

Resonances : from stability to chaos ...

Anne LEMAITRE

naXys - University of Namur

- Full professor in the department of Mathematics in the University of Namur
- Linear algebra, mechanics (point, fluid, celestial), dynamical systems, applied maths
- Research institute naXys (Namur Complex Systems)
- Presently Dean of the Faculty of Sciences
- Scientific and academic : celestial mechanics
- Team : created by J. Henrard in 1970
- Maximum 3 permanent positions up to now
- Up to 15-20 researchers (post-docs + PhD)
- SPACE in naXys : Cosmology - Exoplanets - Solar System

Introduction and survey in our solar system and outside

- 1 Mean Motion Resonances (MMR)
- 2 Other types of resonances (planets and exoplanets)
- 3 Adiabatic invariant
- 4 Dissipations and non gravitational forces
- 5 Spin-orbit resonances : Mercury (BepiColombo)
- 6 Gravitational resonances : space debris
- 7 Chaos indicators
- 8 Applications and examples
- 9 Symplectic integrators

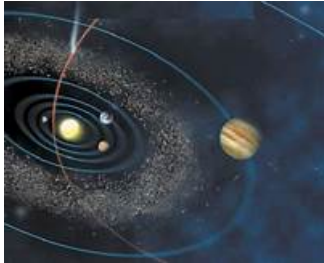
- 9 hours
- Courses and conferences : mixed
- Latex beamer - Powerpoint - Keynote
- French and english : mixed
- Very old and very new results : mixed
- Sorry ..

Resonances in the Solar System

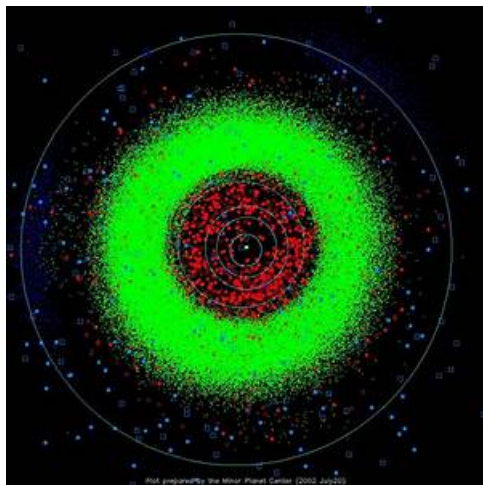


- 8 planets : 4 + 4
- Mainly an elliptic orbit around the Sun
- Mutual gravitational influences

Asteroids



Asteroid Belt



Asteroids population

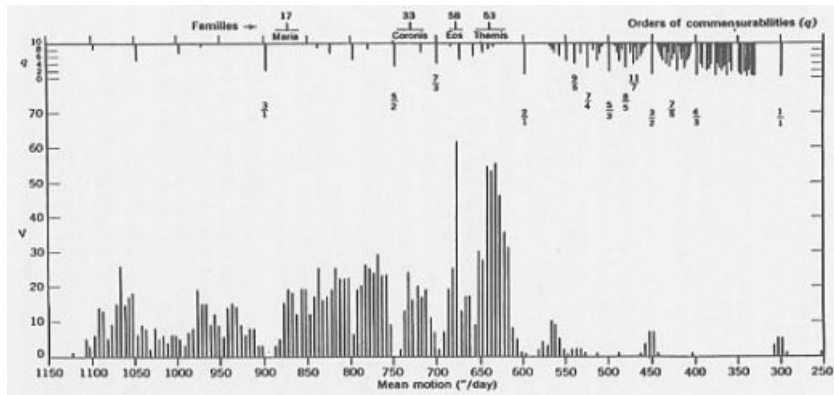
- Minor Planet Center : catalogues
- ASTDYS and NEODYS (Pisa + USA + Valladolid)
- Non numbered asteroids
- Systematic Surveys : magnitude 22
- 2018 : 99 % of asteroids of 1 km
- Masses, spin velocities, densities
- Albedos, thermal conductivity, surface temperatures
- Irregular shapes - binaries
- Effect on planetary ephemerides : 300 included

Asteroids : Kirkwood gaps 1866

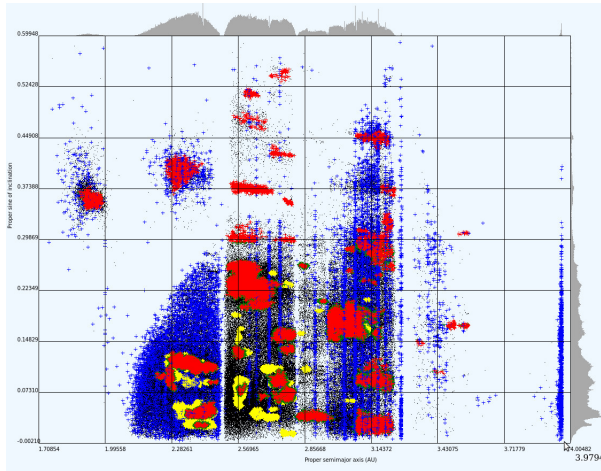
Kirkwood, Daniel (1814–1895)



Asteroids : Kirkwood gaps 1866

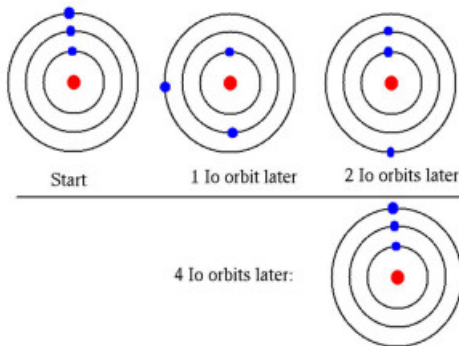


Asteroids : Kirkwood gaps 2017



MMR resonances destabilize and stabilize the objects

Galilean satellites : MMR 1-2-4



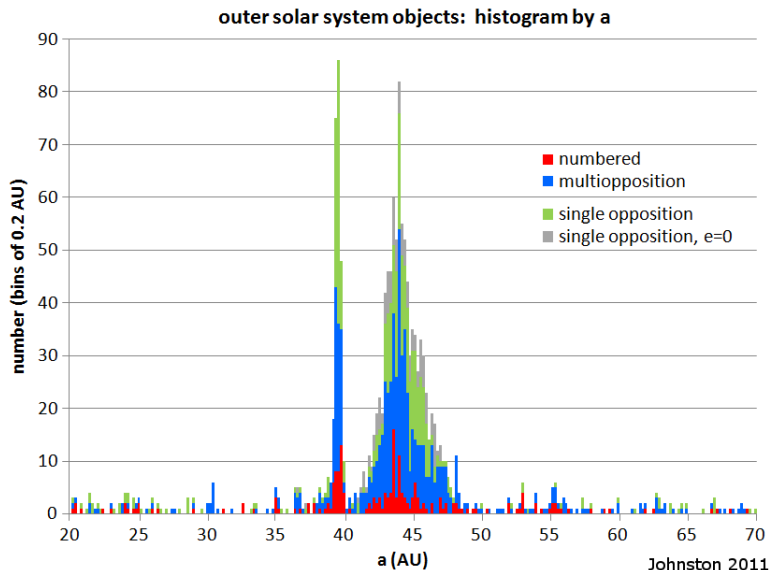
MMR resonances stabilize the objects

MMR : Kuiper belt



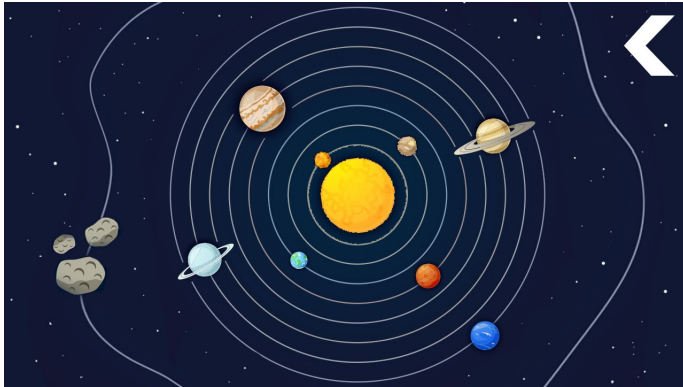
MMR resonances stabilize the objects

MMR : Kuiper belt

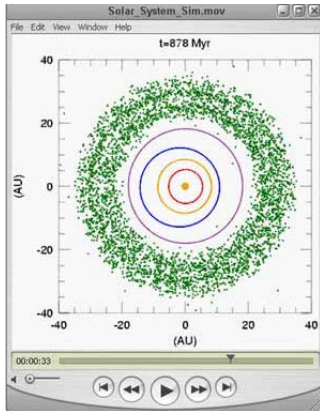


Johnston 2011

MMR : Kuiper belt



MMR : Nice Model



Spin-Orbit resonances : Moon

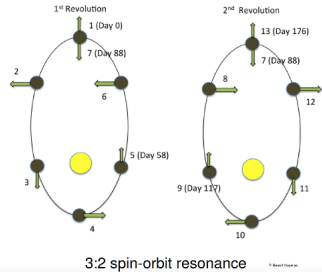


Always the same side facing the Earth
The body has a shape and a spin

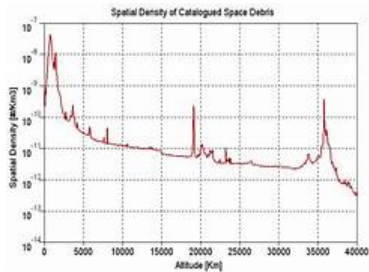
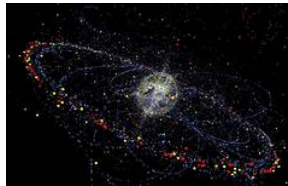
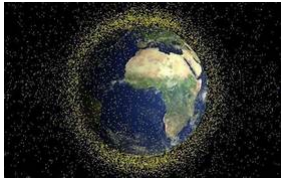
Spin-Orbit resonances : natural satellites



Spin-Orbit resonances : Mercury



Gravitational resonances : space debris



- Mean motion resonances : asteroid belt, Galilean satellites, Kuiper belt
The planets are point masses, orbit the same primary, have commensurable periods
If the planets have no fixed orbits, apparition of secular and secondary resonances
- Spin orbit resonances : Moon, natural satellites, Mercury
The body has a spin, its orbital and rotational periods are commensurable
- Gravitational resonances : GEO, MEO orbits
The primary has a spin, the orbital period of the orbiter is commensurable with the spin of the primary

Mean Motion resonances

- **Hamiltonian formalism**
- **Averaging**
- **Classical variables**
- **Mathematical approach**
- **Different Models and orders**
- **Simplified perturber**

Systemes hamiltoniens

- Hamiltonien : $H = H(q,p)$
- Transformation canonique : $(q,p) \rightarrow (Q,P)$
- Fonction génératrice S : $S = S(q,P)$
- Nouvel hamiltonien

$$\begin{aligned} p &= \frac{\partial S}{\partial q} \\ Q &= \frac{\partial S}{\partial P} \end{aligned}$$

1

$$\begin{aligned} K &= K(Q,P) \\ &= H(q(Q,P),p(Q,P)) \end{aligned}$$

$$\begin{aligned} \frac{dq}{dt} &= \frac{\partial H}{\partial p} & \frac{dQ}{dt} &= \frac{\partial K}{\partial P} \\ \frac{dp}{dt} &= -\frac{\partial H}{\partial q} & \frac{dP}{dt} &= -\frac{\partial K}{\partial Q} \end{aligned}$$

Averaging principles

Very efficient tool

Different angles : fast and slow

Long-term dynamics

By a canonical transform

New averaged canonical variables

Averaging process performed
step by step, up to some order N

$$\mathcal{H}(\phi, I) = \mathcal{H}_0(I) + \epsilon \mathcal{P}(\phi, I, \epsilon)$$

$$\phi = (\phi_1, \dots, \phi_n)$$

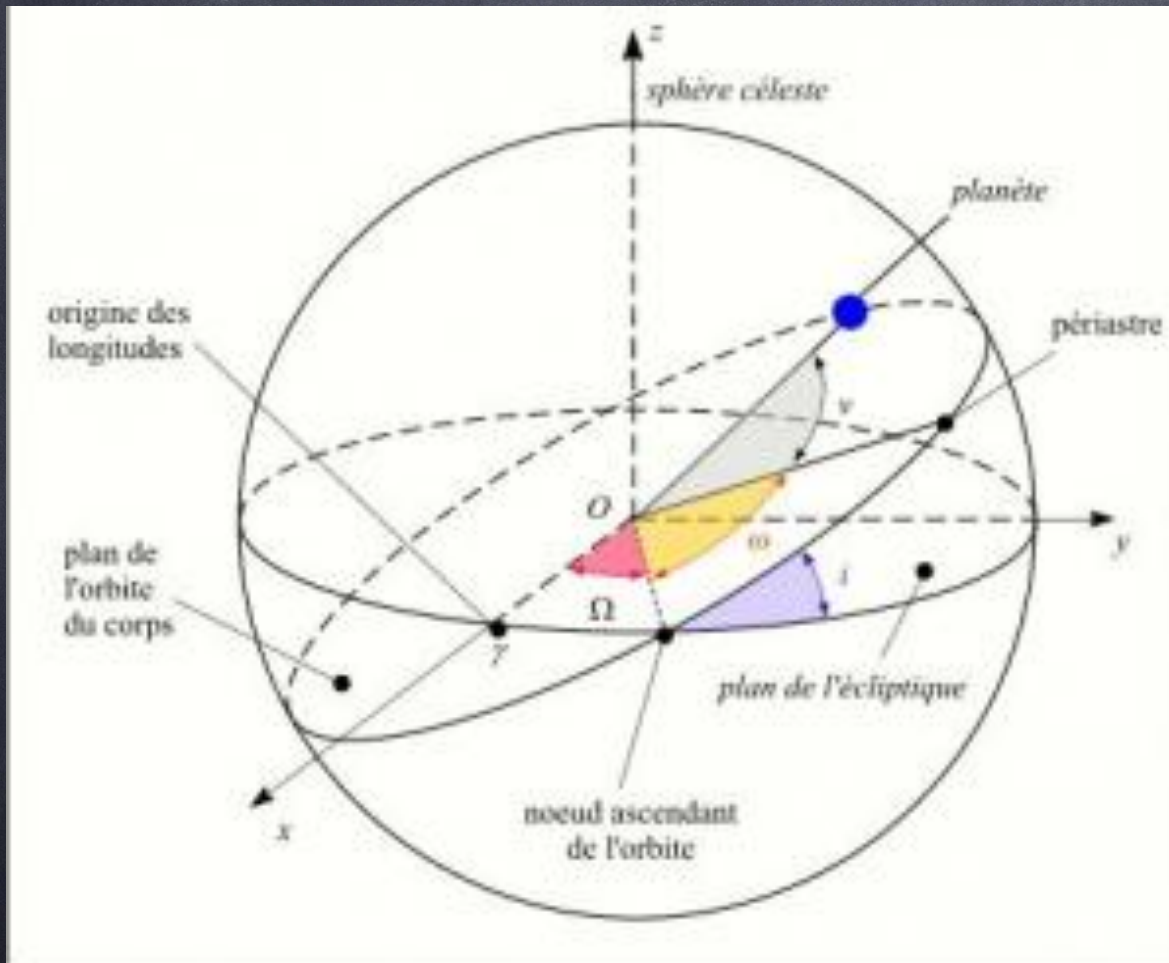
$$I = (I_1, \dots, I_n)$$

$$\mathcal{K}(\psi, J) = \mathcal{H}_0(J) + \epsilon \mathcal{P}'(-, J, \epsilon)$$

$$\frac{dJ_i}{dt} = -\frac{\partial \mathcal{K}}{\partial \psi_i} = 0$$

$$\frac{d\psi_i}{dt} = \frac{\partial \mathcal{K}}{\partial J_i} = \omega_i(J)$$

6 elliptic elements : a , e , i , M , ω , Ω



**5 Constant
and 1 fonction of time
in case of a fixed ellipsis**

Elements de delaunay

6 elliptic elements : $a, e, i, M, \omega, \Omega$

Non canonical set

Canonical variables
and momenta

M = Mean anomaly

ω = argument of pericentre

Ω = longitude of ascending node

L = $\sqrt{\mu a}$

G = $\sqrt{\mu a (1 - e^2)}$

H = $\sqrt{\mu a (1 - e^2)} \cos i$

Non singular Delaunay

Problems when $i = 0$ (node ?) and $e = 0$ (pericentre ?)

$$\lambda = M + \omega + \Omega$$

$$p = -\omega - \Omega$$

$$q = -\Omega$$

$$L = \sqrt{\mu a}$$

$$S = L - G = L (1 - \sqrt{1 - e^2}) \simeq L \frac{e^2}{2}$$

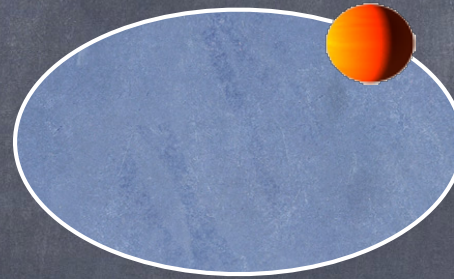
$$T = G - H = G (1 - \cos i) \simeq G \frac{i^2}{2}$$

Problème des deux corps

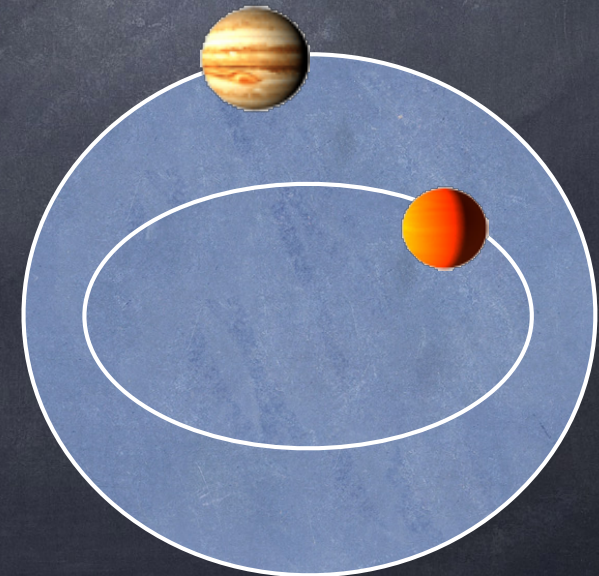
$$\begin{aligned}\mathcal{H} &= H(a, e, i, M, \omega, \Omega) \\ &= H(L, G, H, M, \omega, \Omega) \\ &= H(L, -, -, -, -, -) \\ &= -\frac{\mu^2}{2L^2}\end{aligned}$$

$$\dot{M} = \frac{\mu^2}{L^3} = n$$

Fixed orbit $\rightarrow$ Time dependent orbit
Osculating elements



+ Perturbation 3ième corps
orbite circulaire plane fixe



$$\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{\sqrt{r^2 + r'^2 - 2rr' \cos \gamma}} = \sum_{\ell=0}^{\infty} \frac{r'^{\ell}}{r^{\ell+1}} P_{\ell}(\cos \gamma), \text{ avec } r > r'$$

Troisième corps



$$V = -m_J \frac{G}{a_J} \sum_{l,m} e^l \sum_k P_{l,m}^k \left(\frac{a}{a_J} \right) \cos(k\lambda_J - (k-m)\lambda - m\omega)$$

$$\frac{n}{n_J} = \frac{j+i}{j}$$

$$H_0 = -\frac{\mu^2}{2L^2} + n_J \Lambda_J$$

i = ordre de la résonance

2/1 : j = 1 et i = 1

3/2 : j = 2 et i = 1

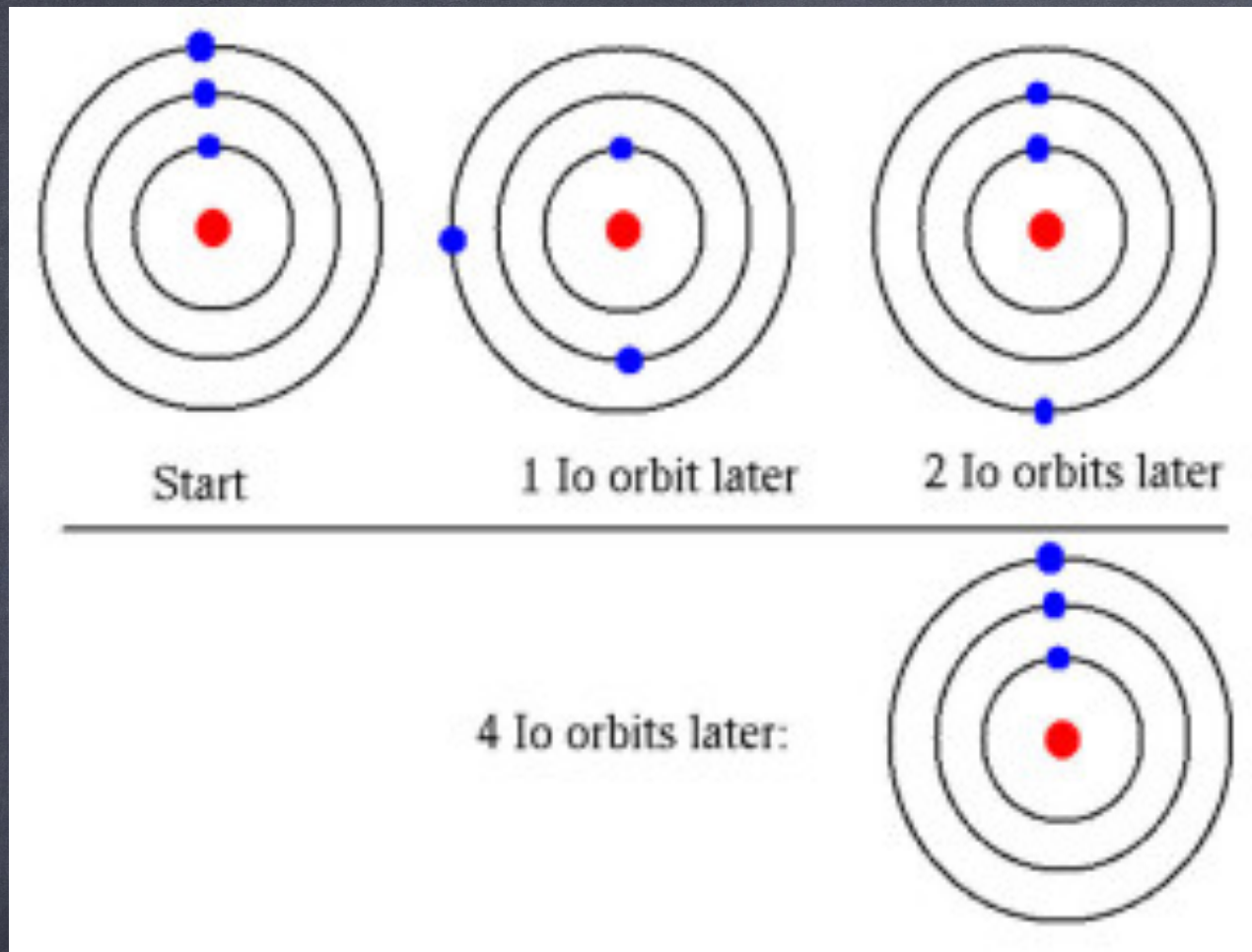
3/1 : j = 1 et i = 2

5/2 : j = 2 et i = 3

7/3 : j = 3 et i = 4

$$\dot{\sigma} = \frac{j+i}{i} n_J - \frac{j}{i} n$$

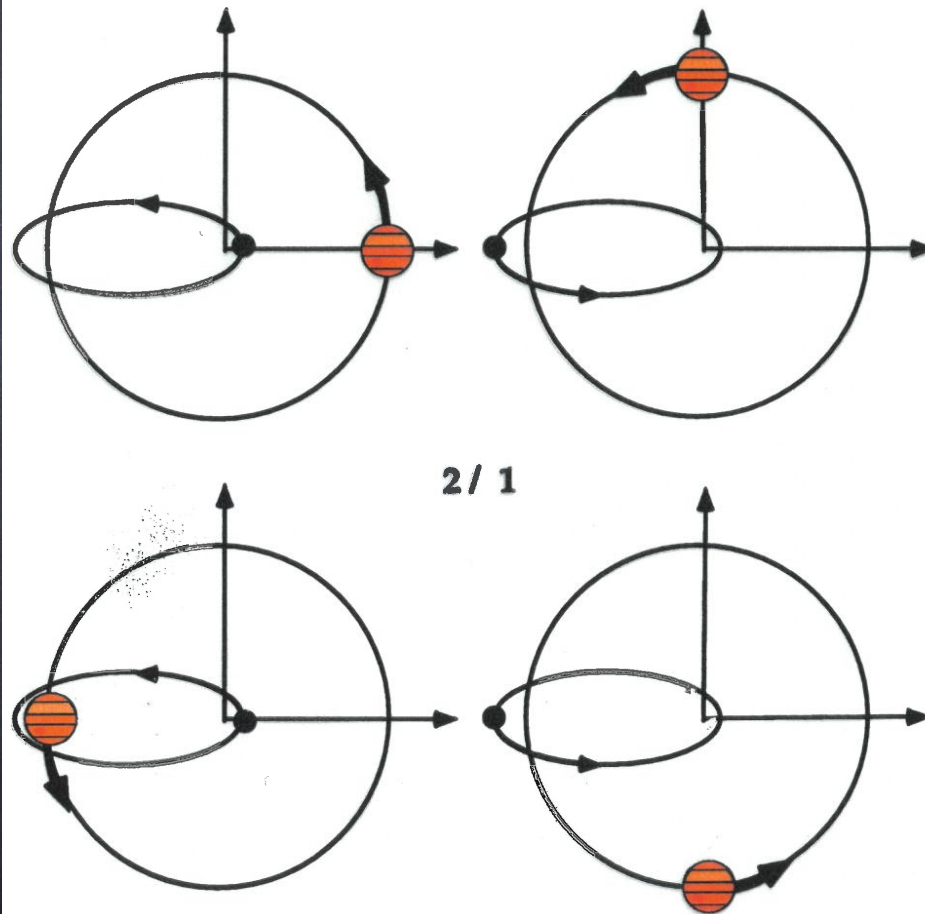
$$\sigma = \frac{j+i}{i} \lambda_J - \frac{j}{i} \lambda - \omega$$



**Case of the Laplace resonance
almost circular orbits : no crossings !**

Mean Motion Resonance :

a life-insurance



2 / 1

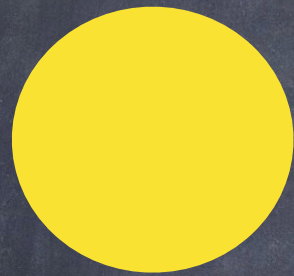
Example of 2:1

Eccentric orbits
Crossing of the other orbit ?

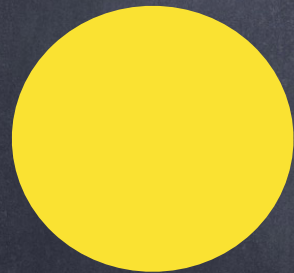
Resonance acting
as a protection mechanism

No close approach

résonance interne ou externe



Internal



external

Moyennisation

$$k^* = m \frac{j+i}{i}$$

$$\begin{aligned} V &= -m_J \frac{G}{a_J} \sum_{l,m} e^l P_{l,m}^{k^*} \left(\frac{a}{a_J} \right) \cos(k^* \lambda_J - (k^* - m)\lambda - m\omega) \\ &= -m_J \frac{G}{a_J} \sum_{l,m} e^l P_{l,m}^{k^*} \left(\frac{a}{a_J} \right) \cos m \left(\frac{j+i}{i} \lambda_J - \frac{j}{i} \lambda - \omega \right) \end{aligned}$$

+ autres termes de
courte période

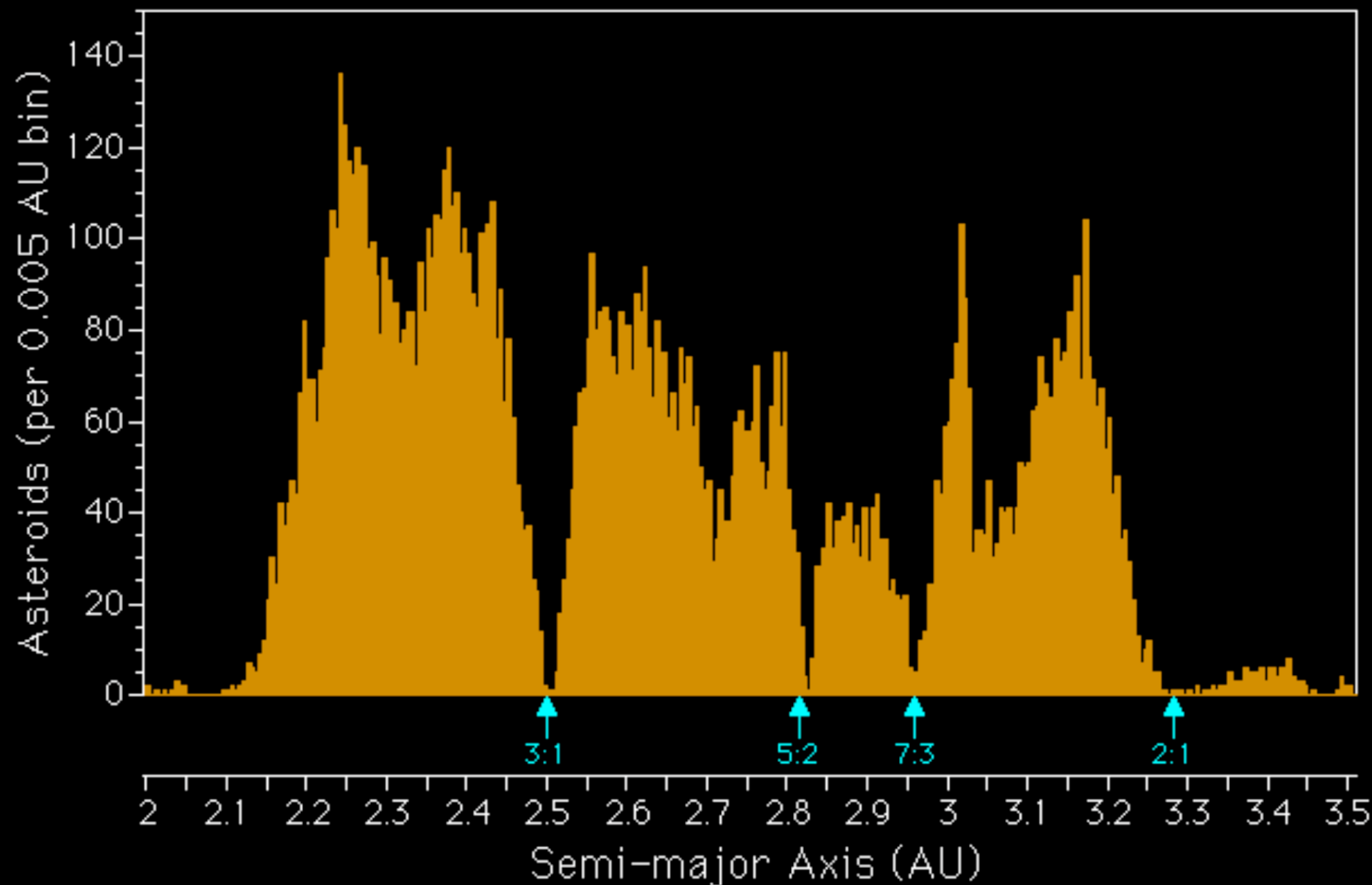
$$\begin{aligned} S &= L - G \\ &= \sqrt{\mu a} (1 - \sqrt{1 - e^2}) \\ &= \sqrt{\mu a} \frac{e^2}{2} \end{aligned}$$

$$\sigma = \frac{j+i}{i} \lambda_J - \frac{j}{i} \lambda - \omega$$

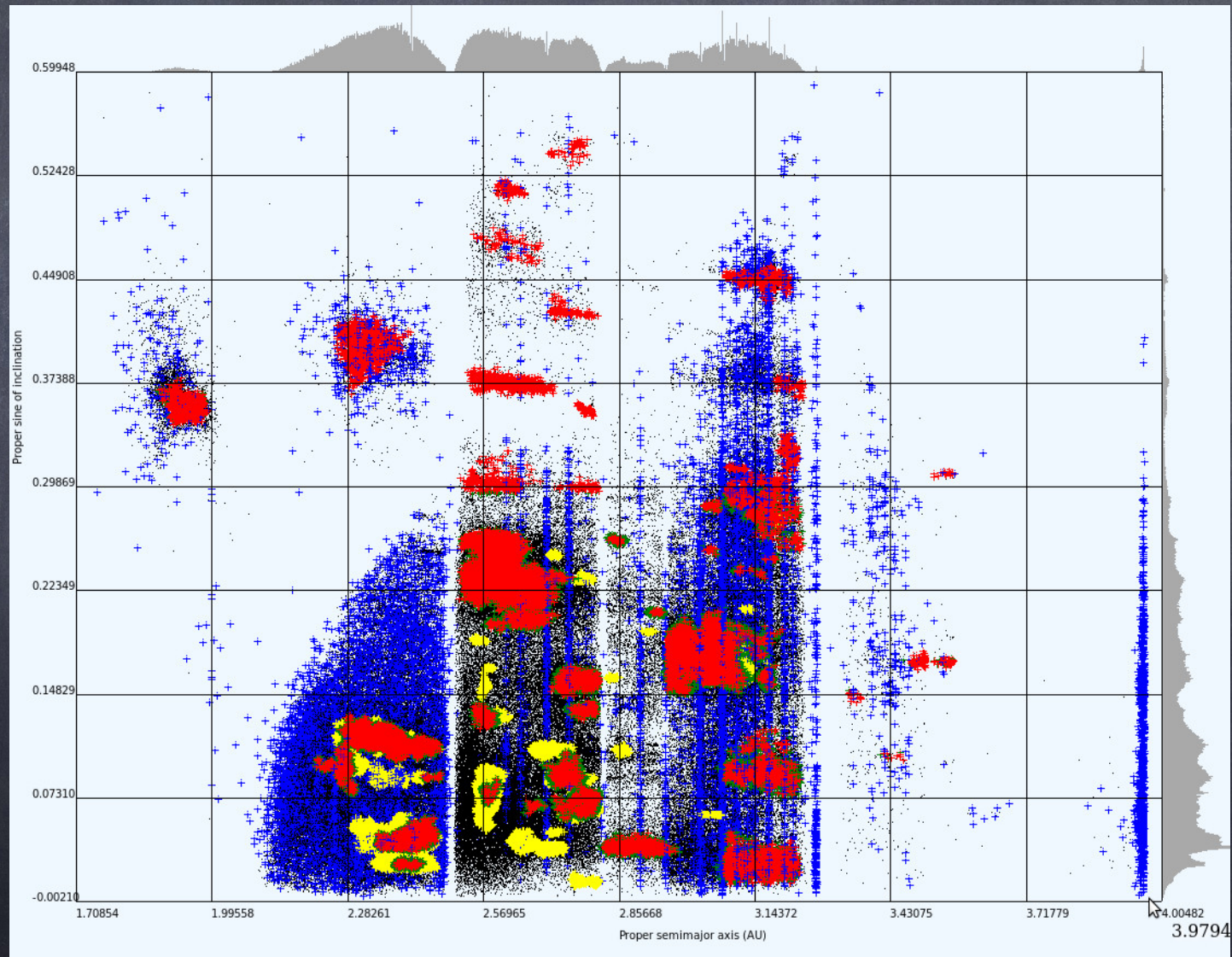
angle de résonance : angle lent

Main Asteroid Belt Distribution

Kirkwood Gaps



Kirkwood maintenanant



$$H_0 = -\frac{\mu^2}{2L^2} + n_J \Lambda_J$$

$$V = -m_J \frac{G}{a_J} \sum_{l,m} e^l P_{l,m}^* \cos m\sigma$$

$$\begin{array}{cc} M & L \\ \omega & G \\ \lambda_J & \Lambda_J \end{array}$$

$$\begin{array}{cc} \lambda & L \\ p = -\varpi & S = L - G \\ \lambda_J & \Lambda_J \end{array}$$

Moyenne sur les courtes
périodes : $N = \text{cste}$

Excentricité petite
(S petit)

Développement en série
et troncature rapide

$$\begin{array}{cc} \lambda & N = L + \frac{j}{i} \\ \sigma = \frac{j+i}{i} \lambda_J - \frac{j}{i} \lambda + p & S = L - G \\ \lambda_J & \Lambda'_J = \Lambda_J - \frac{j+i}{i} S \end{array}$$

$$H_0 = -\frac{\mu^2}{2(N - \frac{j}{i}S)^2} + n_J \frac{j+i}{i} S + \dots$$

Modèles de résonances

$$\mathcal{K} = \beta S^2 + \alpha S + \epsilon \cos \sigma \quad \text{Pendule}$$

$$\mathcal{K} = \beta S^2 + \alpha S + \epsilon \sqrt{2S} \cos \sigma \quad \text{SFMR : } i=1$$

$$\mathcal{K} = \beta S^2 + \alpha S + \epsilon 2S \cos 2\sigma \quad i=2$$

$$\mathcal{K} = \beta S^2 + \alpha S + \epsilon (\sqrt{2S})^3 \cos 3\sigma \quad i=3$$

$$\mathcal{K} = \beta S^2 + \alpha S + \epsilon \sqrt{2S} \cos \sigma + \eta 2S \cos 2\sigma$$

SFMRAS : $i=1$

$$\alpha = \alpha(N) = \alpha(a, e)$$

$$\beta = \beta(N) = \beta(a, e)$$

$$\epsilon = \epsilon(N) = \epsilon(a, e)$$

$$\eta = \eta(N) = \eta(a, e)$$

Modèles mathématiques

$$R = a S$$

$$r = b \sigma + c$$

$$\tau = d t$$

$$K = s H$$

avec

$$b = \pm 1$$

$$c = 0 \text{ ou } \pi$$

$$d \geq 0$$

$$K = \frac{1}{2} R^2 + \delta \cos r$$

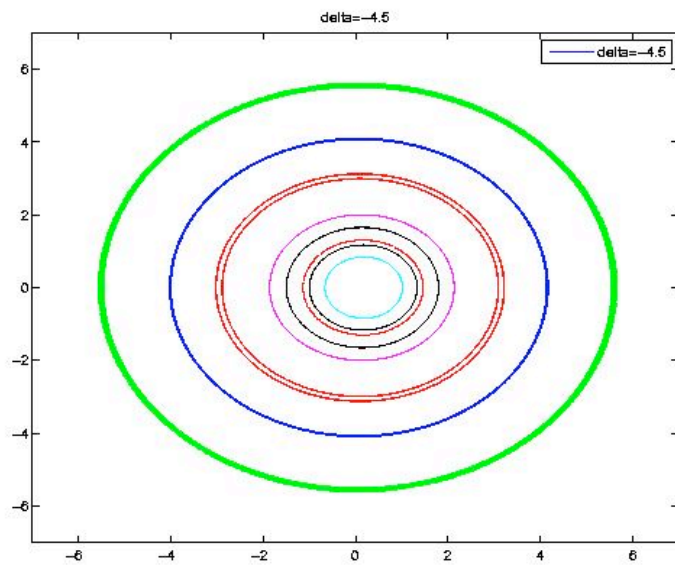
un seul paramètre

$$K = -3(\delta + 1) R + R^2 - 2\sqrt{2R} \cos r$$

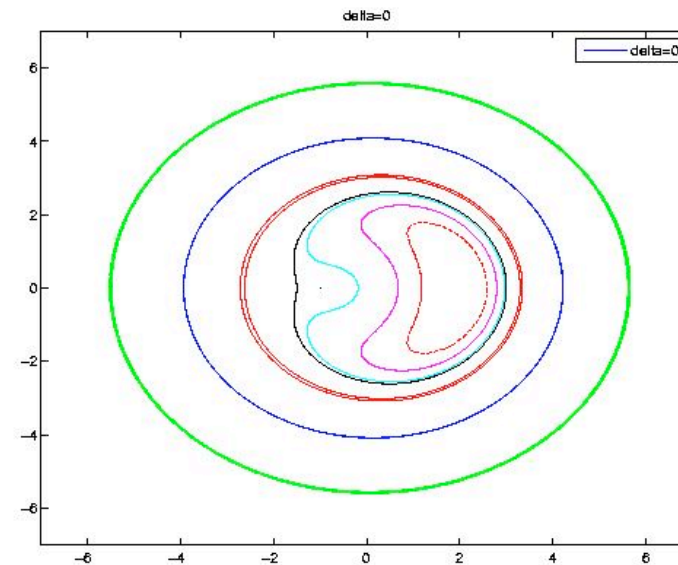
ou

$$K = -3(\delta + 1) R + R^2 - 2\sqrt{2R} \cos r + b 2R \cos 2R$$

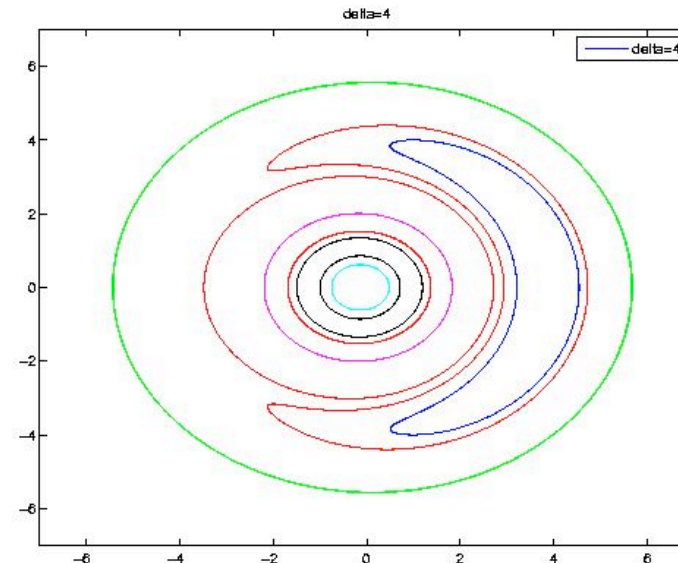
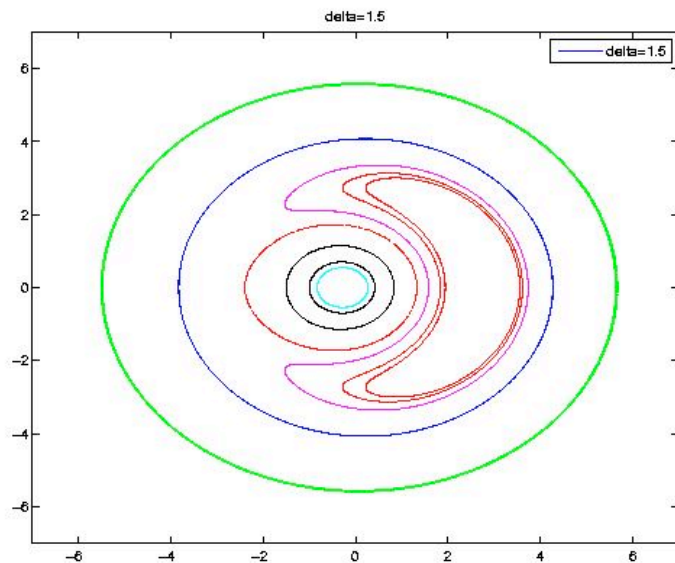
deux paramètres



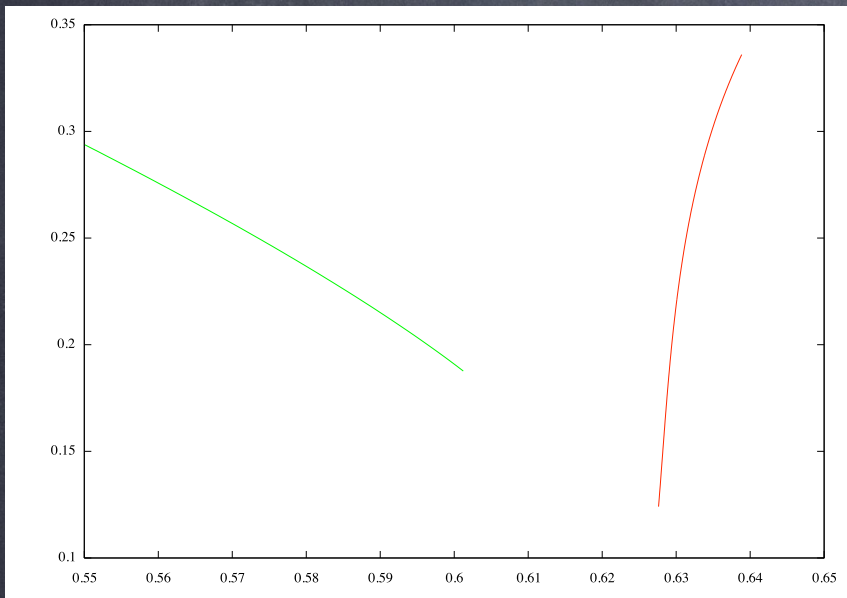
Variations périodiques
de l'excentricité



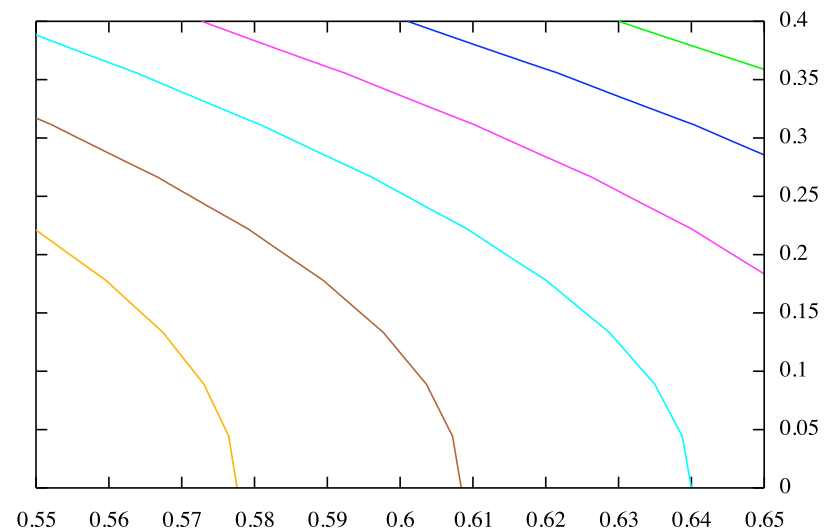
Larges excursions
en excentricité



Forme en V dans l'espace (a,e)



Courbes $N(a,e) = \text{cste}$



Resonances of order 2

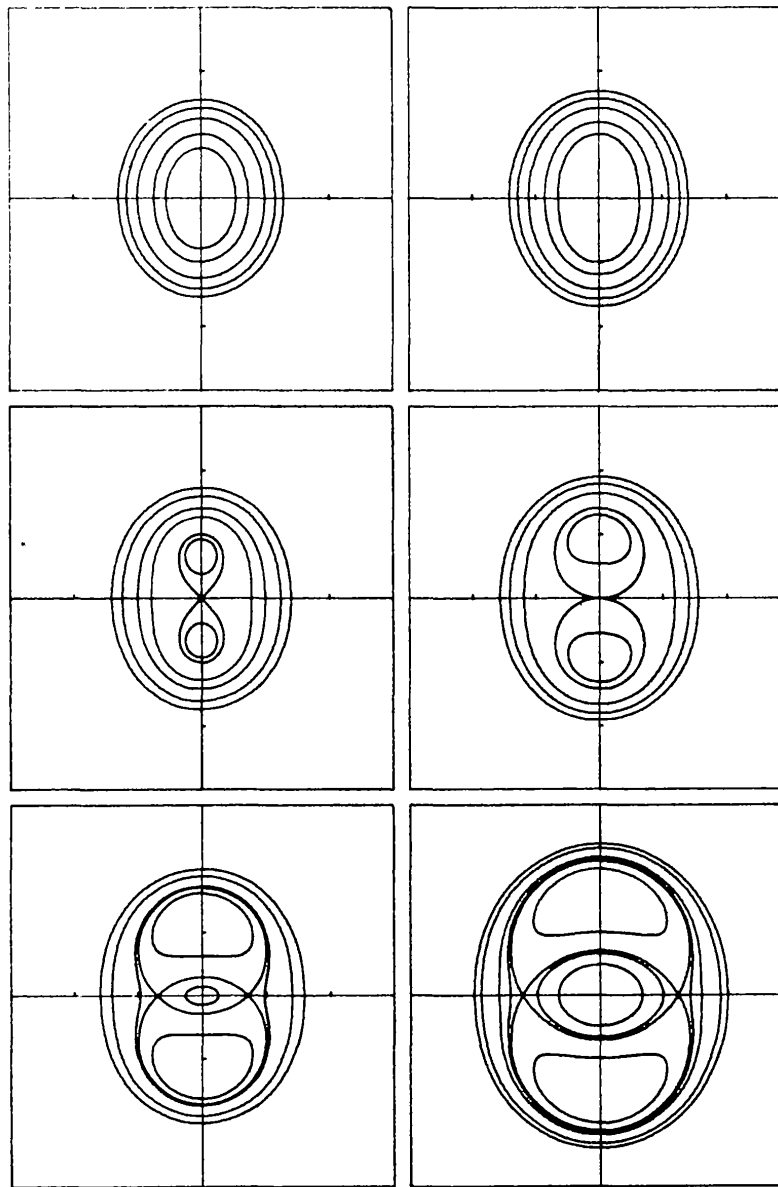


Fig. 1. The phase space for the $(j+2)/j$ case.

Resonances of order 3

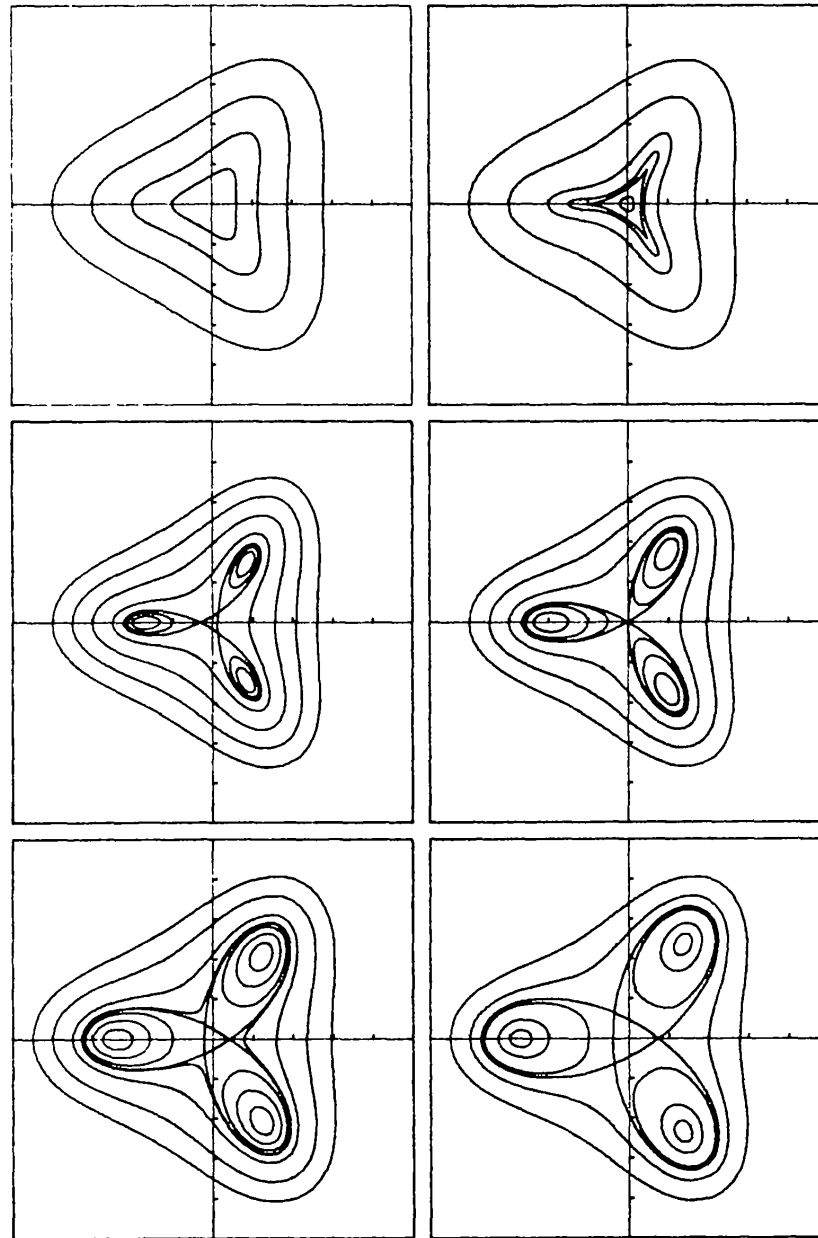


Fig. 4. The phase space for the $(j + 3)/j$ case.

Minor planet in MMR with a planet
Possible large excursions in eccentricity
Crossing of the orbit of another planet (Mars for example)

**Nothing to hope for small eccentricities
blocked in the resonance area**

2 mechanisms

Explanation of the depletion
presence of other resonances

Explanation of the flux of NEO
action of dissipations

More complex dynamics

Perturber : fixed circular orbit ?

More realistic orbit

Elliptic and plane : pericentre

Elliptic and inclined : pericentre and node

Perturbed orbit (by the other planets)

Analytical theory Ephemerides

Full numerical integration of the dynamics

Orbite **elliptique** plane 3ième corps

$$V = -m_J \frac{G}{a_J} \sum_{l,m} e^l \sum_t e_J^t P_{l,m}^{k*} \left(\frac{a}{a_J} \right) \cos(m\sigma + t\nu)$$

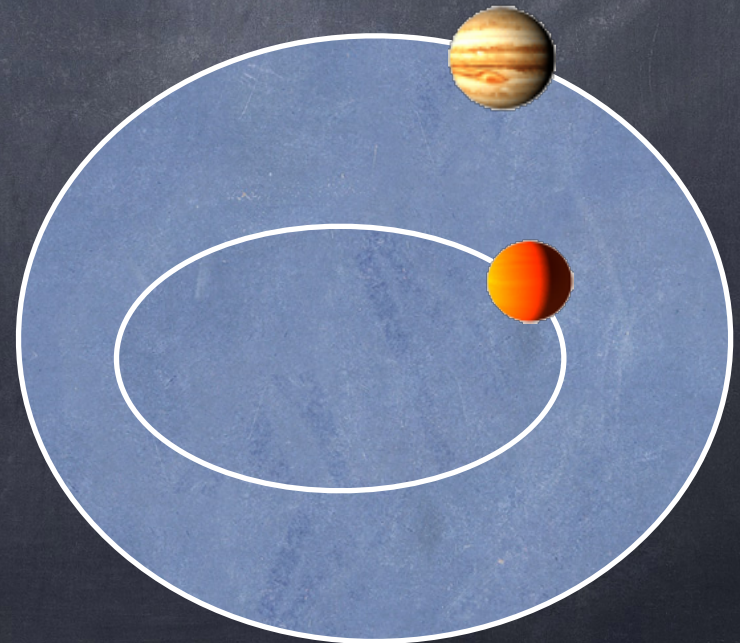
avec $\nu = -\frac{j+i}{i} \lambda_J + \frac{j}{i} \lambda - \omega_J$ **et** $N = \frac{i}{j} L + L - G$

σ conjugué à $S = O(e^2)$

ν conjugué à $N = N(a, e)$

$$\sigma = \frac{j+i}{i} \lambda_J - \frac{j}{i} \lambda - \omega$$

Résonances
secondaires
between the
two angles



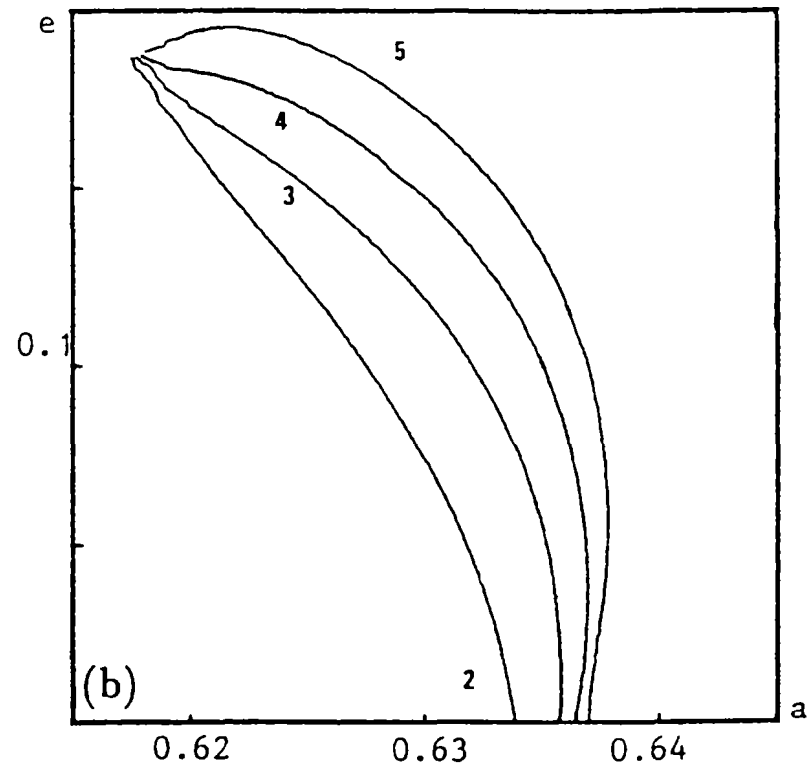
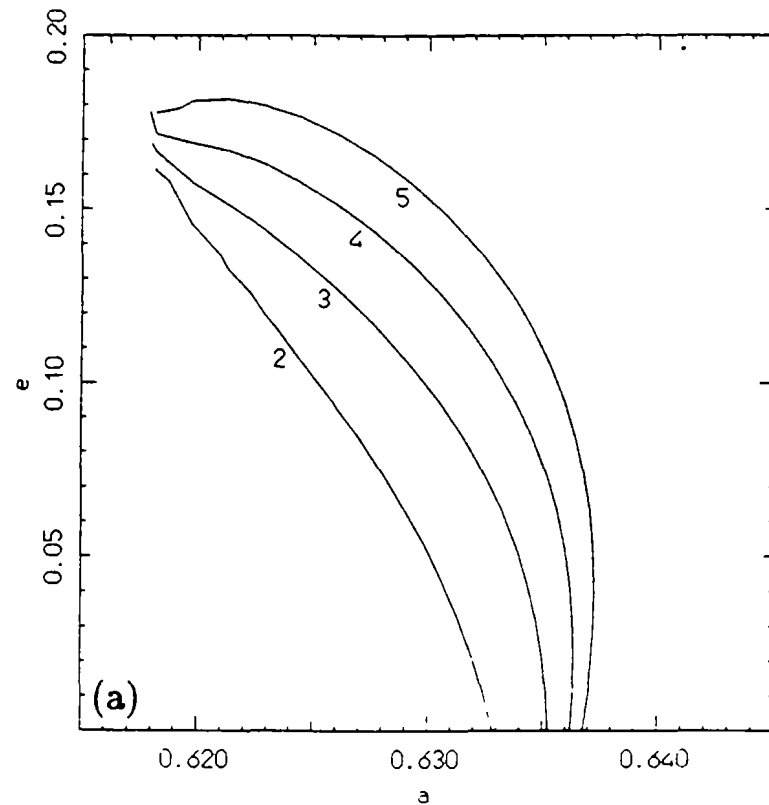


Fig. 1. (a) Secondary resonances $\omega_1/\omega_2 = 2, 3, 4, 5$ in our model. (b) The same in the model of Lemaître and Henrard (1990).

Résonance 2:1

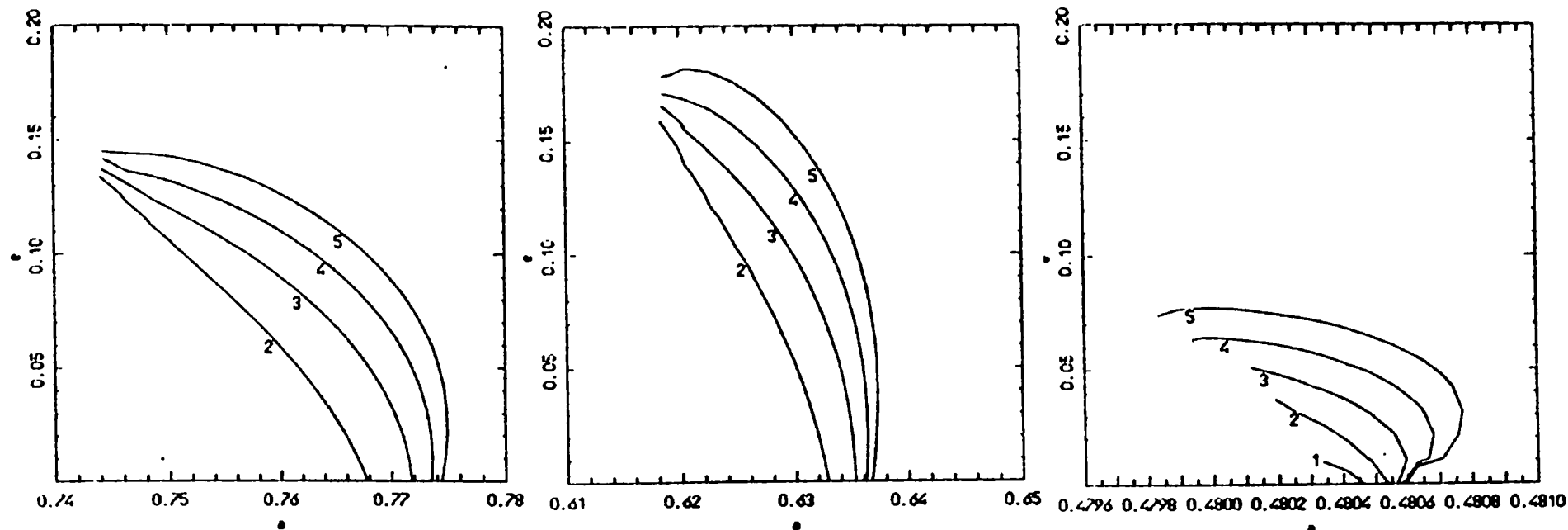
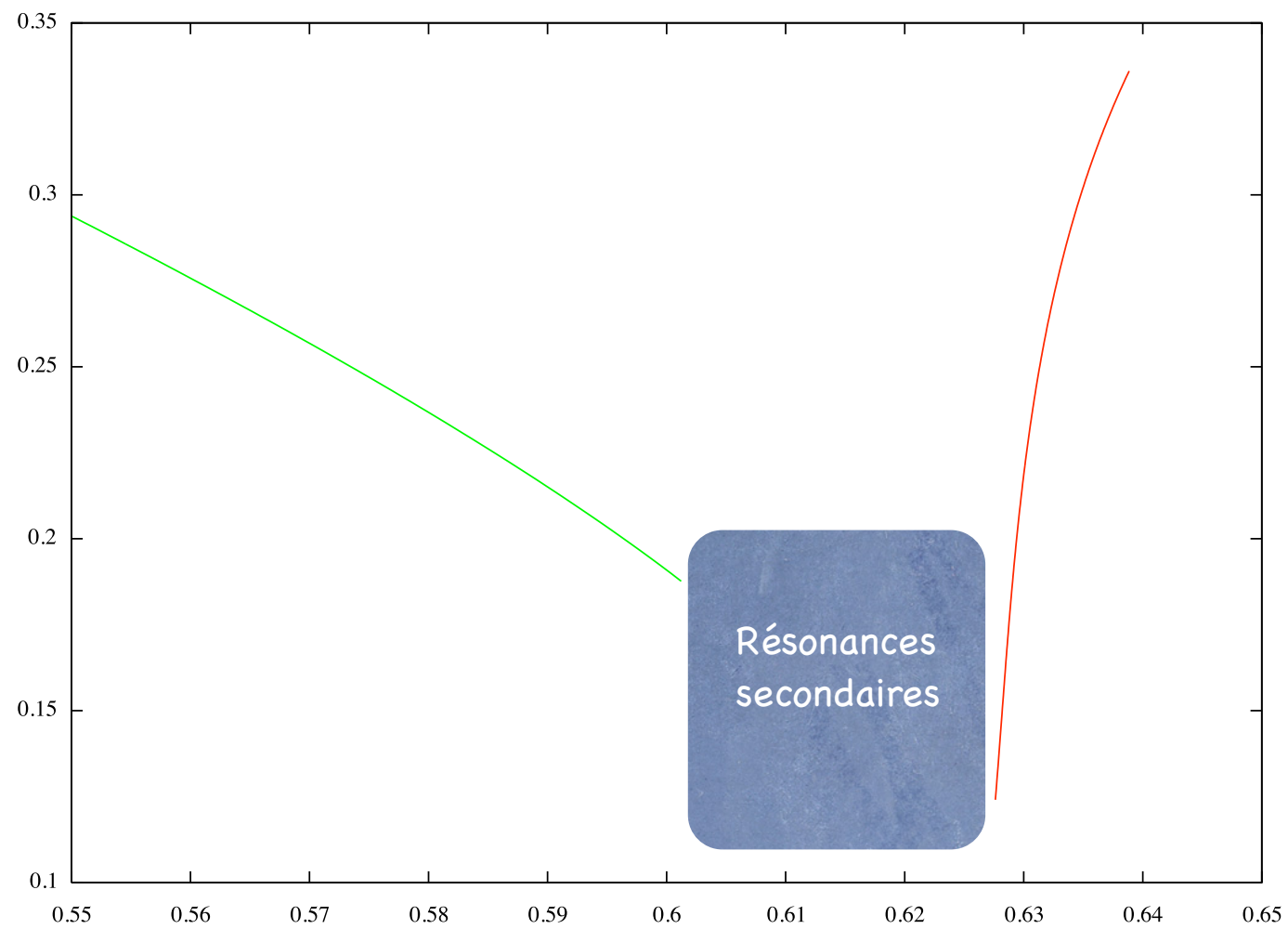


Fig. 3. Secondary resonances $\omega_1/\omega_2 = 1, 2, 3, 4, 5$ (denoted by 1, 2, 3, 4, 5) in the 3:2 (on the left), the 2:1 (at the centre) and the 3:1 (on the right) cases.

résonance 2:1

résonance 3:2

résonance 3:1



Case Umbriel and Miranda

Two satellites of Uranus

Orbital period Umbriel : 4.44 days

Orbital period of Miranda : 1.41 days

Inclination of Miranda high : 4.34 degrees

Passage through the 3:1 resonance in the past

Trapped in a secondary resonance

Umbriel
Miranda

résonance 3:1
en inclinaison
(a,i)

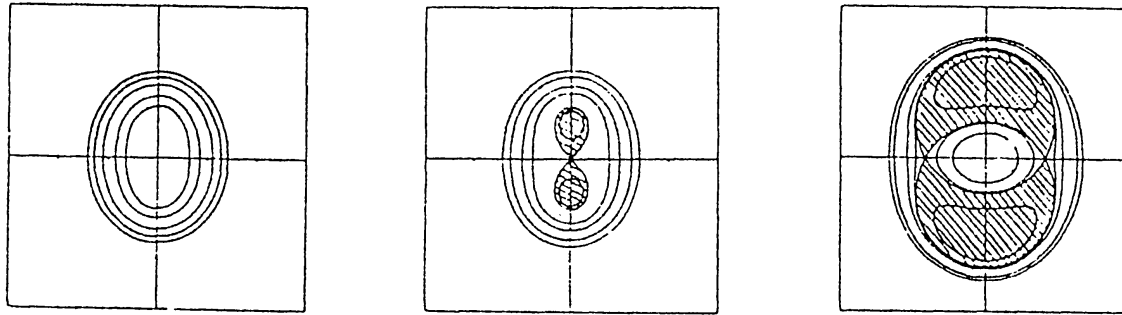


Fig. 1. Typical phase space portraits of the Hamiltonian H_0 in the plane (x, y) . The first figure is for $N < -D/C$, the second one for $-D/C < N < D/C$ and the third one for $N > D/C$. The resonance zone is shaded.

$$\begin{aligned}\sigma &= \frac{1}{2} (-\lambda_M + 3\lambda_U - 2\Omega_M) \\ \nu &= \frac{1}{2} (\lambda_M - 3\lambda_U + 2\Omega_U) \\ S &= \sqrt{\mu a} (1 - \cos i_M) \\ N &= N^* + 2 \sqrt{\mu a_M} + S\end{aligned}$$

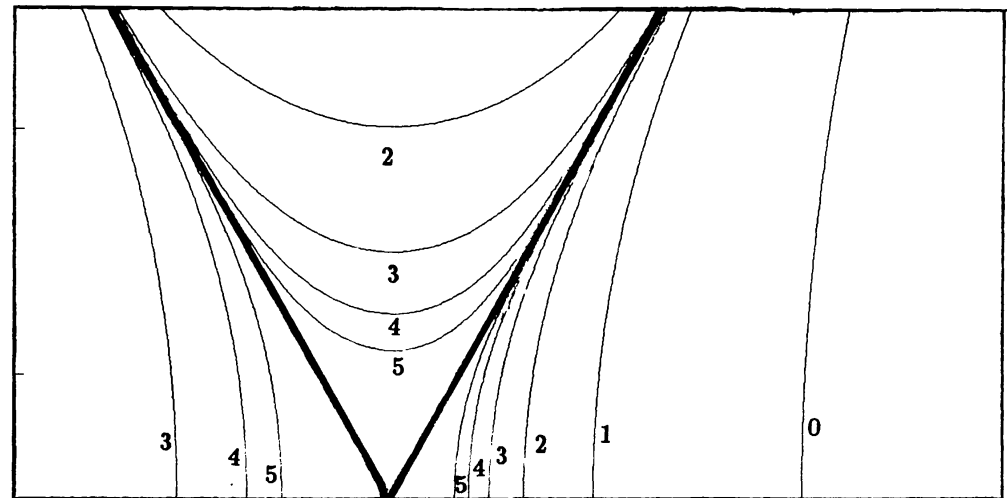


Fig. 2. Location of the secondary resonances in the plane (a_M, i_M) . On the horizontal axis, we have $0.4806 \leq a_M \leq 0.4809$ while, on the vertical axis, we have $0 \leq i_M \leq 0.1$. The $N/1$ secondary resonances ($\omega_2/\omega_1 = N$) are labelled N . The thick V-shaped line is the separatrix of the first degree of freedom.

Résonances de Kozai

Hors résonances en moyen
mouvement

$$\lambda = M + \omega + \Omega$$

$$p = -\varpi$$

$$q = -\Omega$$

$$L$$

$$S = L - G$$

$$T = G - H$$

$$\sqrt{a}$$

$$\frac{e^2}{2}$$

$$\sin^2 \frac{i}{2}$$

orbites planétaires
circulaires
+ moyenne sur les
angles rapides

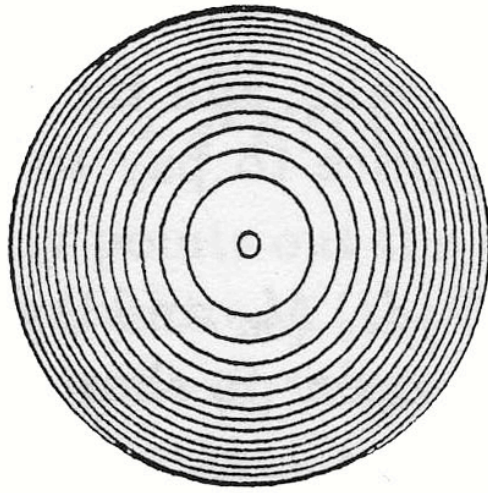
$$V = V(\lambda, p, q, L, S, T, \lambda_i)$$

$$K_0 = K_0(g, e, Q)$$

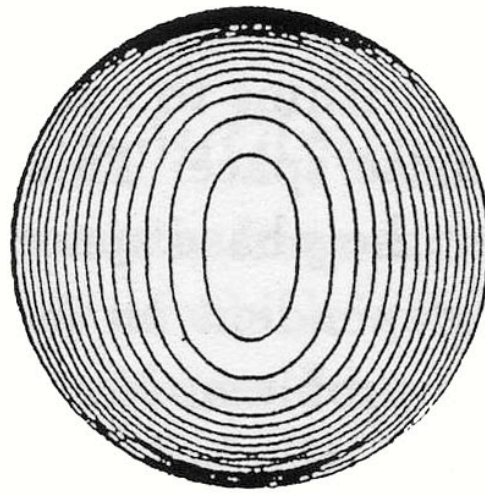
Kozai ou Morbidelli

$$\begin{aligned} Q &= L - H \\ &= L(1 - \sqrt{1 - e^2} \cos i) \\ &\simeq \frac{L}{2}(e^2 + i^2) \end{aligned}$$

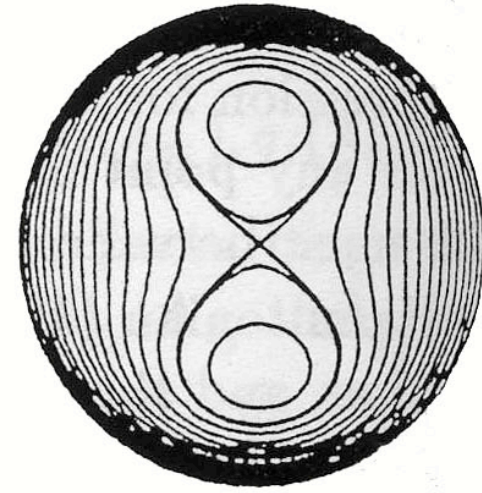
$$Q = Q_0 \rightarrow i_{max}^2 = \frac{2Q_0}{L}$$



(a) $l=10$ $e<.15$



(b) $l=20$ $e<.325$



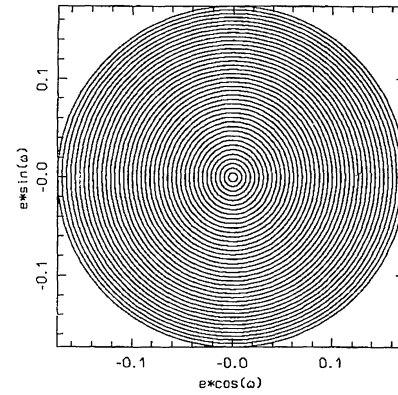
(c) $l=30$ $e<.5$

coordonnées polaires : (excentricité, péricentre)

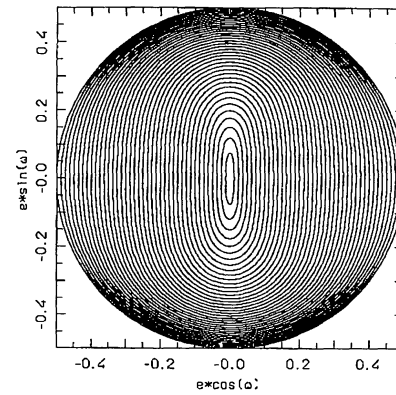
$$Q = Q_0 \rightarrow i_{max}^2 = \frac{2Q_0}{L}$$

L'astéroïde a un péricentre
qui ne circule plus mais est en
libration autour de 90° ou 270°

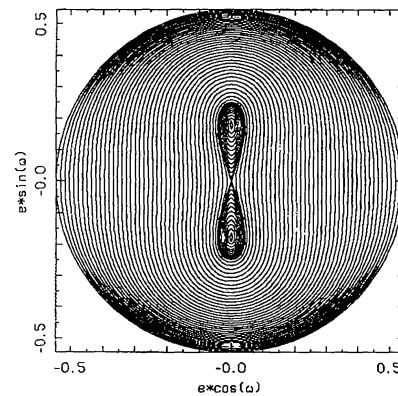
a = 3.00 i_{max} = 10.00 (a)



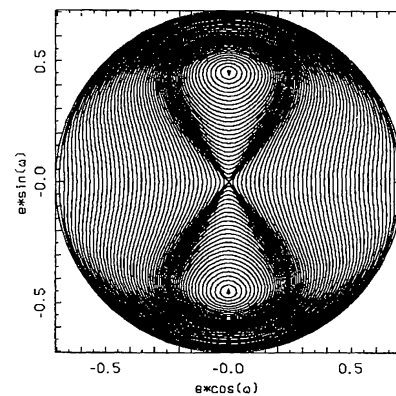
a = 3.00 i_{max} = 30.00 (b)



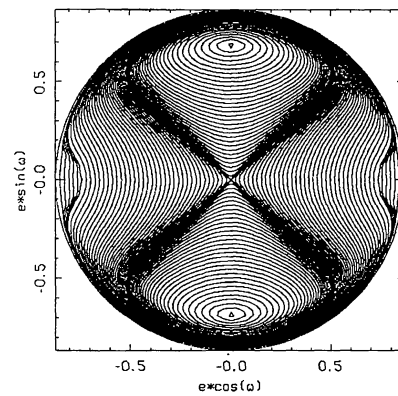
a = 3.00 i_{max} = 33.00 (c)



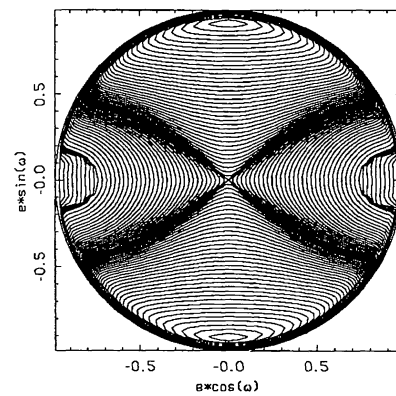
a = 3.00 i_{max} = 45.00 (d)



a = 3.00 i_{max} = 60.00 (e)



a = 3.00 i_{max} = 80.00 (f)



Morbidelli

Résonances séculaires

$$\mathcal{H}_0 = -\frac{\mu^2}{2L^2} + \sum_{i=5,6} n_i \Xi_i \sum_{i=5,6} g_i \Lambda_i + \sum_{i=5,6} \nu_i \Gamma_i$$

$$\dot{\varpi}_i = g_i$$

$$\dot{\Omega}_i = \nu_i$$

$$\dot{\lambda}_i = n_i$$

$$\lambda = M + \omega + \Omega$$

$$L$$

$$\sqrt{a}$$

$$p = -\varpi$$

$$S = L - G$$

$$\frac{e^2}{2}$$

$$q = -\Omega$$

$$T = G - H$$

$$\sin^2 \frac{i}{2}$$

$$V = V(\lambda, p, q, L, S, T, \lambda_i, \varpi_i, \Omega_i)$$

$$\dot{\omega}_J = g_5 \neq 0$$

$$\dot{\omega}_S = g_6 \neq 0$$

$$\dot{\Omega}_J = s_5 \neq 0$$

$$\dot{\Omega}_S = s_6 \neq 0$$

Hors résonances en moyen
mouvement

Moyenne sur les angles rapides

$$\bar{V} = \bar{V}(-, p, q, L, S, T, -, \varpi_i, \Omega_i)$$

$$\sigma = \varpi - \varpi_i$$

$$= -p - \varpi_i$$

$$\sigma = \Omega - \Omega_i$$

$$= -q - \Omega_i$$

Orbites des 3ièmes corps non kepleriennes

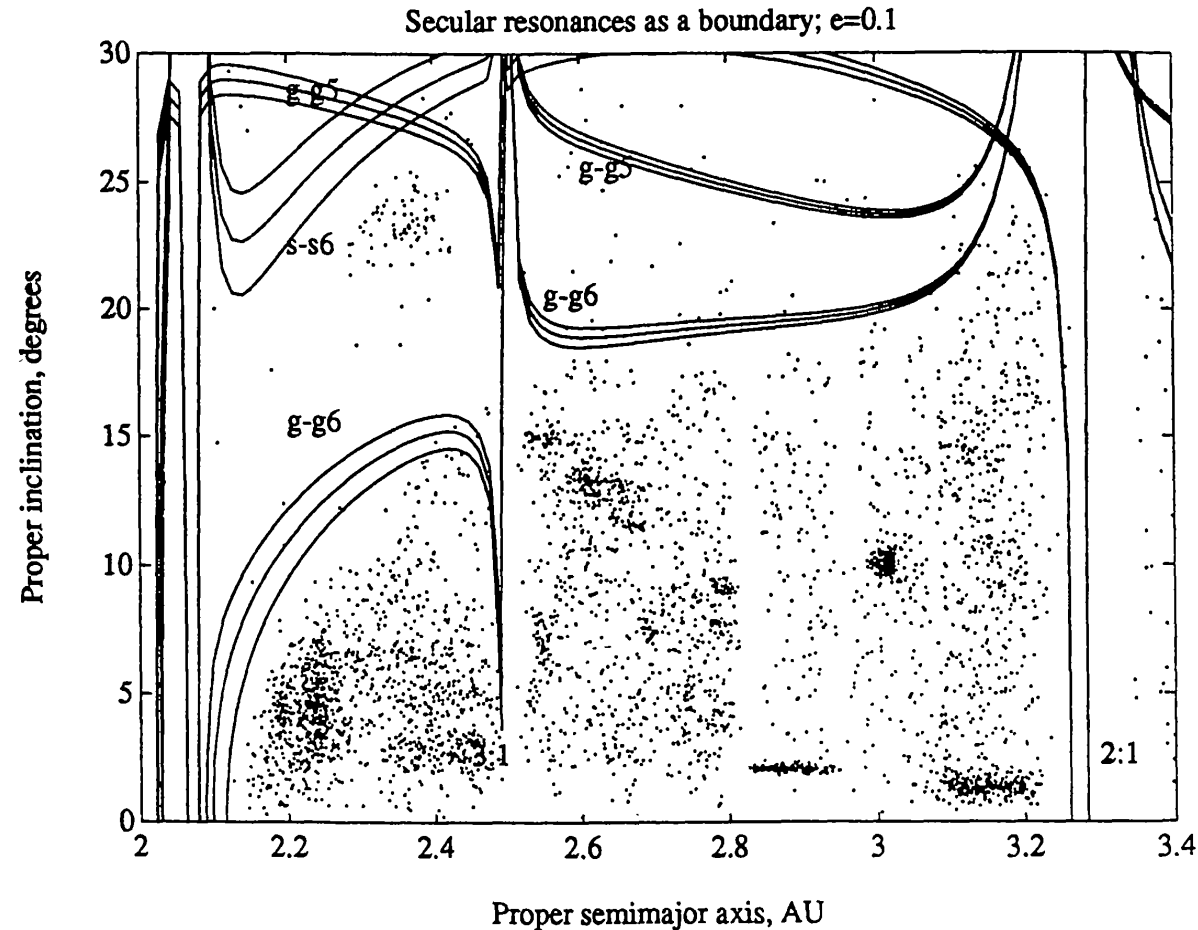
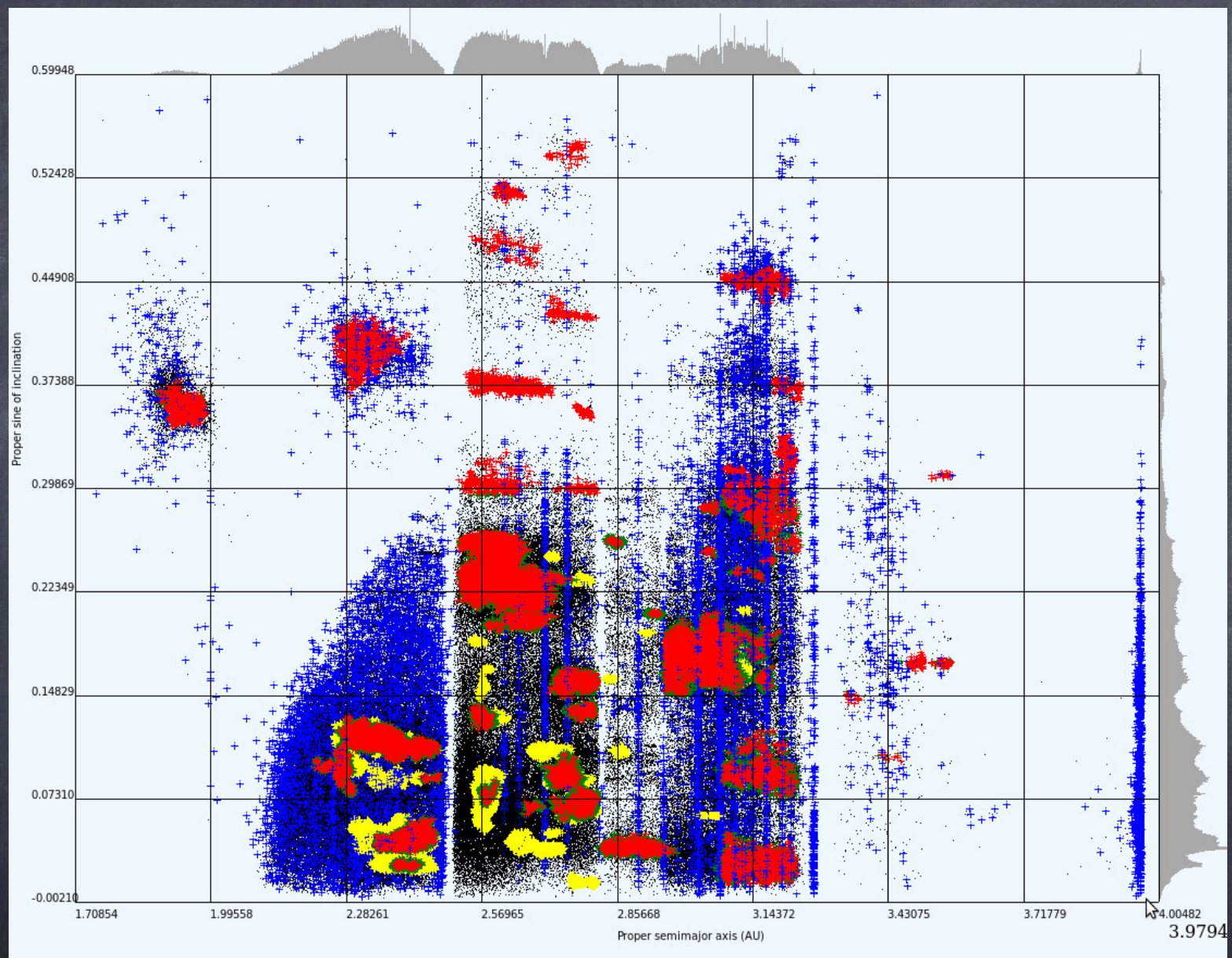
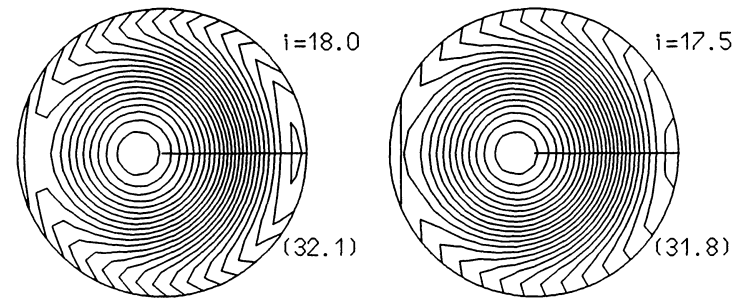
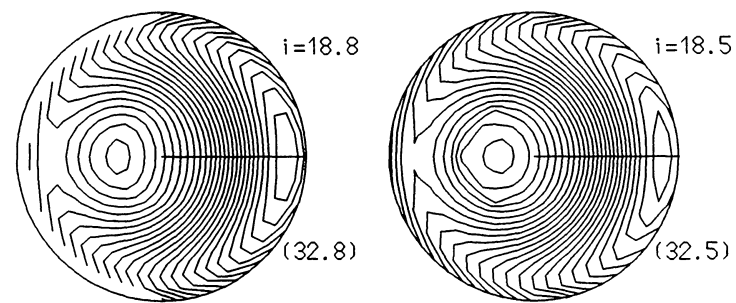
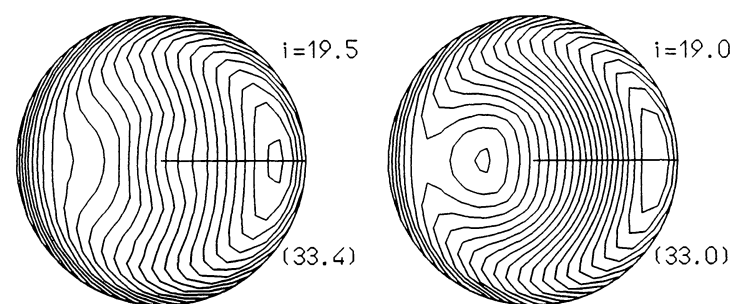
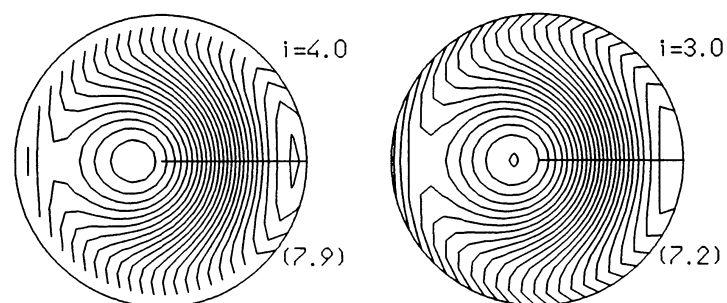
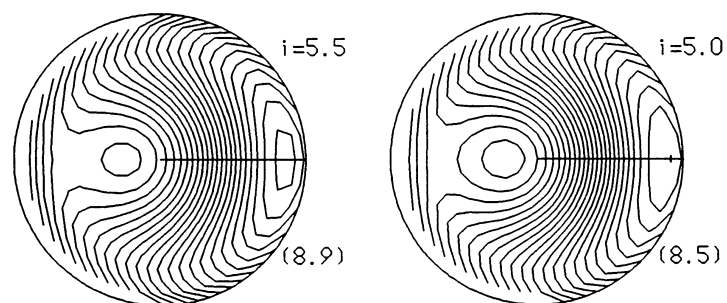
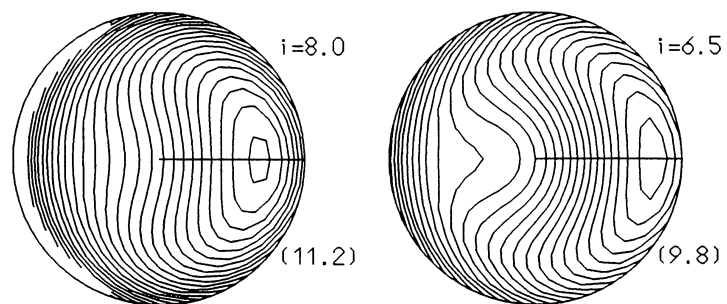


Fig. 1. The location of the main linear secular resonances for proper $e = 0.1$ according to Milani and Knežević (1990). The location of the resonances is strongly affected by the interaction with the mean motion resonances $3/1$ and $2/1$. The contour lines around the secular resonances correspond to a width of $+/- 1''/\text{yr}$.





Résonance séculaire en
péricentre

$(e, -\sigma)$

$e = 0$ est au centre

ν_6

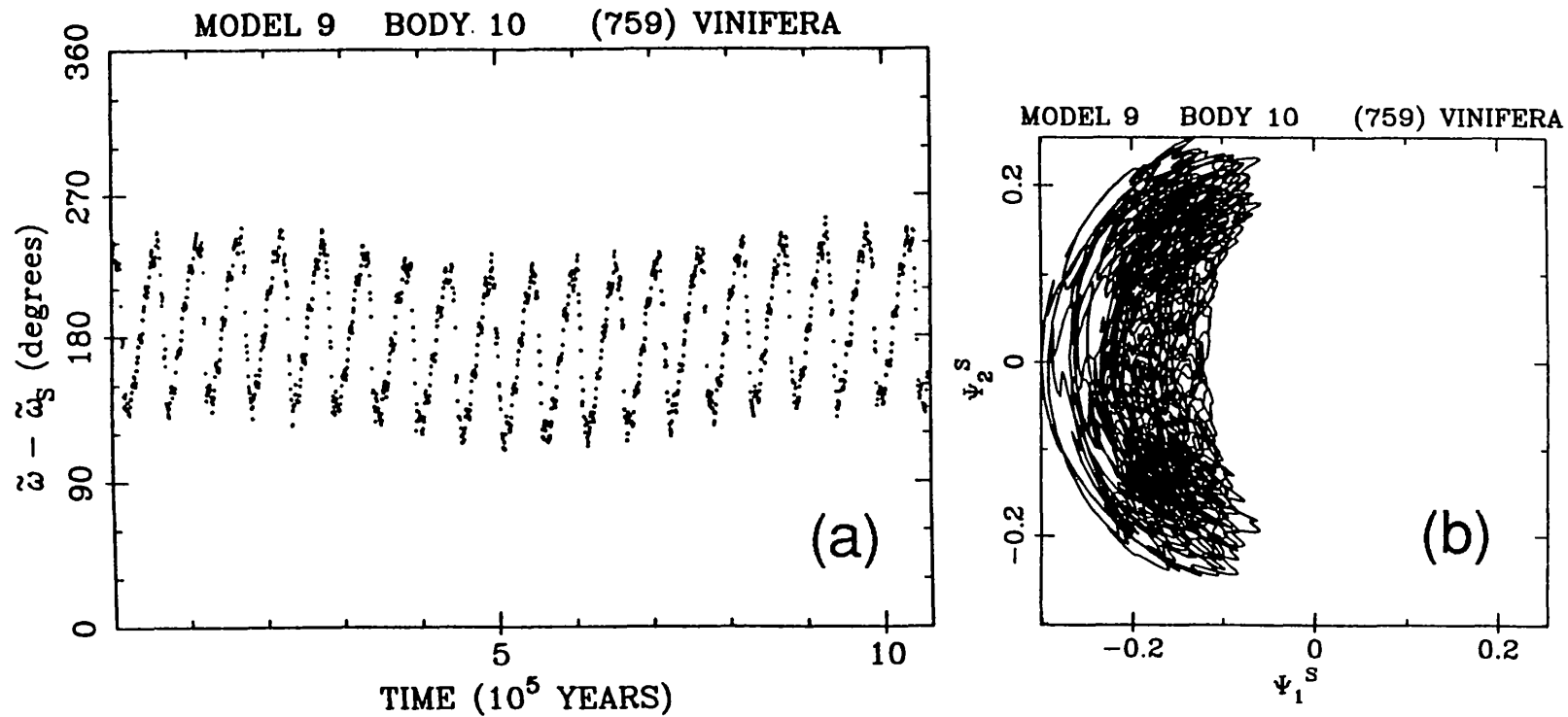


Fig. 4. Numerical simulations on 759 Vinifera. On the left (4a) $\bar{\omega} - \bar{\omega}_S$ is given as a function of time; on the right (4b) the motion is illustrated in polar coordinates $e, \bar{\omega} - \bar{\omega}_S$. (Ch. Froeschlé and H. Scholl, 1987b).

$$\sigma = \varpi - \varpi_S$$

Echelles de temps
1 million d'années

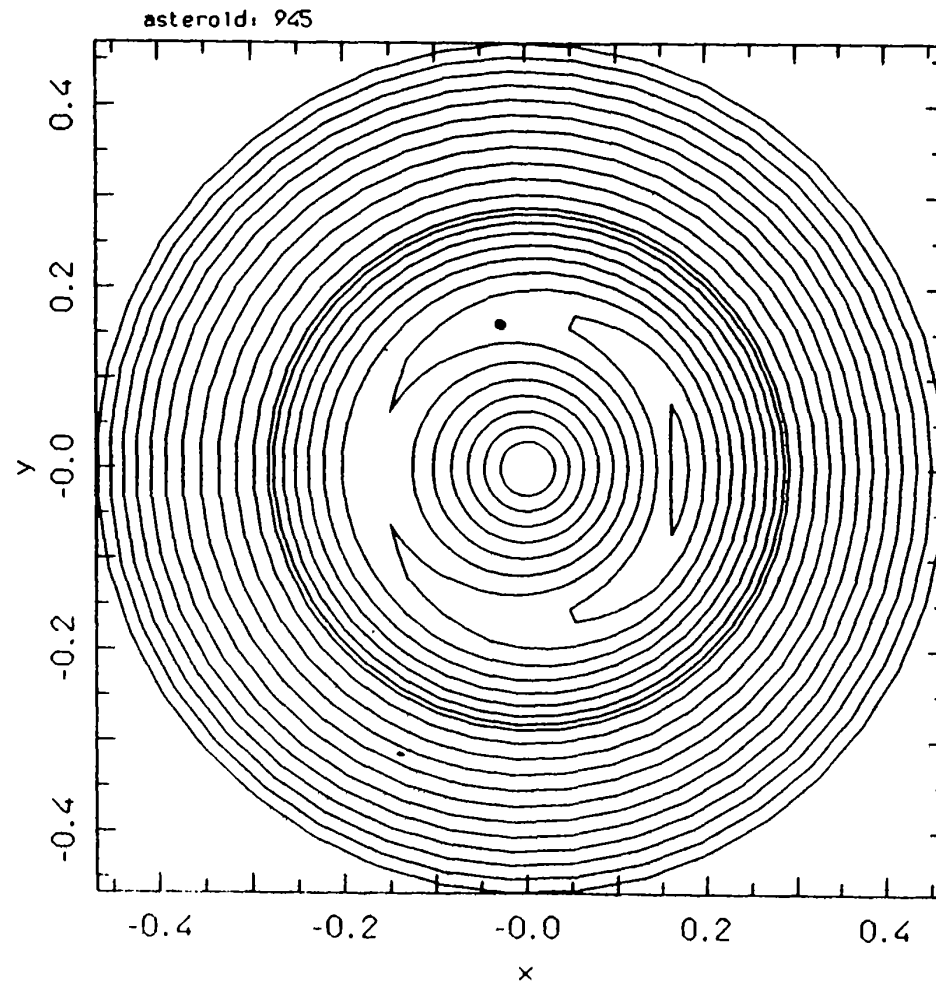


Fig. 5. The phase space of the $g = g_5$ resonance for the asteroid 945 Barcelona. The dot marks the present position of the asteroid in the semi-proper elements space.

RÉSONANCE SÉCULAIRE

$$\sigma = \Omega - \Omega_J$$

$i = 0$ est au centre

$(i, -\sigma)$

ν_{16}

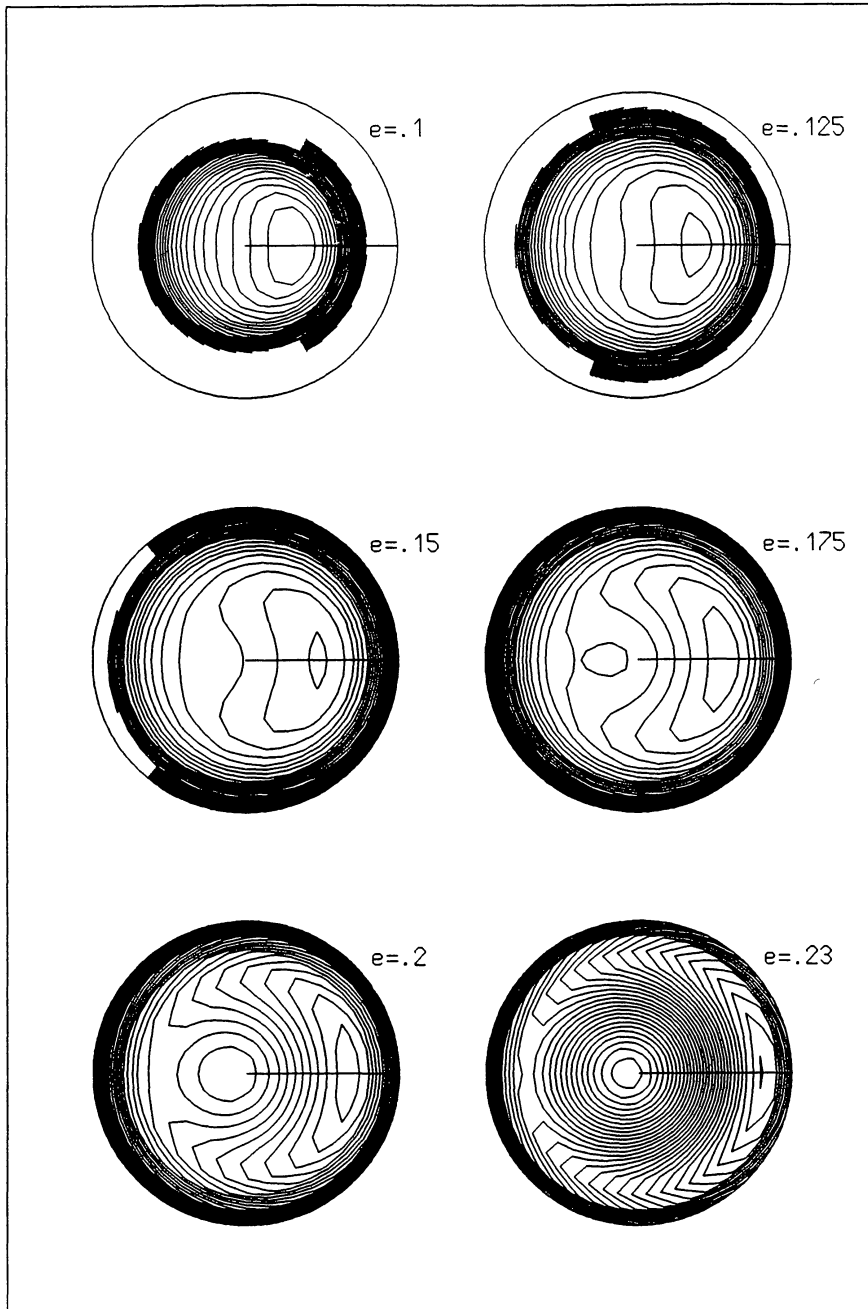


Fig. 13. The ν_{16} resonance for $a = .38$ times that of Jupiter (1.976 A.U.).

Résonances séculaires à l'intérieur des résonances en moyen mouvement

Superposition de
plusieurs
dynamiques

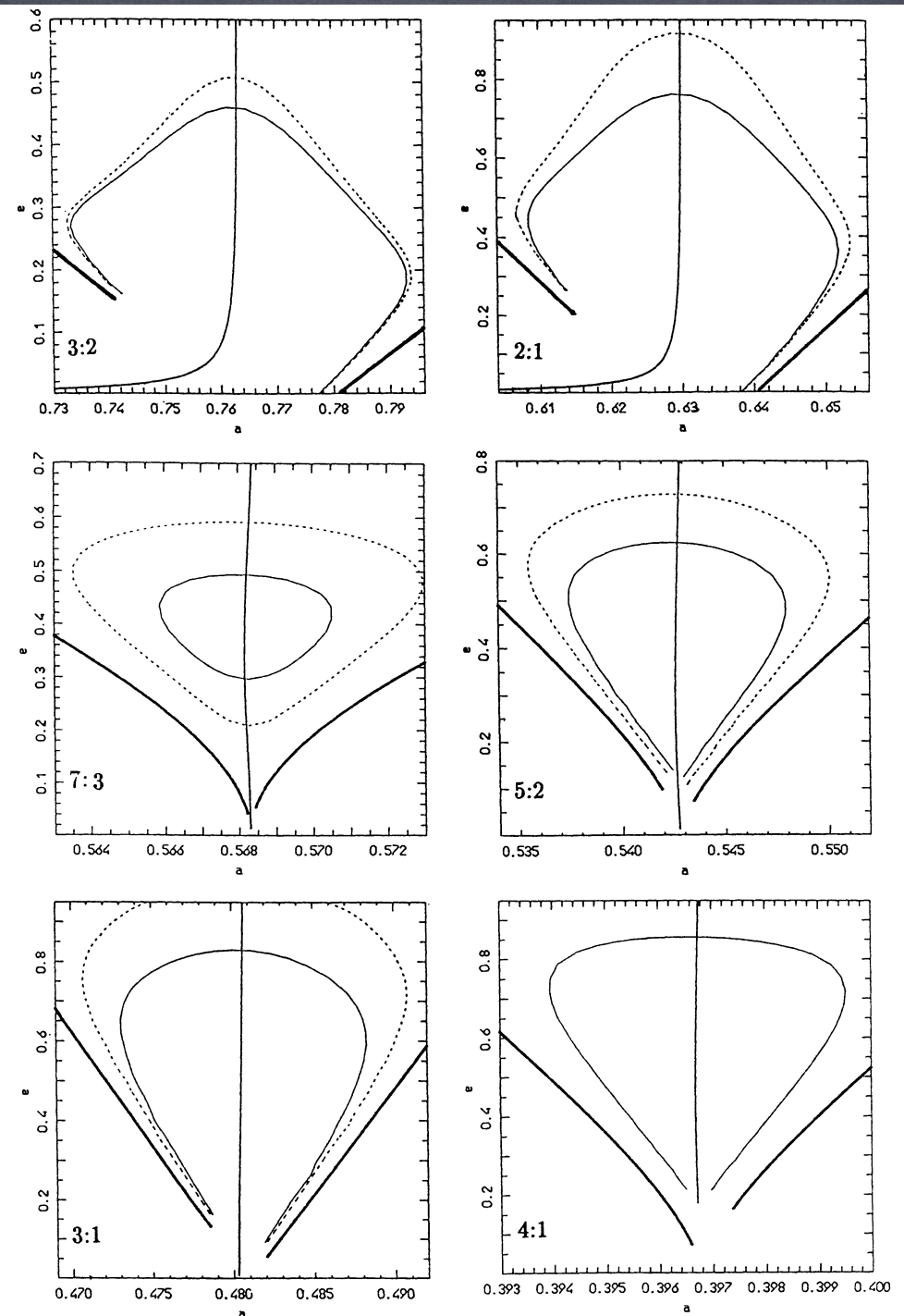


Fig. 4. Secular resonances ν_5 (solid line) and ν_6 (dashed line).

Orbites des 3ièmes corps non kepleriennes

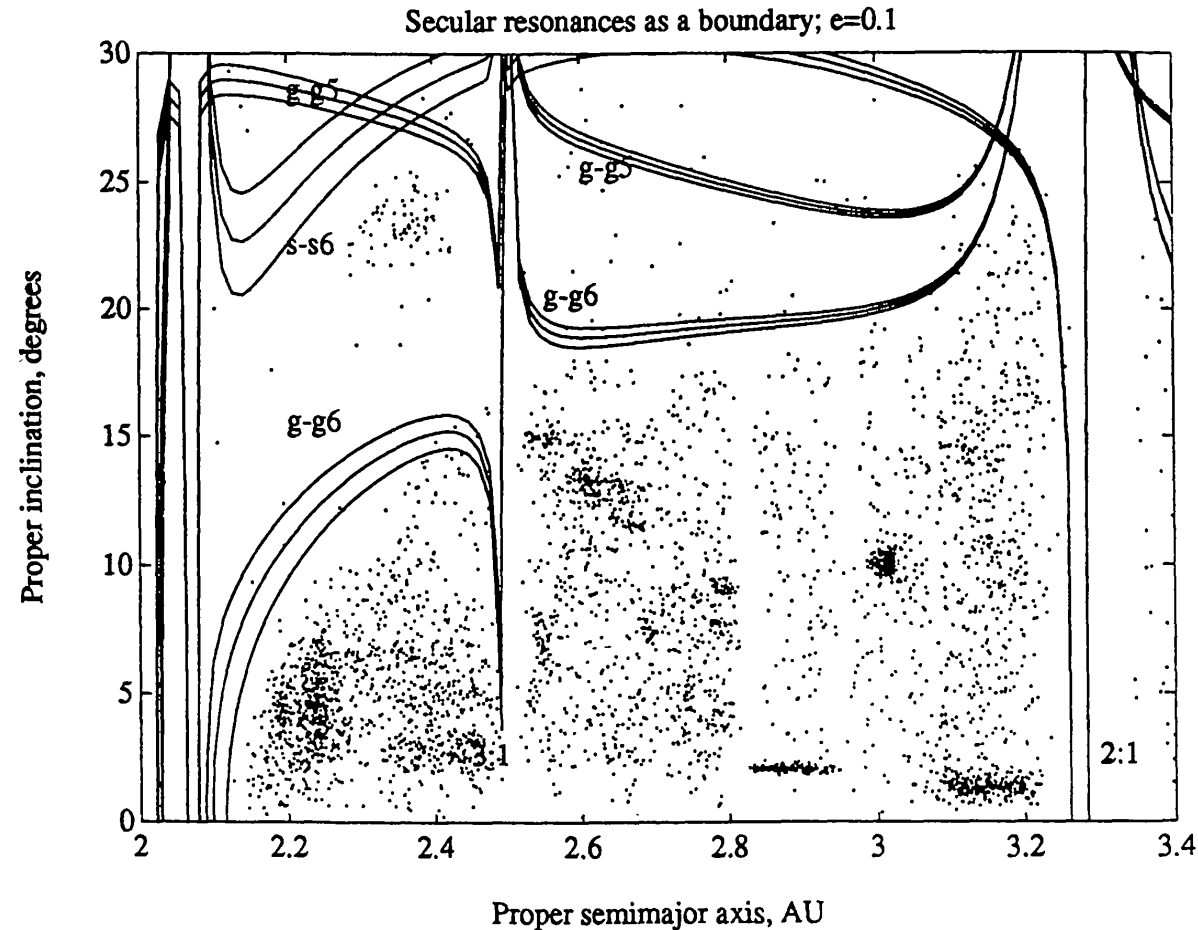
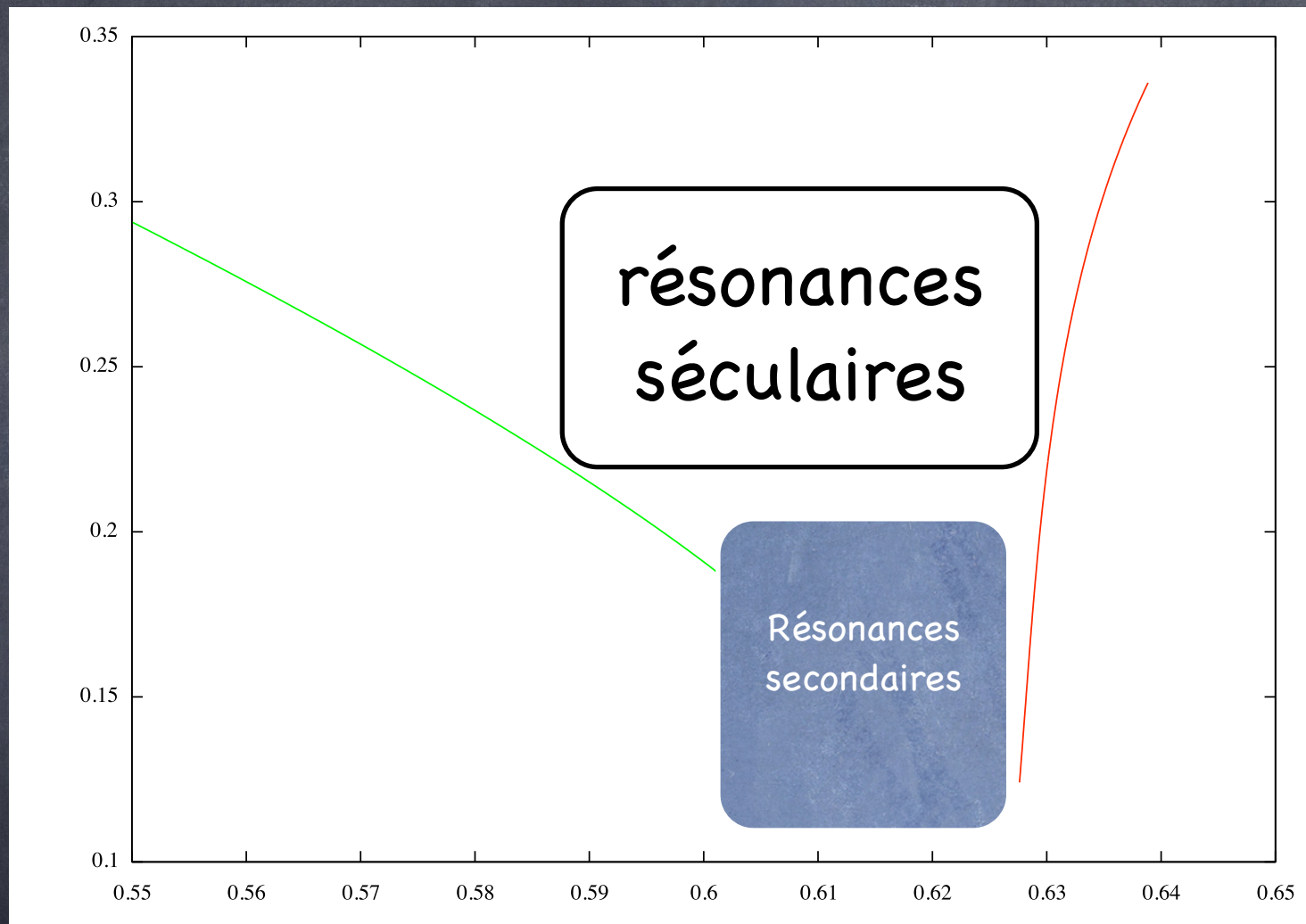
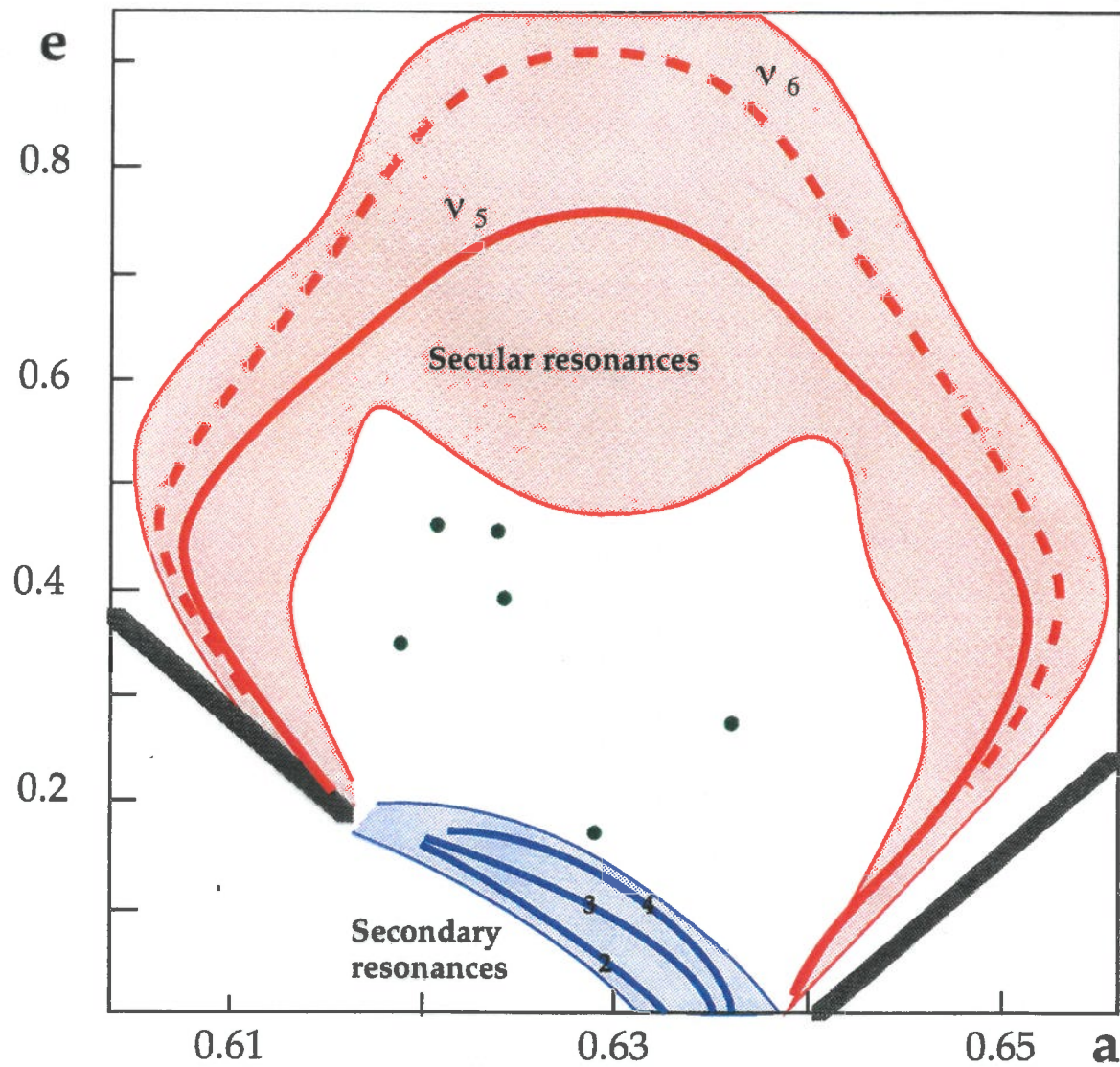


Fig. 1. The location of the main linear secular resonances for proper $e = 0.1$ according to Milani and Knežević (1990). The location of the resonances is strongly affected by the interaction with the mean motion resonances $3/1$ and $2/1$. The contour lines around the secular resonances correspond to a width of $\pm 1''/\text{yr}$.

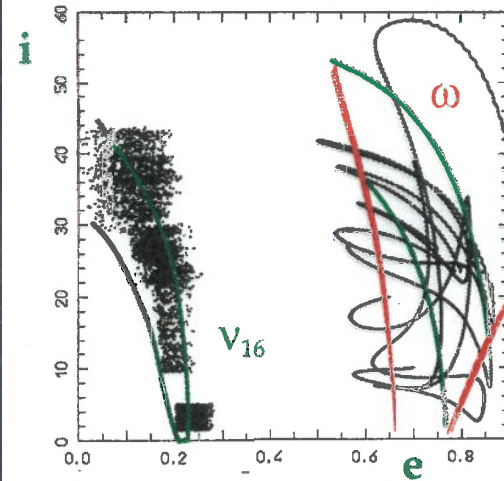


Secular resonances in the planar problem:
analytic localization for the 2/1 Hecuba gap

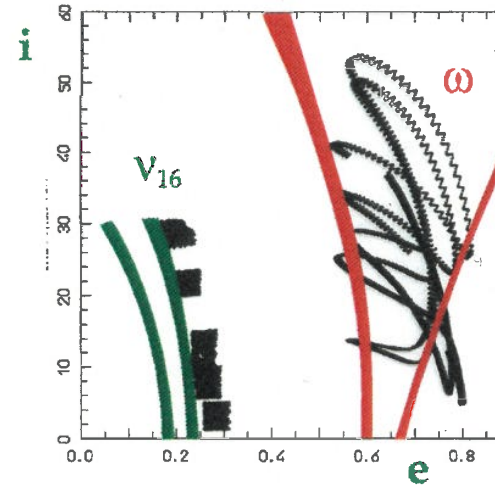


3D-Localization of secular resonances₆ (numerical simulations)

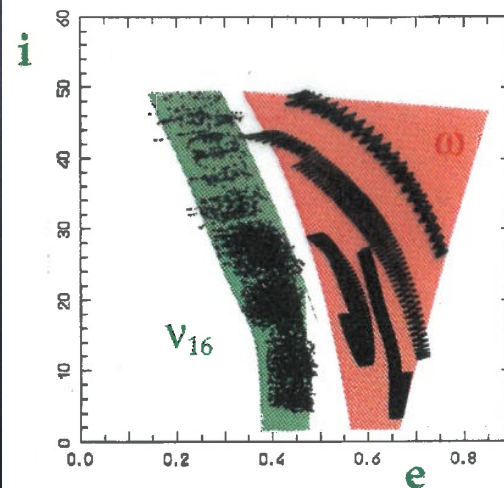
$\Delta\sigma \approx 20^\circ$



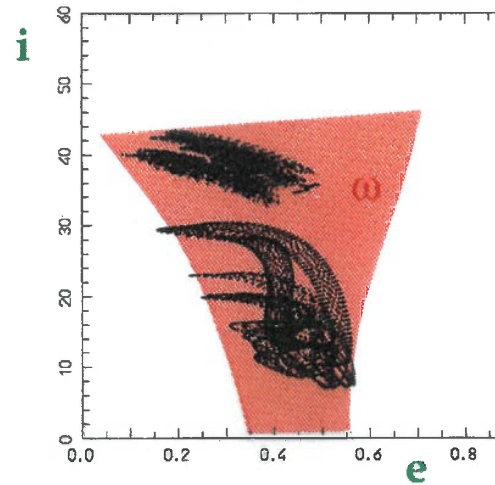
$\Delta\sigma \approx 60^\circ$



$\Delta\sigma \approx 100^\circ$



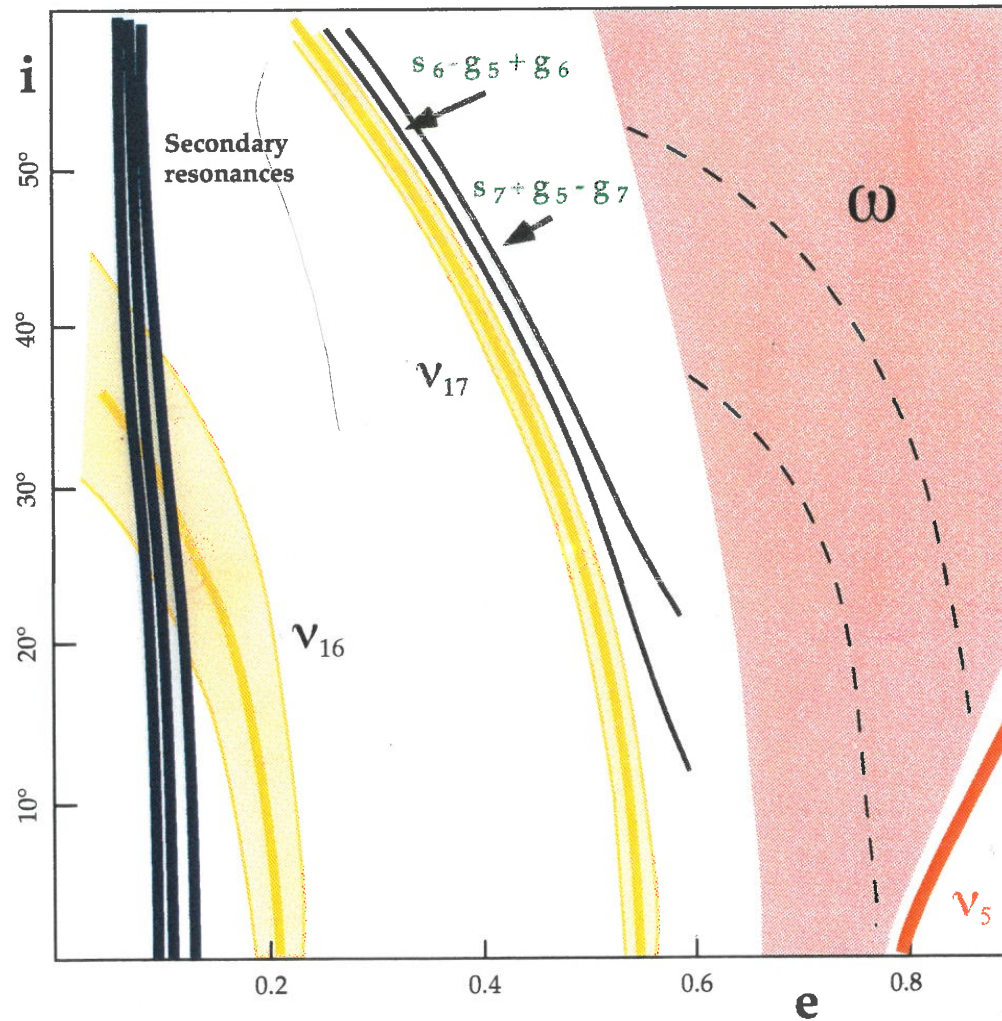
$\Delta\sigma \approx 130^\circ$



3D - Localization of resonances

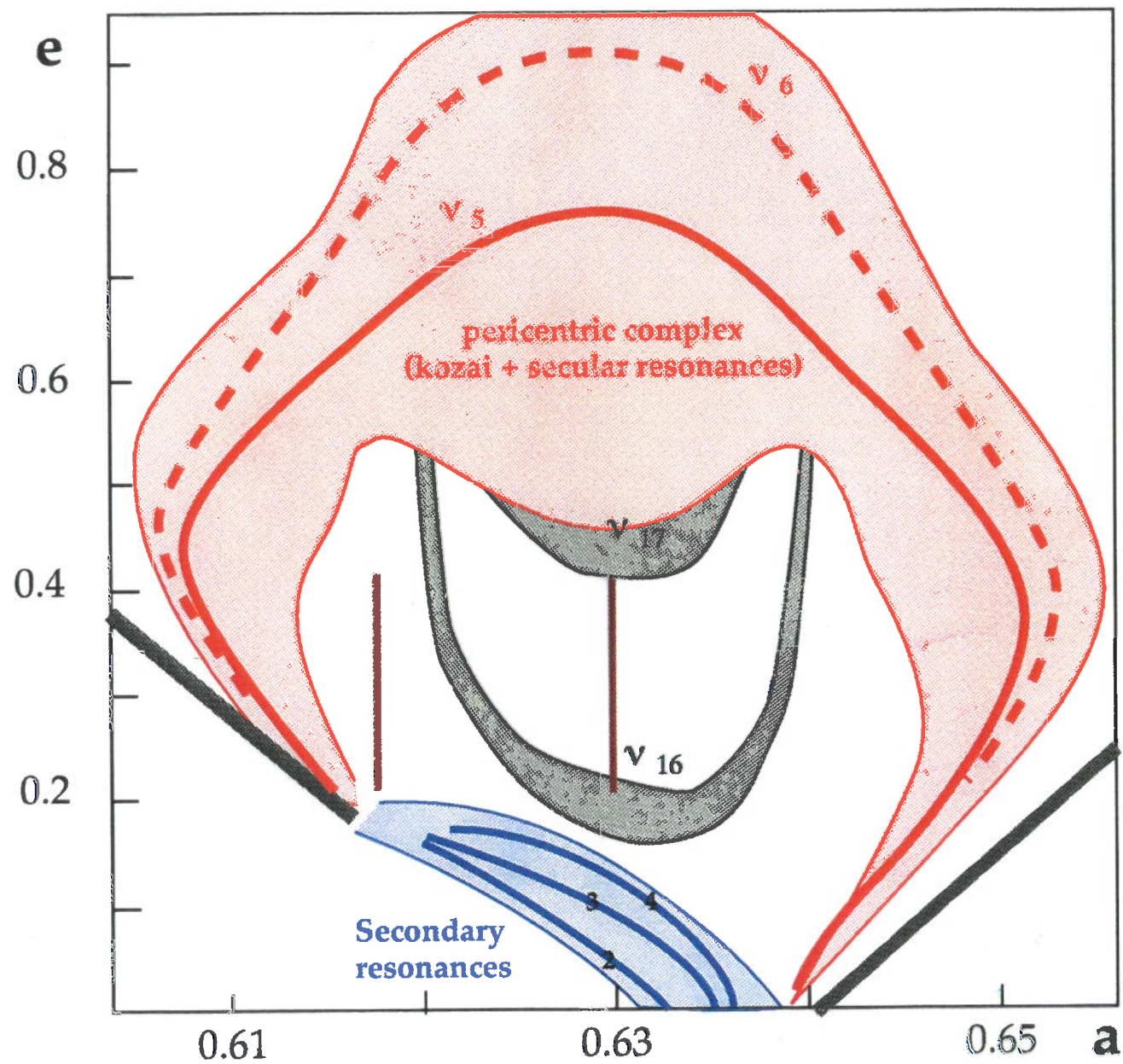
for small values of the amplitude of libration of σ
(semi-analytical predictions)

Moons and Morbidelli

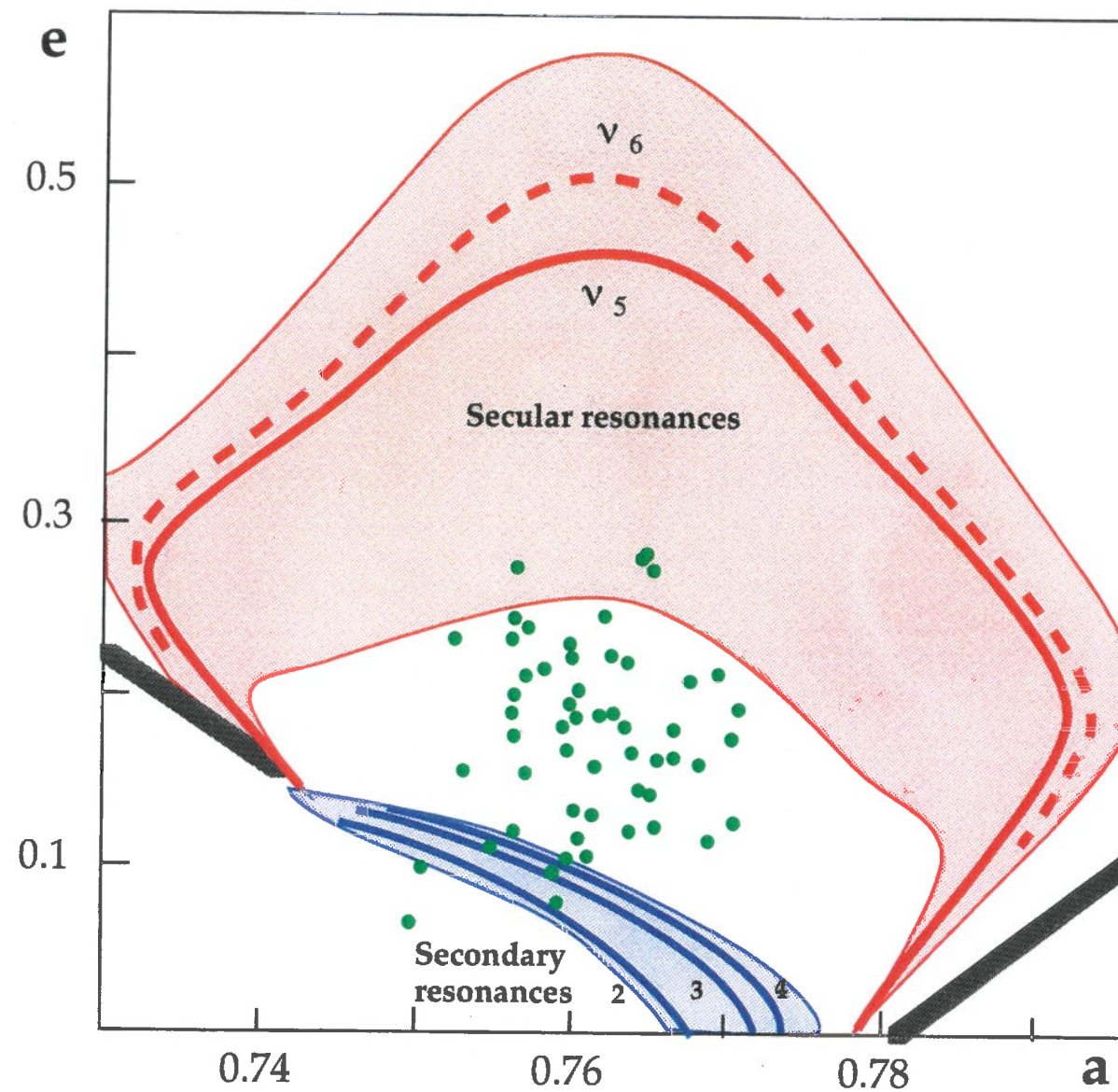


Hecuba gap - 2/1 res.

Origin of chaos in the 2/1 Hecuba gap



Secular resonances in the planar problem:
analytic localization for the 3/2 Hilda group



Minor planet in MMR with a planet

Possible large excursions in eccentricity

Crossing of the orbit of another planet (Mars for example)

Nothing to hope for small eccentricities
blocked in the resonance area

2 mechanisms

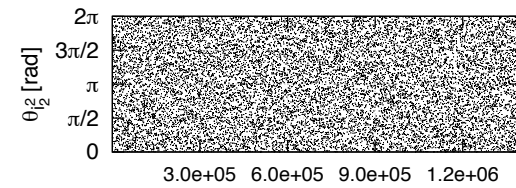
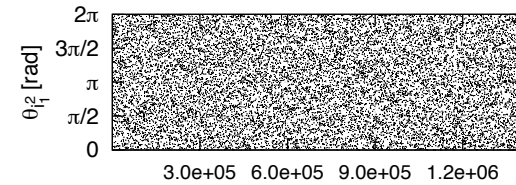
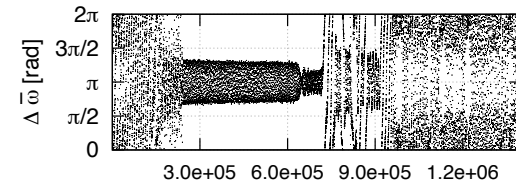
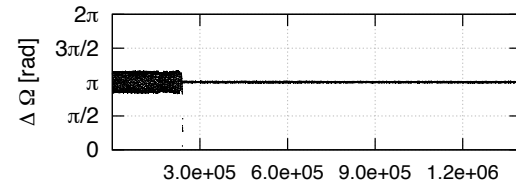
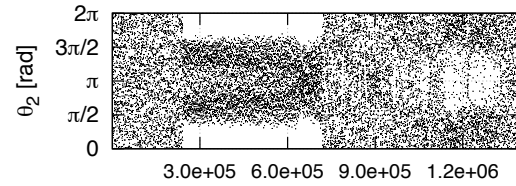
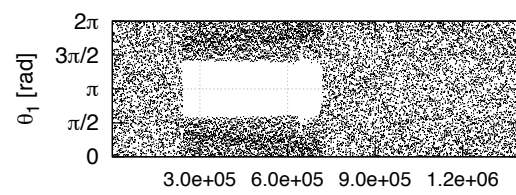
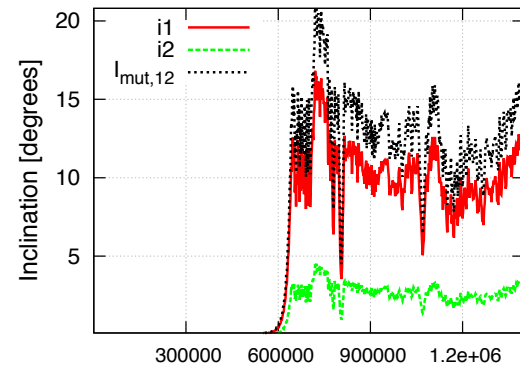
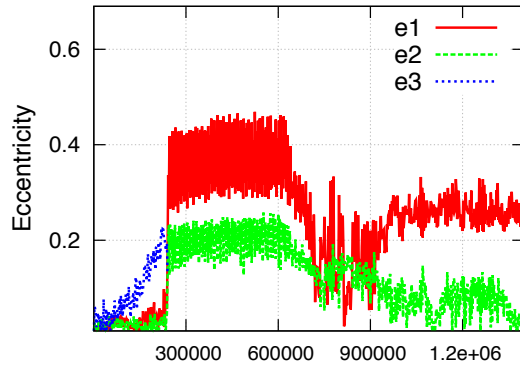
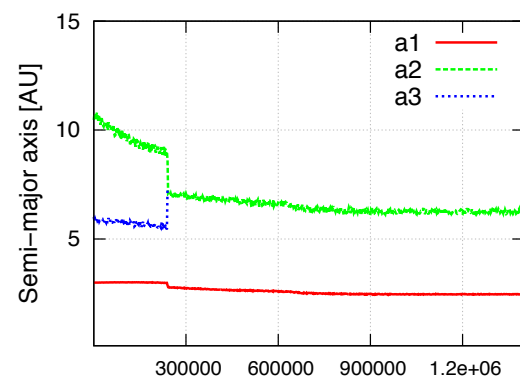
Explanation of the depletion
presence of other resonances OK

Explanation of the flux of NEO
action of dissipations ???

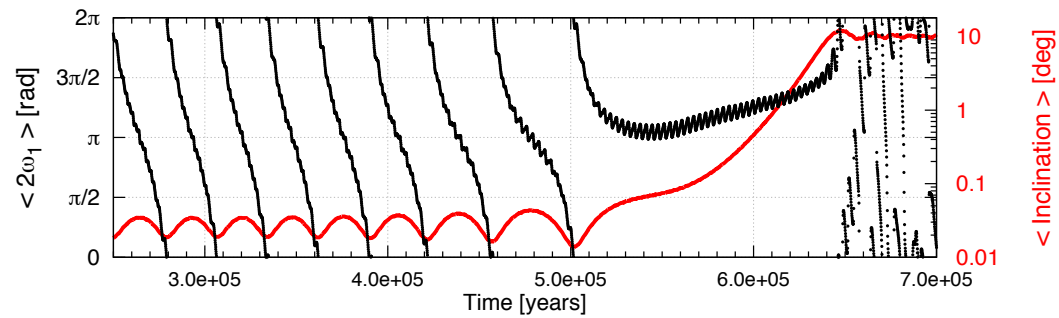
Except for very stable zones (inexistent ?)
the eccentricity increases

Very recent PhD thesis
Exoplanets forming in a disk
Huge simulations on long times
Purely numerical
Search for high inclinations cases
Kozai dynamics

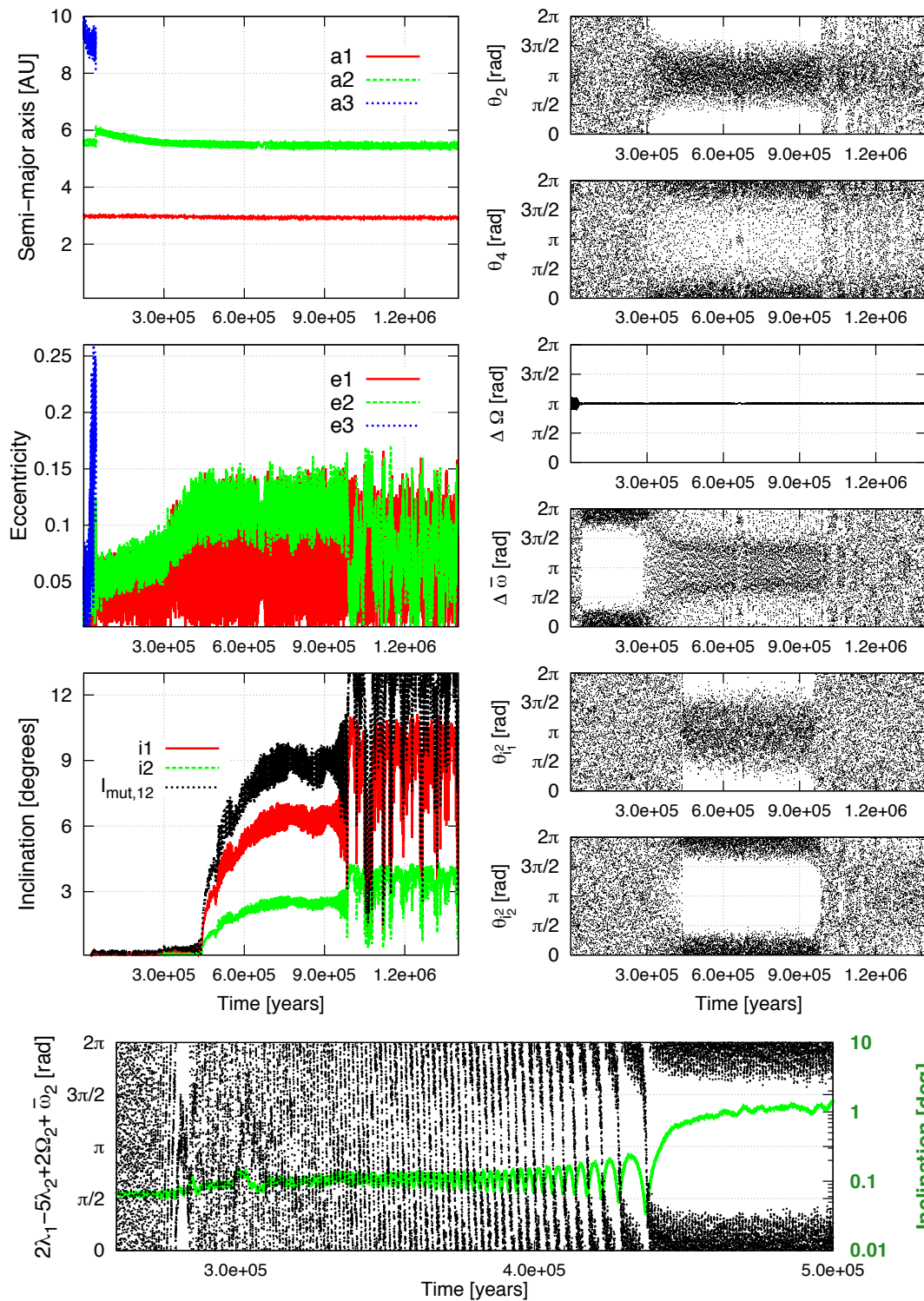
Sotiris Sotiriadis
PhD in October 2017

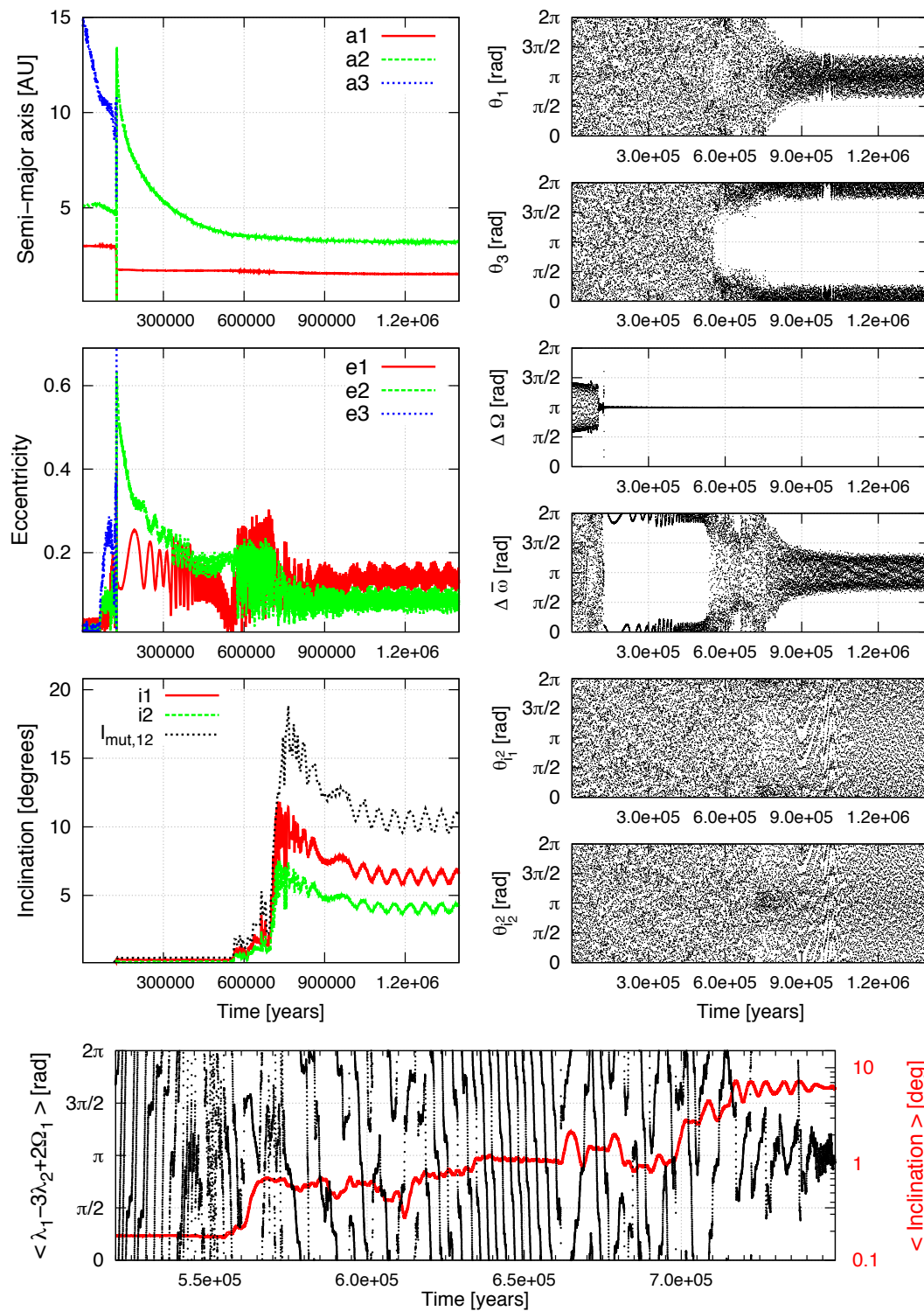


Time [years]



Superpositions MMR + Kozai + secondary





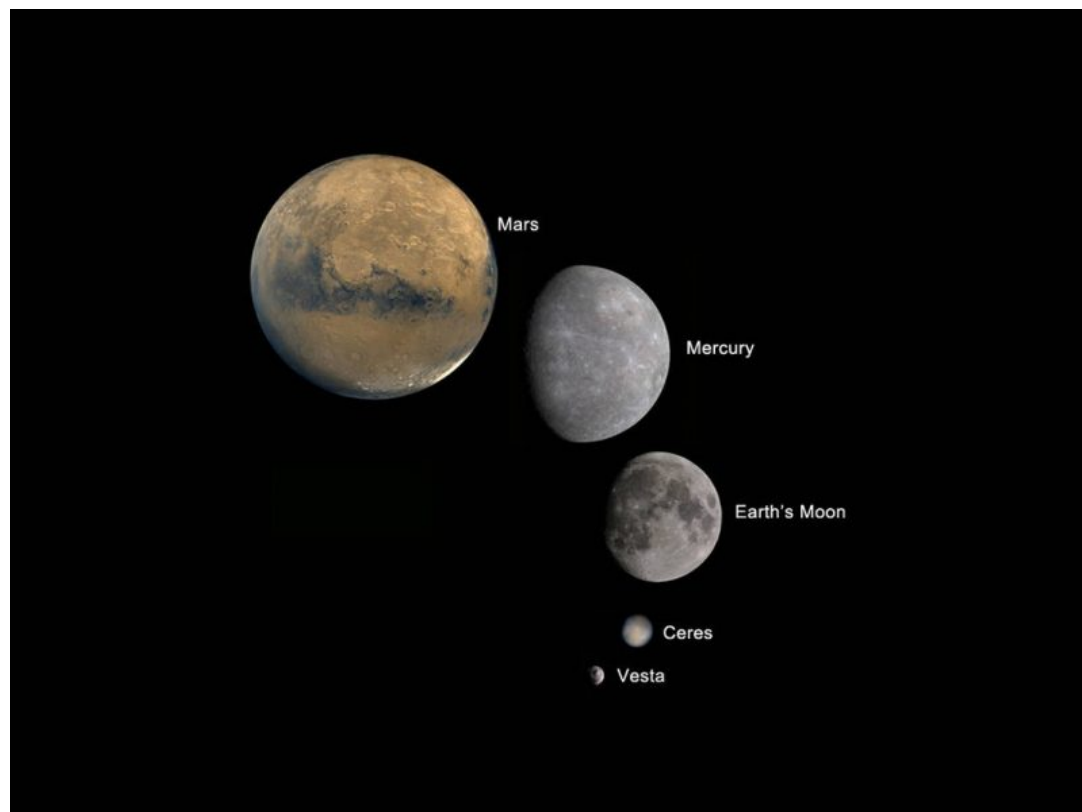
Goal: existence of systems
with high inclination

Secular resonances with Ceres and Vesta

Georgios Tsirvoulis^{a,*}, Bojan Novaković^b

^a*Astronomical Observatory, Volgina 7, 11060 Belgrade 38, Serbia*

^b*Department of Astronomy, Faculty of Mathematics, University of Belgrade, Studentski trg
16, 11000 Belgrade, Serbia*



New resonances for small asteroids

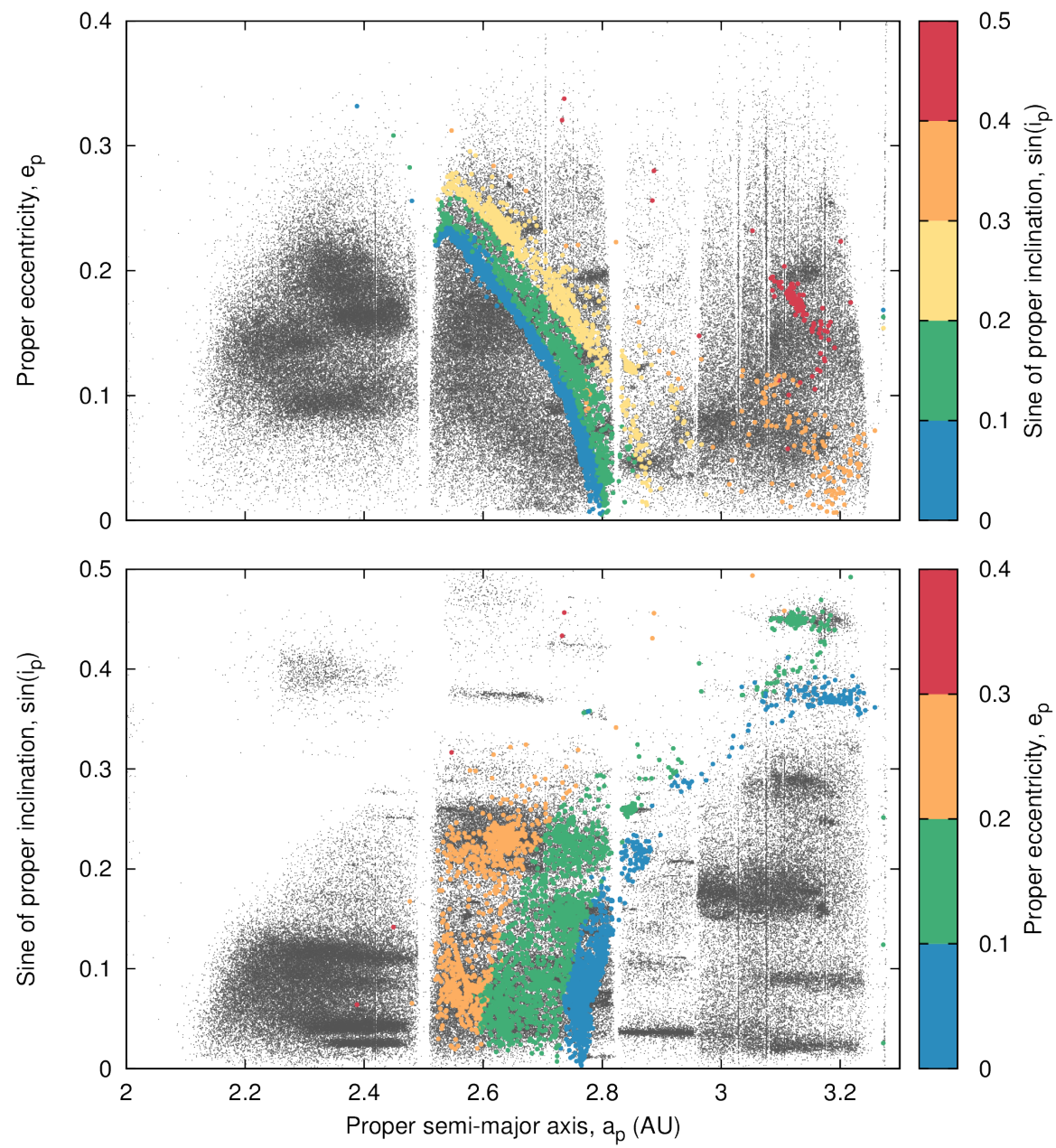


Figure 2: The location of the v_{1c} secular resonance on the (a_p, e_p) plane (top), and on the $(a_p, \sin i_p)$ plane (bottom). The gray dots represent all main belt asteroids, and the colored points the resonant ones for different inclinations (top panel) and eccentricities (bottom panel) according to the respective color codes given in the legend.

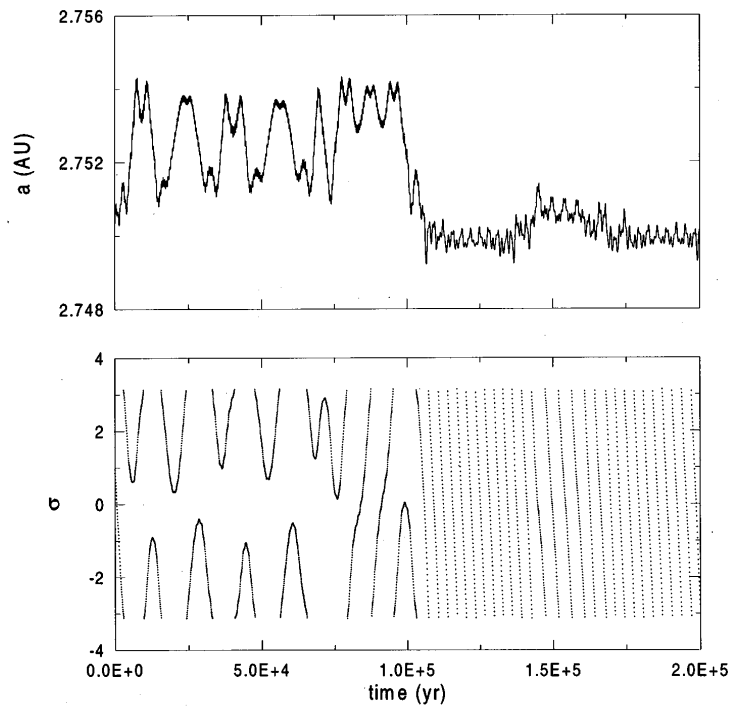
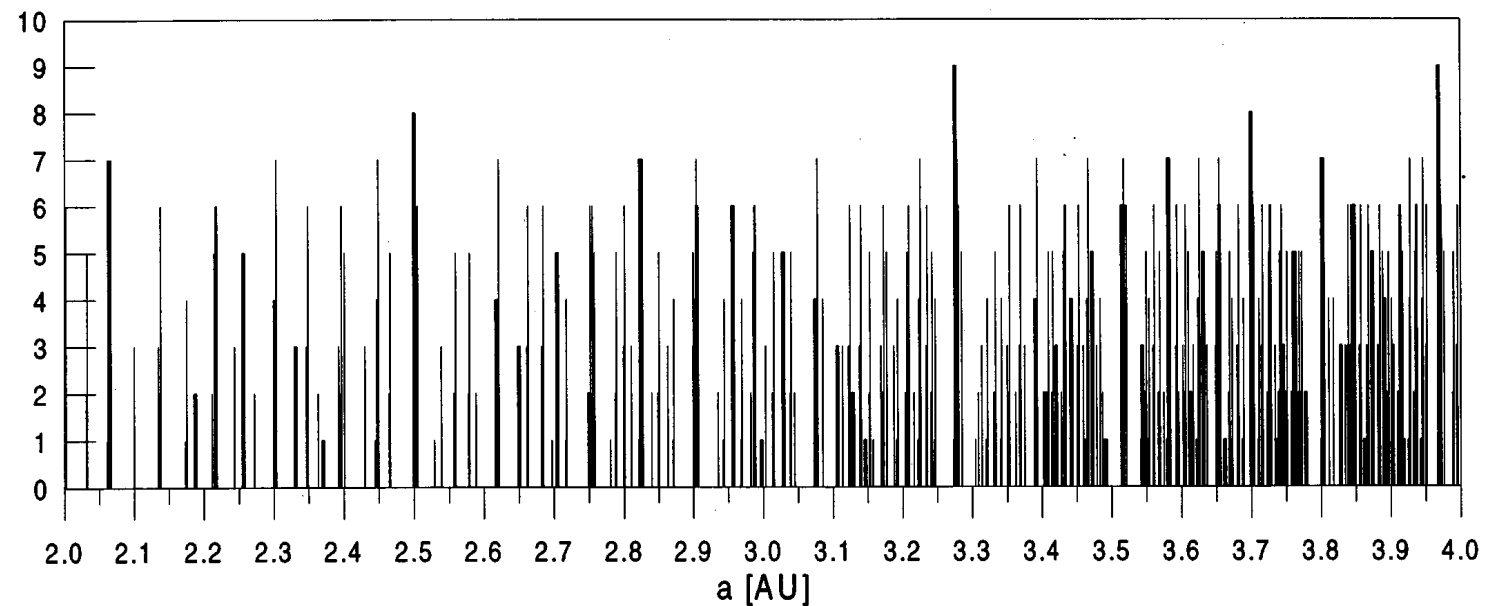


FIG. 2.—Same as Fig. 1, but for the asteroid 485 Genua. In this case, $\sigma = 3\lambda_J - \lambda_S - \lambda - \varpi$. The asteroid is initially inside the 3 - 1 - 1 three-body resonance.

MMR résonances faisant
intervenir 3 angles
2 planètes : J and S

NESVORNÝ & MORBIDELLI



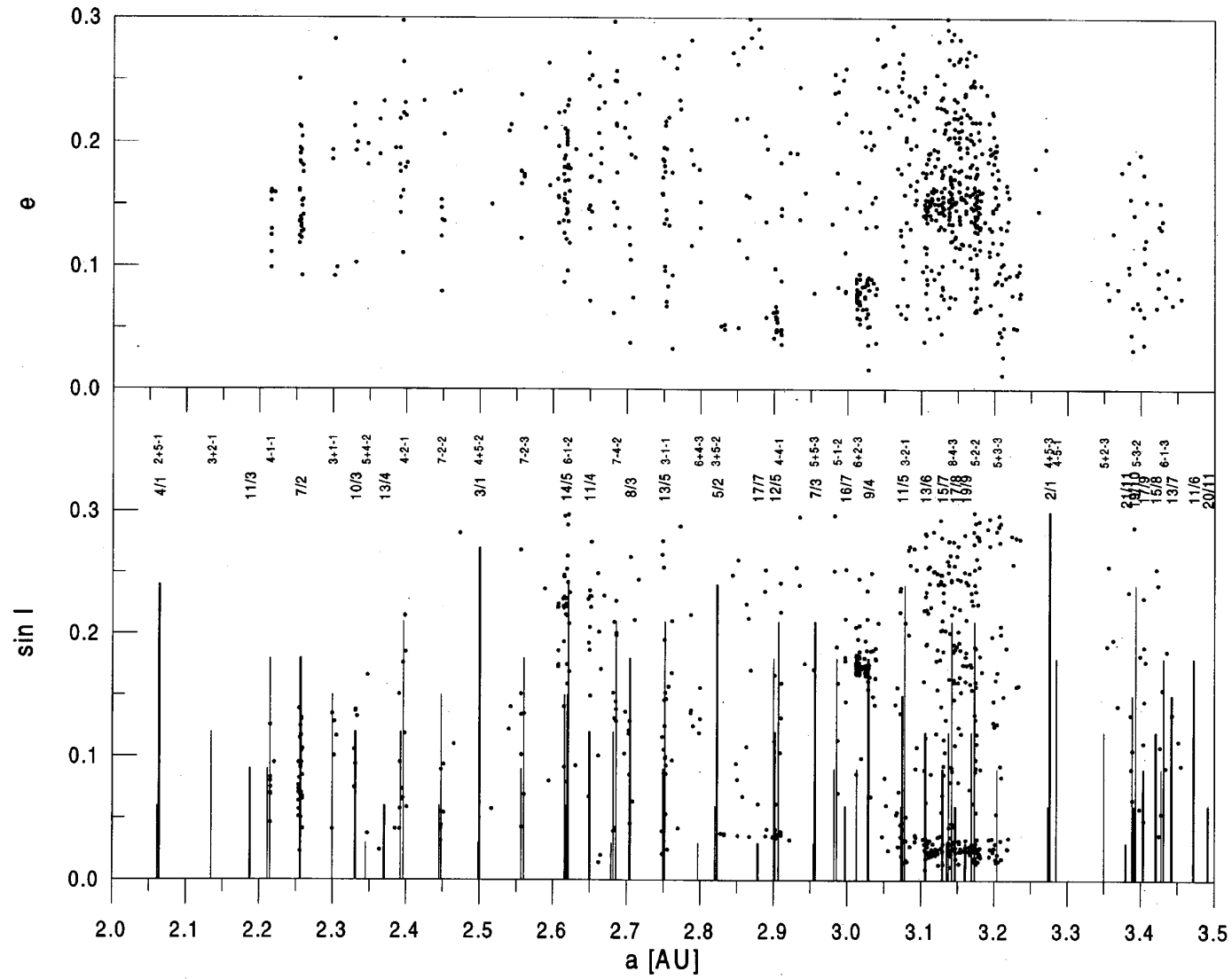


FIG. 5.—Chaotic asteroids with the fast Lyapunov indicator larger than 10^8 (*points*) and the location of the main ordinary and three-body mean motion resonances. The height of the resonant bars is scaled as in Fig. 3, but only some of the resonances are reported.