Mixing conditions for 2-D SFTs

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  - A $\mathbb{Z}^2$ SFT X is **topologically mixing** if any GA rectangular patterns w, w' can coexist in the same element of X in any orientation where they are far enough apart.

Examples

- The **domino tiling** $\mathbb{Z}^2$ SFT D is the set of all ways of tiling the plane with dominoes.

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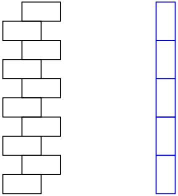

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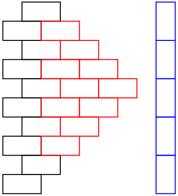

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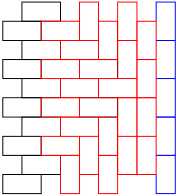

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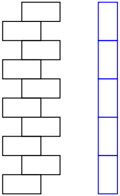

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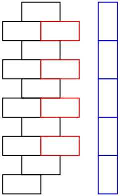

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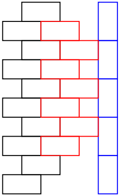

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$$\boxed{\begin{array}{ccc} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{array}} \qquad \boxed{\begin{array}{cccc} 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{array}}$$

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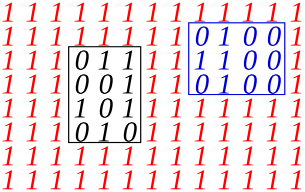

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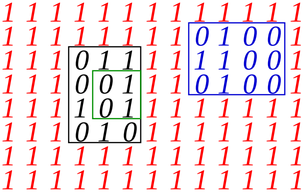

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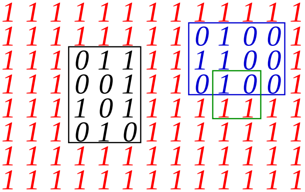

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  - If there is one uniform distance sufficient for coexistence for all GA w, w' with any shapes (not just rectangles), then X is **strongly irreducible**.

Examples

- The $\mathbb{Z}^2$ **"golden mean shift"** $G^{(2)}$ is the $\mathbb{Z}^2$ SFT with $A = \{0, 1\}$ and $\mathcal{F} = \{11, \begin{smallmatrix}1\\1\end{smallmatrix}\}$.

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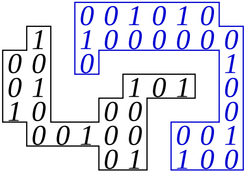

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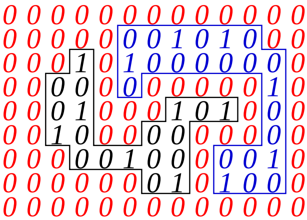

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- Can force existence of dense periodic points, decidability, computability of entropy