamical systems help?

- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems.
- ▶ What is necessary to rise complexity?
- ▶ DS tools allow to make **accurate predictions** (e.g., phase and response functions)...



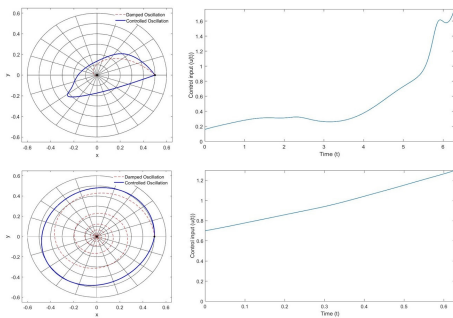
$$PRF(x) = \nabla\theta(x) \cdot v,$$

$$ARF(x) = \nabla\Sigma(x) \cdot v.$$

[Cabr e et al, JDE 2003], [Freire, Gasull, G., BScMat 2007], [G., Huguet, SIADS 2009], [Castej n, G., Huguet, JMathNeuro 2013],  
 [P rez-Cervera et al, JNonSci 2019], [Castej n, G., CNSNS 2020],...

# How do dynamical systems help?

- ▶ Simplified models allow to inspect which mechanisms are involved in relevant neuroscience problems.
- ▶ What is necessary to rise complexity?
- ▶ DS tools allow to make accurate predictions (e.g., phase and response functions) and **exert control**.



[Moreno, TFM 2019; Martínez-Añón, TFM 2020]



## How do dynamical systems help?

- ▶ **Simplified models** allow to inspect which **mechanisms** are involved in relevant neuroscience problems (bursting, mix-mode oscillations, mix-mode bursting oscillations, spike-adding. . .).
- ▶ What is necessary to rise complexity?
- ▶ DS tools allow to make **accurate predictions** (e.g., phase and response functions) and **exert control**.
- ▶ Underlying models are a solid ground also for **parameter estimation**.

## 2. Parameter estimation: At the edge of DS

## Parameter estimation: at the edge of DS

- ▶ Now, we start thinking on **real data**, where only  $v$  and few other outcomes are **measurable**.
- ▶ Electrophysiological experiments are crucial to know the set  $\Upsilon$  (number and type of ionic channels) and estimate the parameters from data... **but** it is not always possible.

$$\begin{cases} C \frac{dv}{dt} = -g_L(v - V_L) - \sum_{\iota \in \Upsilon} g_\iota w_\iota^{p_\iota} (v - V_\iota) + I_{app}, \\ \frac{dw_\iota}{dt} = \frac{W_\iota(v) - w_\iota}{\tau_\iota(v)}, \quad |\Upsilon| \approx \# \text{ ionic channels.} \end{cases}$$

Challenging questions arise:

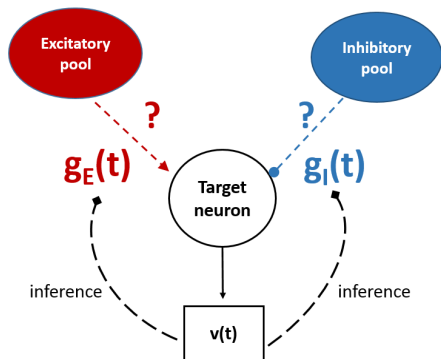
- ▶ How can we **estimate inputs** from partial observations?
- ▶ How can we **disambiguate models** from partial observations?
- ▶ How can we **determine the underlying system**,... when data sources are uncertain (so, not clear biophysics)?

# Estimation of synaptic inputs

Inverse problem:

From a measurable (e.g., membrane potential) we aim at inferring the **temporal contribution** of the **synaptic current** and discerning global **excitation** from global **inhibition**.

- ▶ Note that we assume that **other variables are not measurable** (e.g., partial observations).
- ▶ The time course of synaptic conductances arriving at a single cell provides **local information** about its **functional connectivity**, often an indicator to validate more general theories (e.g., un/balanced networks).

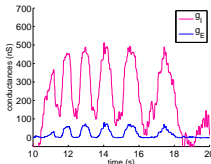
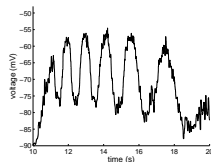


# Estimation of synaptic inputs

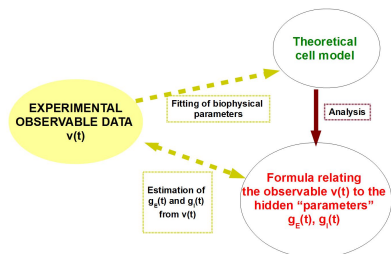
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# Estimation of synaptic inputs: mathematical formulation



## Theoretical cell model:

$$\begin{cases} C\dot{v} = f(v, \mathbf{w}; \mu_1) + I_{app} - I_{syn}, \\ \dot{\mathbf{w}} = h(v, \mathbf{w}; \mu_2) \end{cases} \quad \text{where } v \in \mathbb{R}, \mathbf{w} \in \mathbb{R}^n, \mu_i \in \mathbb{R}^{p_i}$$

- ▶  $f, h$  **nonlinear**, ionic currents, ion channels' dynamics;
- ▶  $I_{app}$ , externally applied current (control parameter);
- ▶  $I_{syn}(t) = g_E(t)(v(t) - V_E) + g_I(t)(v(t) - V_I)$ , synaptic input.

Main goals:

1. How to estimate  $I_{syn}(t)$  given  $v(t)$ ?
2. How to estimate  $g_E(t)$  and  $g_I(t)$  given  $v(t)$ ? (**ill-posed**)

# Estimation of synaptic inputs in spiking regimes

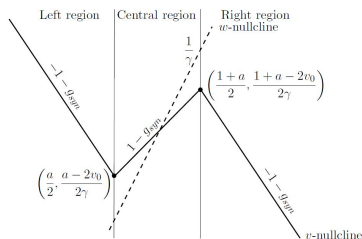
We consider a piecewise version of the Fitzhugh-Nagumo model, the **McKean model**, given by

$$\begin{cases} C \dot{v} = f(v) - w - w_0 + I_{app} - I_{syn}(v), & C \ll 1, \\ \dot{w} = v - \gamma w - v_0, \end{cases} \quad (1)$$

where

$$I_{syn}(v) = g_{syn}(t) (v - V_{syn})$$

$$f(v) = \begin{cases} -v, & v < a/2; \\ v - a, & a/2 \leq v \leq (1+a)/2; \\ 1 - v, & v > (1+a)/2. \end{cases}$$



# Estimation of synaptic inputs in spiking regimes

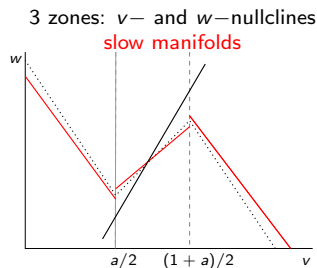
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where

$$I_{syn}(v) = g_{syn}(t) (v - V_{syn})$$

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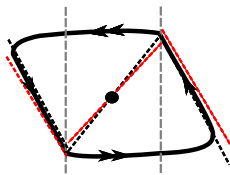




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where  $I_{syn}(v) = g_{syn}(t)(v - V_{syn})$ .

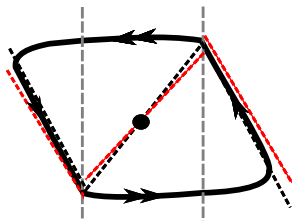


Main goal:

Express the period  $T$  of the periodic orbit as a function of  $g_{syn}$  and  $I_{app}$ ,  
i.e.  $T(I_{app}, g_{syn})$ ,

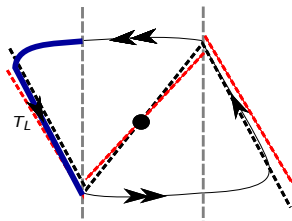
# McKean w/ synaptic input: period of limit cycle

We approximate  $T(I_{app}, g_{syn})$  by  $\hat{T}(I_{app}, g_{syn})$  using analytical computations and **slow-fast** approximations.



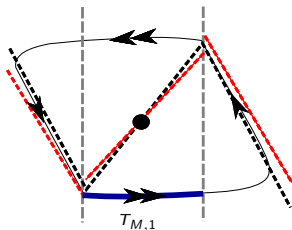
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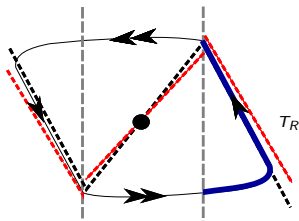
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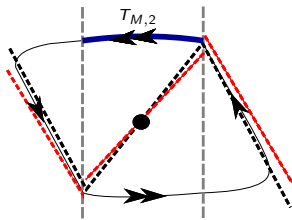
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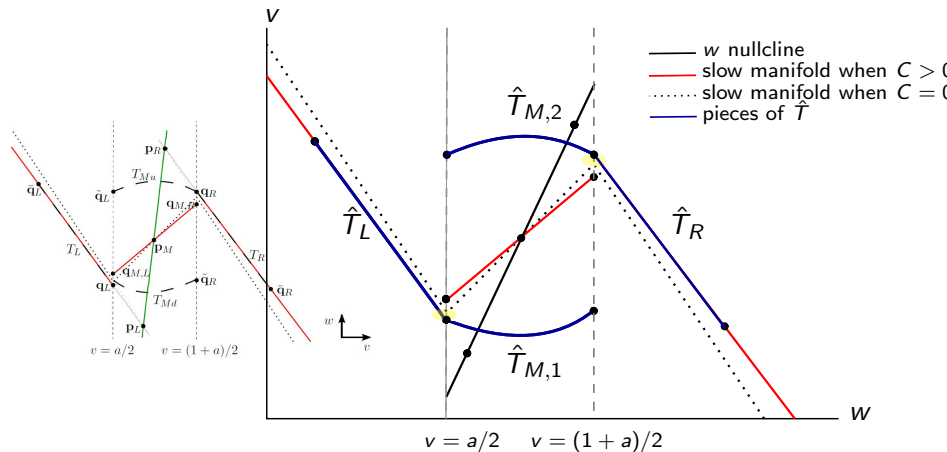
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# McKean w/ synaptic input: period of limit cycle

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Analytical approximation  $\hat{T}(I_{app}, g_{syn})$  of the period

[Fernández-García et al., SIADS 2015; G., Prohens, Teruel, Vich, SIADS 2017]

Analytical approximation of the period,  $\hat{T}(g_{syn})$ 

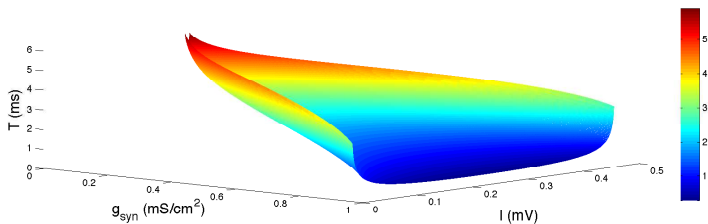
$$\hat{T} = \hat{T}_L + \hat{T}_{M,2} + \hat{T}_R + \hat{T}_{M,1}. \text{ That is,}$$

$$\hat{T}(g_{syn}) = \sum_{i=1}^2 \left[ \frac{1}{\lambda_{s,L}} \ln \left( \left| \frac{\gamma(I_{app} - I_i)}{\gamma(I_{app} - I_i) - K_L} \right| \right) + \frac{1}{\lambda_{q,M}} \ln \left( \left| \frac{\gamma(I_{app} - I_i) + K_M}{\gamma(I_{app} - I_i) + K_{M,i}} \right| \right) \right]$$

- ▶  $\lambda_{s,L}$  and  $\lambda_{q,M}$  are the slow lateral and the fast middle **eigenvalues**, respectively. They have a **nonlinear dependence on**  $g_{syn}$
- ▶  $K_L$ ,  $K_M$ ,  $K_{M,1}$  and  $K_{M,2}$  are given functions. They have a **nonlinear dependence on**  $g_{syn}$
- ▶  $I_1$  and  $I_2$  are the minimum and maximum values of external current. They **depend linearly on**  $g_{syn}$



# Use of the period expression



$\hat{T}$  is “highly” nonlinear but monotone with respect to  $g_{syn}$ .

Solving  $\hat{T}(I^*, g_{syn}) = T^*$ , given  $I^*$ , we estimate  $g_{syn}$ .

## Procedure to estimate $g_{syn}(t)$

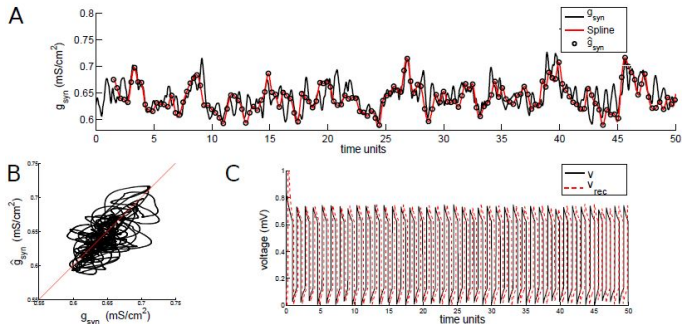
1. Record  $v(t)$  for an specific  $I^*$  such that the neuron oscillates.
2. Extract a sequence of times  $\{T^{(k)}\}_{k=1}^N$ , where each  $T^{(k)}$  corresponds to the time length of the  $k$ -th interspike interval.



3. Solve  $\hat{T}(I^*, g_{syn}^{(k)}) = T^{(k)}$ , for each  $T^{(k)}$ , to find  $g_{syn}^{(k)}$ .
4. Assign at each  $g_{syn}^{(k)}$  the time  $t^{(k)}$ ; the  $(k + 1)$ -th peak.
5. Interpolate  $(t^{(k)}, g_{syn}^{(k)})$  and obtain  $\hat{g}_{syn}(t)$ .

Refinement: use subperiods together with the partial approximations of  $\hat{T}_L$ ,  $\hat{T}_{M,2}$ ,  $\hat{T}_R$  and  $\hat{T}_{M,1}$ .

# Example: an estimation with realistic data



Actual and estimated synaptic input received by a cell from the computational model of the layer 4C $\alpha$  of the primary visual cortex by [McLaughlin et al. (2000), Tao et al. (2004)].

Voltage reconstruction.

High frequency synaptic input hinders estimations, but still, there is a good approximation in mean!

[G., Prohens, Teruel, Vich, SIADS 2017; Vich et al., Mathematics 2020]

# Estimation of synaptic inputs

Using sliding mode control

We consider the **Morris-Lecar model**:

$$\begin{cases} C_m \dot{v} &= -g_L (v - V_L) - g_K w (v - V_K) - g_{Ca} m_\infty(v) (v - V_{Ca}) + I_{app} - I_{syn}, \\ \dot{w} &= \phi \frac{w_\infty(v) - w}{\tau_w(v)}. \end{cases}$$

$$m_\infty(v) = \frac{1 + \tanh\left(\frac{v - V_1}{V_2}\right)}{2}, \quad w_\infty(v) = \frac{1 + \tanh\left(\frac{v - V_3}{V_4}\right)}{2}, \quad \tau_w(v) = \left( \cosh\left(\frac{v - V_3}{2V_4}\right) \right)^{-1}.$$

- ▶  $I_{app}$  : represents a **constant** externally applied current
- ▶  $I_{syn}(t) = g_E(t)(v - V_E) + g_I(t)(v - V_I)$  represents the **synaptic input**.

## A state observer for $w$ and synaptic input estimation

We consider a copy of the Morris-Lecar model with an extra input term instead of the synaptic input. Namely,

$$\begin{cases} C_m \dot{v} &= -g_L (v - V_L) - g_K \hat{w} (v - V_K) - g_{Ca} m_\infty(v) (v - V_{Ca}) + I_{app} + u, \\ \dot{\hat{w}} &= \phi \frac{w_\infty(v) - \hat{w}}{\tau_w(v)}. \end{cases}$$

Since  $v$  is the only measured output, it can be used in the  $(\hat{v}, \hat{w})$  dynamics. Let us define  $e_v = v - \hat{v}$  and  $e_w = w - \hat{w}$ . Combining the two systems:

$$\begin{aligned} C_m \frac{de_v}{dt} &= -g_K (v - V_K) e_w - I_{syn} - u, \\ \frac{de_w}{dt} &= -\frac{\phi}{\tau_w(v)} e_w. \end{aligned} \tag{4}$$

## The control action

The control action  $u = U \cdot \text{sign}(e_v)$  with

$$U \geq |-g_K (v - V_K) e_w - I_{syn}|, \quad \text{for all } t,$$

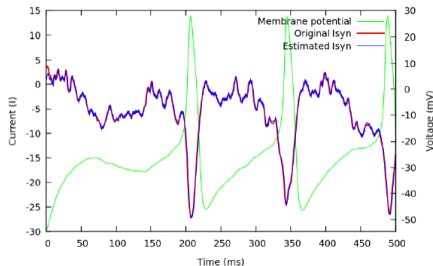
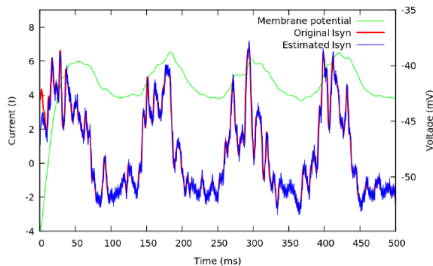
forces  $e_v \dot{e}_v < 0$  and  $e_v(t)$  to **converge to zero** in finite time, presumed that

- ▶ **Filtering** input  $u(t)$  provides an approximation of  $I_{app} - I_{syn}$  and so,  $I_{syn}$  ( $I_{app}$  is known).
- ▶ The error dynamics  $e_w$ , converges to zero since  $e_w = 0$  is the unique equilibrium point and  $e_w^2$  is a **Lyapunov function**

$$\frac{de_w^2}{dt} = -\frac{\phi}{\tau_w(v)} e_w^2 \leq -\phi e_w^2.$$

- ▶ It is an **off-line** procedure.

# Estimation of synaptic inputs using sliding mode control; examples



Actual and estimated synaptic input for  $I_{app} = 30$  (left) and  $I_{app} = 45$  (right).

[L. Tresserras, TFM 2020; Fossas, G., Tresserras, in progress]

### 3. Empiric models: Beyond DS



# Empiric models

Paradigm:

What to do when we do not have a model? Mainly, because of lack of knowledge about the biophysical structure of the *source* (neuron, population, . . . ).

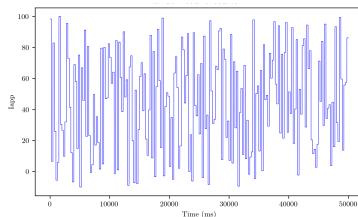
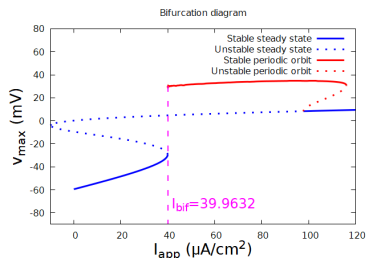
- ▶ We can do a **statistical model fitting**.
- ▶ **Machine learning** and other methods from AI can be helpful.

# Black-box models: a proof-of-concept

Consider again the **Morris-Lecar model**:

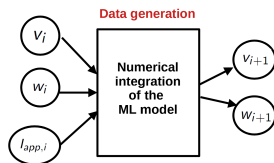
$$\begin{cases} C_m \dot{v} &= -g_L (v - V_L) - g_K w (v - V_K) - g_{Ca} m_\infty(v) (v - V_{Ca}) + I_{app}(t), \\ \dot{w} &= \phi \frac{w_\infty(v) - w}{\tau_w(v)}. \end{cases}$$

- ▶ We integrate the system with external input  $I_{app}(t)$ . The **design** of  $I_{app}(t)$  is important: we want it to *visit all the states* of the system.



## Black-box models: a proof-of-concept

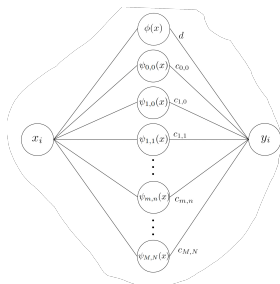
- ▶ Let  $x_i := \{(v_i, w_i, l_{app,i})\}_{i=0}^{M-1}$  the set of **initial conditions** and  $y_i := \{(v_{i+1}, w_{i+1})\}_{i=0}^{M-1}$  the set of corresponding **outputs**. In the sequel, we ignore the model and imagine the paired vectors  $x$  and  $y$  as **raw data** from an experiment.
- ▶ Next, we use some method of **machine learning** (artificial neural networks, genetic algorithms, ...) to learn the input-output relationship.
- ▶ We use a less sophisticated method: a **regression** method that does not require training: we construct a **wavenets**, a network based on Mallat's multiresolution analysis.



## Black-box models: a proof-of-concept

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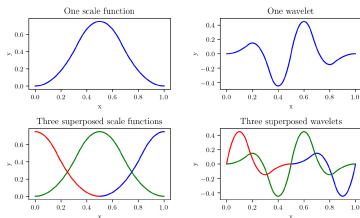
- ▶ We use a less sophisticated method: a **regression** method that does not require training: we construct a **wavenets**, a network based on Mallat's multiresolution analysis.

# Black-box models. The wavenet

- ▶ The goal is to express **approximate**  $y = \hat{f}(x)$ , with

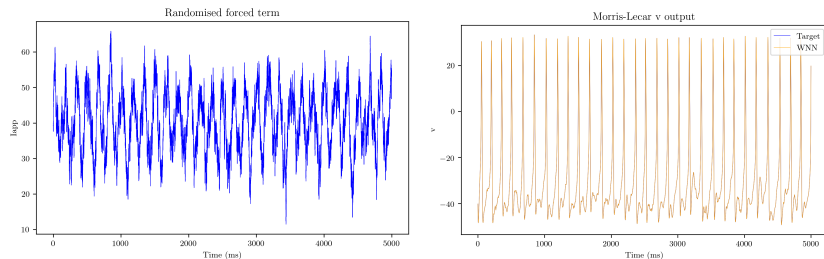
$$\hat{f}(x) = \sum_{k=1}^K d_k \phi_k(x) + \sum_{r=0}^R \sum_{s=0}^S c_{m,n} \psi_{s,r}(x),$$

where  $\phi_0$  is the so-called **scale function** and the  $\psi_{s,r}$  are the resulting **wavelets** up to some **resolution level**  $R$  (see Mallat's theory).



- ▶ We have introduced modifications (not shown here) to gain computational efficiency (python packages), reduce the resolution levels (basis expanded to function frames),...

# Black-box models. Results



Prediction versus actual voltage using a stochastic  $I_{app}$  input (different from the *training* (step-wise)  $I_{app}(t)$ ).

## Black-box models. Last comments

- ▶ Excellent approximation but ... it is **too ad hoc**, we **lose** the possibility of analysis (mechanisms, control,...), are there **experimental (timing) constraints**?
- ▶ Is there **any biophysical information encoded** in the network? In general, when approximating input/output relationships in a dynamical system, do the **coefficients contain information about the original parameters** of the system?
- ▶ Preliminary explorations show some monotonic relationship between original parameters and groups of weights in the wavenet, but nothing conclusive. In fact, it seems a very naive question, we are *shooting in the dark*.
- ▶ Intermediate solution:

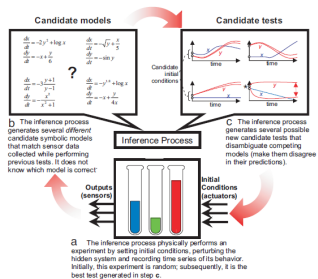
## Black-box models: last comments

- ▶ Design AI schemes to identify parameters:  $|\Upsilon|$ ,  $\{g_\iota\}_{\iota \in \Upsilon}$ ,  $\{V_\iota\}_{\iota \in \Upsilon}$ , parameters involved in  $W_\iota$  and  $\tau_\iota$ .

$$\begin{cases} C \frac{dv}{dt} = -g_L(v - V_L) - \sum_{\iota \in \Upsilon} g_\iota w_\iota^{p_\iota} (v - V_\iota) + I_{app}, \\ \frac{dw_\iota}{dt} = \frac{W_\iota(v) - w_\iota}{\tau_\iota(v)}, \quad |\Upsilon| \approx \# \text{ ionic channels.} \end{cases}$$

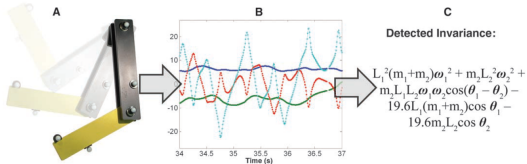


# Black-box models: last comments. Food for DS



[Bongard, Lipson, PNAS 2007]

▶ Go to control-induced oscillations



**Fig. 1.** Mining physical systems. We captured the angles and angular velocities of a chaotic double-pendulum (A) over time using motion tracking (B), then we automatically searched for equations that describe a single natural law relating

these variables. Without any prior knowledge about physics or geometry, the algorithm found the conservation law (C), which turns out to be the double pendulum's Hamiltonian. Actual pendulum, data, and results are shown.

[Schmidt, Lipson, Science 2009]

# Summary

1. Models of dynamical systems in neuroscience.
  - **Simplified models and DS tools** allow to inspect which **mechanisms** are involved in relevant neuroscience problems (bursting, mix-mode oscillations, mix-mode bursting oscillations, spike-adding. . . ) or explain facts like what is **necessary to rise complexity**.
  - DS tools allow to make **accurate predictions** (e.g., phase and response functions) and **exert control**.
2. Parameter estimation: At the edge of DS.
  - Underlying DS models are a solid ground also for **parameter estimation**.
3. Empiric models: Beyond DS.
  - AI methods give excellent approximations but we **lose** the possibility of analysis (mechanisms, control,...).
  - Is there **any biophysical information encoded** in the network? In general, when approximating input/output relationships in a dynamical system, do the **coefficients contain information about the original parameters** of the system?
  - Future work: **Design AI schemes to identify model parameters** within a family of systems. New food for DS!

## Collaborators

- ▶ **Estimation of parameters using control and empiric models:** David Aquilué (Master student, Univ. Paris-Saclay), Pau Fisco (Master student, UPC), Enric Fossas (IOC, UPC), Néstor Roqueiro (UFSC, Brazil), Lluç Tresserras (UPC).
- ▶ **Phase and amplitude response functions:** Oriol Castejón (former PhD student), Gemma Huguet (Dept. Mathematics, UPC), Kevin Martínez-Añón (PhD student, CRM), Román Moreno (PhD student, Dept. Mathematics, UPC).
- ▶ **Estimation of synaptic inputs:** Rafel Prohens (UIB), Antonio Teruel (UIB), Cati Vich (UIB).
- ▶ **Slow-fast dynamics:** Mathieu Desroches (INRIA), Elif Köksal-Ersöz (INRIA), John Rinzel (NYU), Joël Tabak (Exeter Univ.); M.D. (INRIA), Enrique Ponce (US), R.P. (UIB), Serafim Rodrigues (BCAM), A.T. (UIB).
- ▶ **Separatrix map, bistable perception:** Amadeu Delshams (Dept. Mathematics, UPC), Gemma Huguet (Dept. Mathematics, UPC).
- ▶ **Parameter disambiguation** (not in this talk): Susanne Ditlevsen (U. Copenhagen), Horacio Rotstein (NJIT).
- ▶ **Estimation of network parameters (genetic algorithms)**(not in this talk): David Aquilué (TFG student, UPC), Antonio J. Pons (UPC).
- ▶ **Topological data analysis** (not in this talk): Nils Gutiérrez (Master student, UPC), Esther Ibáñez-Marcelo (ISI-Torino).

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Thanks for your attention