

Dynamics Near the Librations Points

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Recent Trends in Nonlinear Science

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Course Content

- Lecture 0. Motivation. Some interplanetary space missions
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Applications
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- Lecture 5. The phase space around the equilibrium points of the CR3BP

The slides of the lectures will not be published in the RTNS workshop website. If you are interested in them, please put in touch with either Elena Fantino (elena.fantino@kustar.ac.ae) or Gerard Gómez (gerard@maia.ub.es)

Lecture I. The n -body problem, the restricted n -body problem and other restricted models

I.1 The n -body problem

I.1.1 The general n -body problem

I.1.2 The n -body problem as a perturbation of Kepler's problem

I.1.3 The n -body problem as an autonomous Hamiltonian system

I-2 Autonomous models of the n -body problem ($n = 3$)

I.2.1 The restricted 3-body problem (RTBP)

I.2.2 Equilibrium points and zero-velocity surfaces (curves)

I.2.3 The restricted 3-body problem as an autonomous Hamiltonian system

I.2.4 A limit case: Hill's problem

I.2.5 The restricted 3-body problem for a solar sail

I.3 Non-autonomous models of the n -body problem

I.3.1 The bicircular 4-body problem (BCP)

I.3.2 The quasi-bicircular 4-body problem (QBCP)

I.3.3 The n -body problem as a perturbation of the RTBP

Lecture I. References

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Lecture II. Computational tools. Equilibrium points, periodic and quasi-periodic orbits

- 2.1 Continuous and discrete dynamical systems
- 2.2 Equilibrium and fixed points
- 2.3 Periodic orbits
- 2.4 2D Invariant tori
- 2.5 Computation of stable and unstable invariant manifolds
- 2.6 Dynamical substitutes of the equilibrium points

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Lecture III. Computational tools. The Lindstedt-Poincaré method. Applications

- 3.1 Problem setting and an academic example
- 3.2 Application of the LP method to the CR3BP
- 3.3 Lissajous orbits
- 3.4 Halo orbits
- 3.5 Quasi-halo orbits
- 3.6 Orbits and manifolds
- 3.7 Elliptic HCW equations

Lecture III. References

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2. D.L. Richardson. A Note for a Lagrangian Formulation for Motion about the Collinear Points. *Celestial Mechanics and Dynamical Astronomy*, 22(3):231–236, 1980.
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Lecture V. The phase space around the equilibrium points of the CR3BP

- V.1 The equilibrium points of the circular restricted 3-body problem
- V.2 The linear dynamics around the collinear equilibrium points
- V.3 The linear dynamics around the equilateral equilibrium points
- V.4 Nonlinear dynamics near the collinear points
- V.5 Reduction to the centre manifold
- V.6 The triangular equilibrium points

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