Models for return maps near a generalized homoclinic tangency for 3D-diffeomorphisms

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(joint work with J. A. Rodríguez, J. C. Tatjer*, E. Vigil)

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Unfolding homoclinic tangencies: The 2D-case revisited

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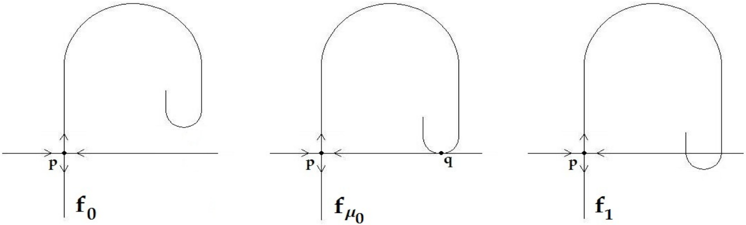

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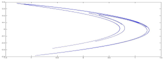
Persistent strange attractors (but not infinitely many) MORA,  VIANA...

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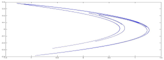

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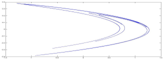
VIANA... 

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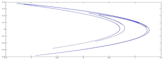

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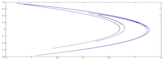
VIANA... 

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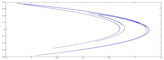

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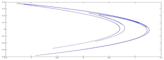

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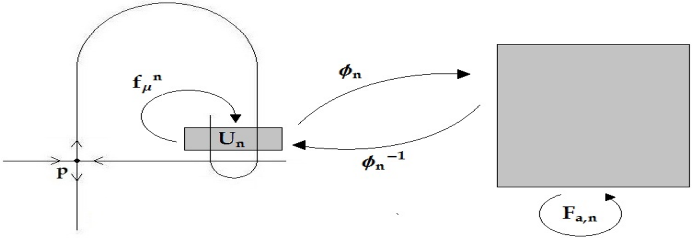
Most of the above results have a common mathematical tool:

The existence of a family of limit return maps associated to the unfolding of homoclinic tangency

Unfolding homoclinic tangencies:The 2D-case revisited

The limit return map for the 2D-case



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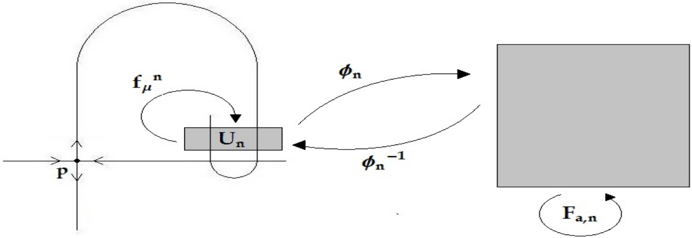

- Reparametrization near the parameters for which there is the homoclinic tangency,

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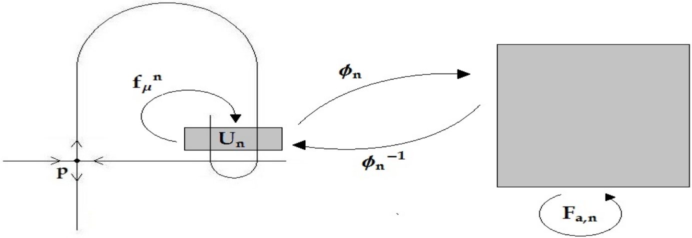

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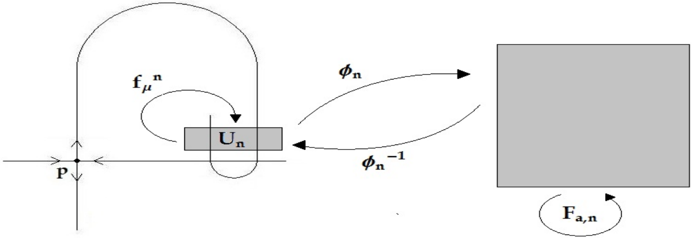
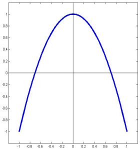


- Reparametrization near the parameters for which there is the homoclinic tangency,
- Change of scale near a point of the homoclinic orbit.

$F_{a,n} = \Phi_n \circ f_\mu^n \circ \Phi_n^{-1}$ $\quad lim_{n\to\infty} F_{a,n} = (f_a(x), 0)$ $\quad f_a(x) = 1 - ax^2$

Unfolding homoclinic tangencies:The 2D-case revisited

The family $f_a(x) = 1 - ax^2$ is the well-known one-dimensional quadratic family



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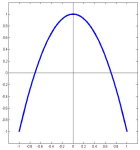

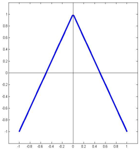

For many values of the parameter a (for instance, for $a = 2$), the quadratic map is conjugate to the piecewise linear map (one-dimensional tent map) $\lambda(x) = 1 - a|x|$

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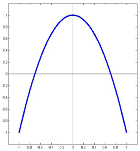

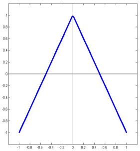

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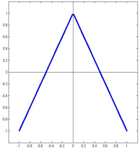
In any case, a natural approach to understand the dynamics of the quadratic family is to obtain a complete description of the dynamics of the one dimensional tent maps.

Unfolding homoclinic tangencies:The 2D-case revisited

Hence, the long travel (more than fifty years...) from a single map



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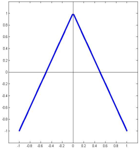
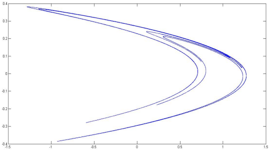


to a strange attractor (those ones arising in the unfolding of a homoclinic tangency)



is complete

Off the record

These ideas were the starting point for proving that the proper HENON MAP

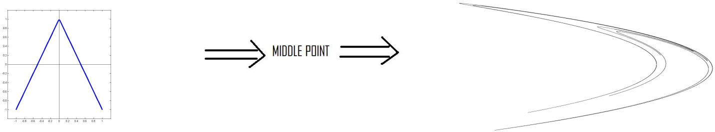
$$H_{a,b}(x,y) = (1 - ax^2 + y, bx)$$

exhibits a STRANGE ATTRACTOR for la large range of parameters (a, b). In:
Benedicks, M. and Carleson, L.- The dynamics of the Henon map. Ann. Math, 1991
the authors strongly use the idea that, after a simple change of coordinates, the Henon family can be written as

$$H_{a,b}(x,y) = (1 - ax^2 + \sqrt{b}y, \sqrt{b}x)$$

which is, for $0 < b << 1$ very close to $(f_a(x), 0) = (1 - ax^2, 0)$.

Unfolding homoclinic tangencies:The 2D-case revisited



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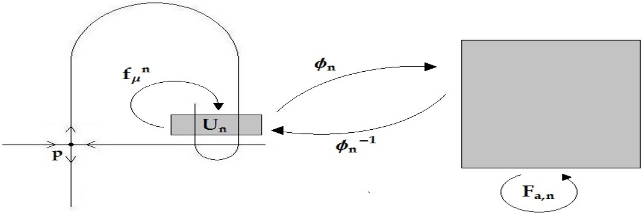

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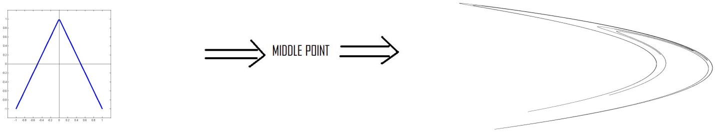

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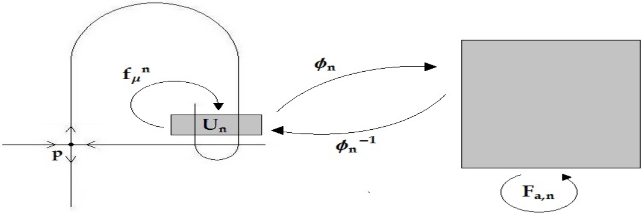

PALIS, TAKENS, NEWHOUSE, MORA, VIANA, COLLI, YOCCOZ, TEDESCHINI-LALI, YORKE, ALLIGOOD, JAKOBSON, DIAZ, ROCHA, ROMERO, MAÑE, PACIFICO, ROVELLA, URES,...and many many others...

THE MAIN OBJECTIVE OF THIS TALK

Dessing the same kind of travel for the case of the unfolding tangencies in the 3D-framework

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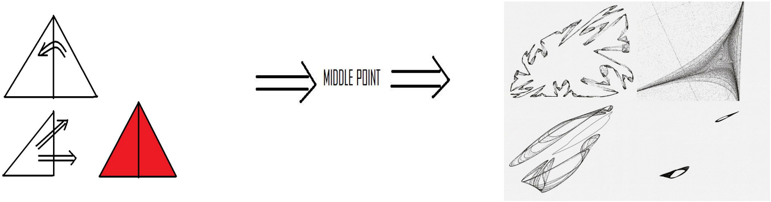

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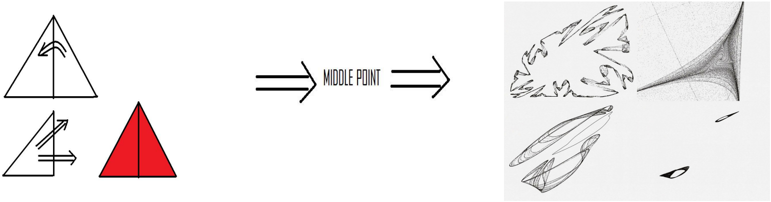

MIDDLE POINT: Families of limit return maps for the unfolding of certain homoclinic bifurcation in dimension three.

THE 3-D MIDDLE POINT

Let us start this travel just in the middle point: Families of limit return maps for the unfolding of certain class of homoclinic tangencies in 3-D.

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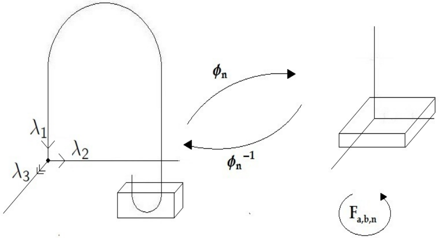

$$|\lambda_1 \lambda_2 \lambda_3| < 1$$
$$|\lambda_1| < 1 < |\lambda_2| < |\lambda_3|$$

J. C. Tatjer.- Three-dimensional dissipative diffeomorphisms with homoclinic tangencies. Ergodic Theory and Dynamical Systems, 21. 2001.

THE 3-D MIDDLE POINT

In our case the family of limit return maps is

$$\lim_{n\to\infty} F_{a,b,n} = f_{a,b}(x, y, z) = (z, a + by + z^2, y)$$

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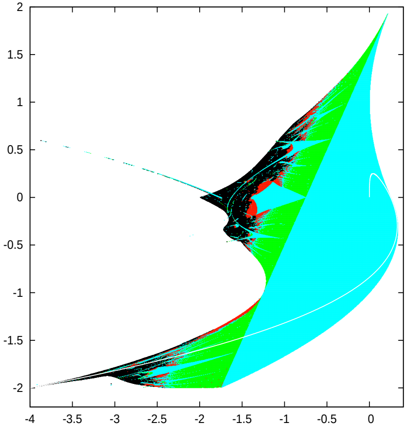
Therefore, our family of limit return maps reduce (as expected) to a bidimensional transformation given by

$$T_{a,b}(x,y) = (a+y^2, x+by),$$

Let us start by drawing the subset of parameters (a,b), for which the map $T_{a,b}$ has an attractor.

The space of parameters

Region in the plane a, b for which $T_{a,b}$ has attractors



- Blue: attracting periodic point.
- Green: Attracting invariant union of closed curves.
- Red: 1D strange attractor.
- Black: 2D strange attractor.

Some of the pathologies

Some of the attractors numerically detected for $T_{a,b}$. Details can be found in

Tatjer and P.- Attractors for return maps near homoclinic tangencies of three-dimensional dissipative diffeomorphisms. Discrete and continuous dynamical systems, series B, 2007.

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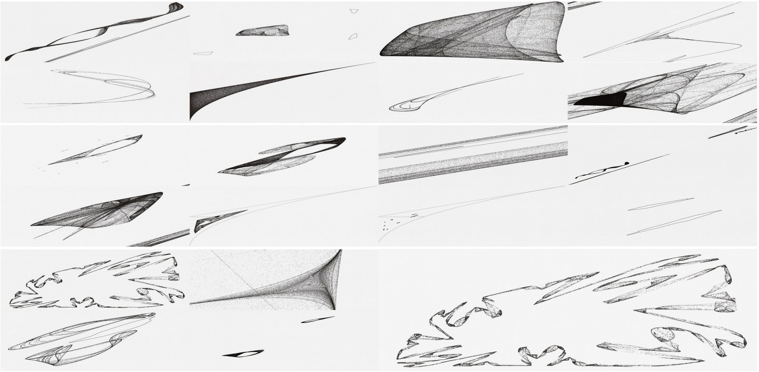
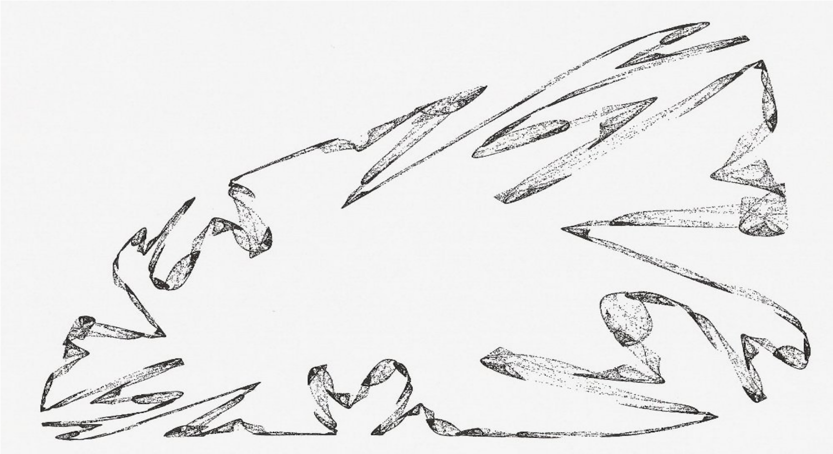
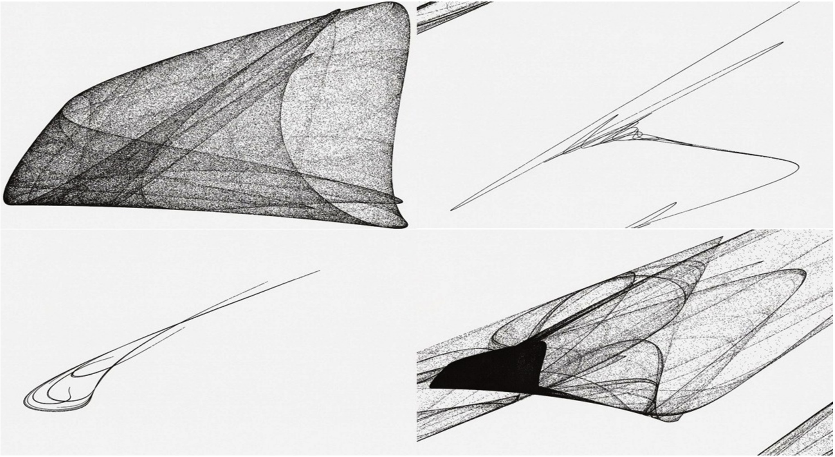
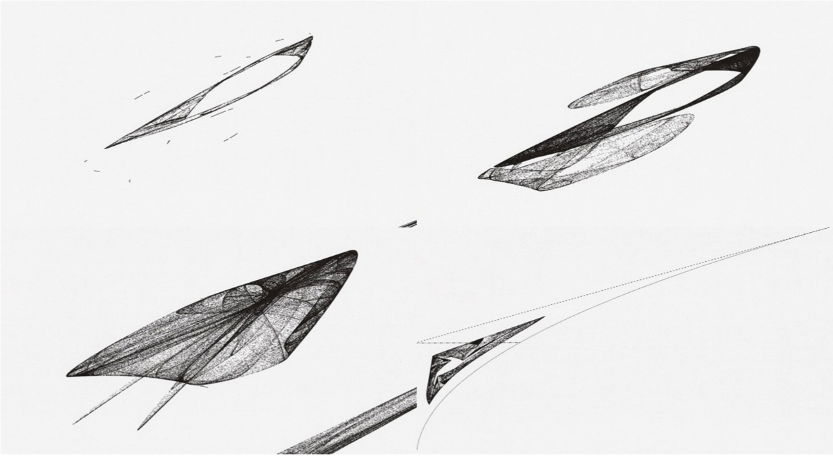
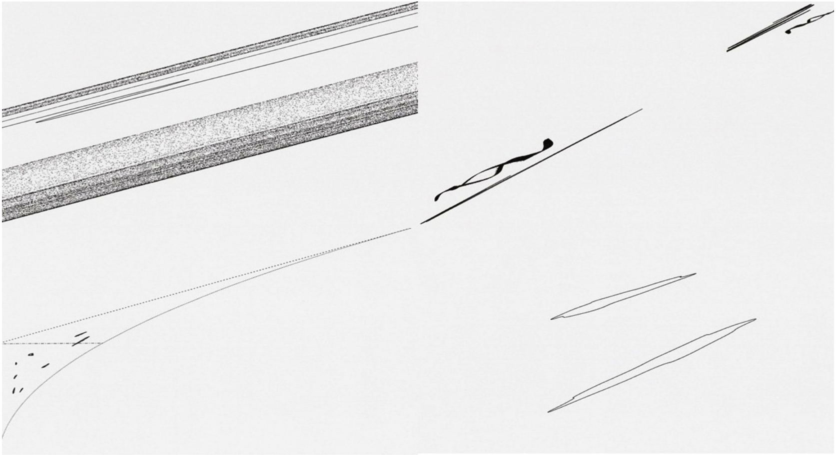
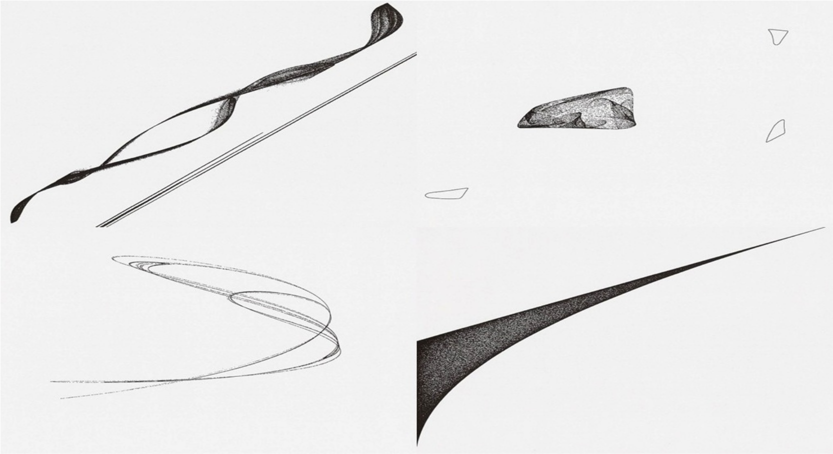










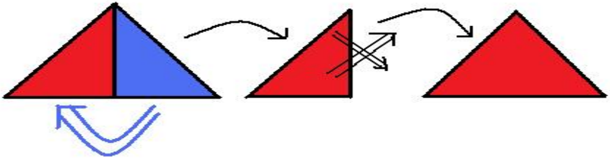


A special value of the parameters $(a, b) = (-4, -2)$

$T_{-4,-2}$ is conjugate to the following trasformation: Let
$T_0 = \{(u, v) : 0 \le u \le 1, 0 \le v \le u\}$ (the red one),
$T_1 = \{(u, v) : 1 \le u \le 2, 0 \le v \le (2 - u)\}$ (the blue one) and
$T = T_0 \cup T_1$. Then, $T_{-4,-2}$ is conjugate to

$$\Lambda = A \circ \mathcal{S},$$

whit $\mathcal{S}(u, v) = \begin{cases} (u, v) & \text{if} \quad (u, v) \in T_0 \\ (2 - u, v) & \text{if} \quad (u, v) \in T_1 \end{cases}$, $A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.



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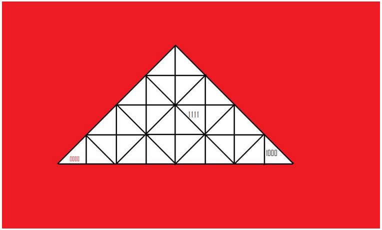

Tatjer, J. C. & P.- Dynamics near homoclinic bifurcations of three-dimensional dissipative diffeomorphisms. Nonlinearity, 2006.

1. The mat Λ (and therefore $T_{-4,-2}$) is conjugate to the one sided shift with two symbols.
2. For almost all (u, v) the Lyapunov exponent of Λ along the orbit of (u, v) is positive (in fact, $\frac{1}{2} \log 2$) in all nonzero direction. The same holds for the return map $T_{a,b}$.
3. The map Λ (or $T_{-4,-2}$) has an absolutely continuous ergodic invariant measure.

LET US CALL Λ, THE BIDIMENSIONAL TENT MAP

THE BIDIMENSIONAL TENT MAP

As far as we know, this is the first example of a bidimensional map with a strange attractor with two positive Lyapounov exponents, which neither it nor any of its powers is C^1-conjugate to a "skew-product" map $SK(x, y) = (h_1(x), h_2(x, y))$
Moreover,

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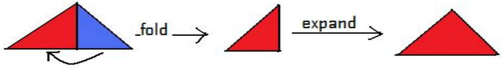

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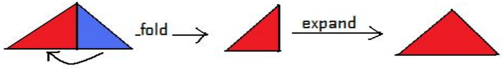

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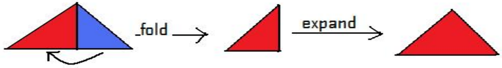

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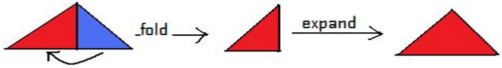

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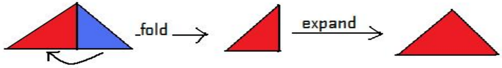

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HE IS NOW HERE.....

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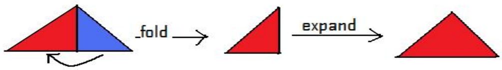

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BUT...... SOMEONE SAID NO.....
HE IS NOW HERE.....
PROBABLY ON THE FRONT ROW...

BAKER MAP IS NOT A GOOD NAME

He said to us that, more than fifty years ago, someone used the name of BAKER MAP to refer to certain conservative bidimensional transformation.

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BUT........................... This original (and so, the authentic) Baker map is not continuous. In other words... The baker needs a knife and the bread dough has to be surgery.

ALTERNATIVE NAMES FOR OUR MAP

Therefore, we thought in using one of the following alternatives for our (continuous, non operated, pacific...) transformation:

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THE NATURAL BAKER MAP (Natural in the sense that the dough never has visited the operated room)

EXPANDING BAKER MAP IS THE BEST CHOICE

Fortunately, Joan Carles Tatjer uses his famous good sense and proposes the best choice to baptize our map under the name of EXPANDING BAKER MAP (although the baker probably is not too much happy).

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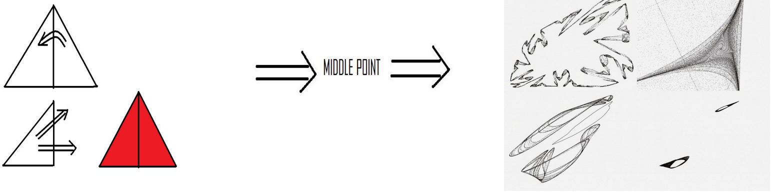
This is, of course, the starting point of the long travel...we have to make



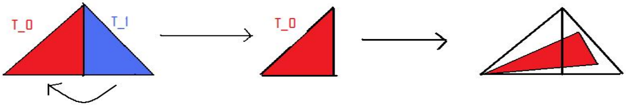
LONG FAMILIES OF EXPANDING BAKER MAPS (EBM)

We extend the EBM, $\Lambda = A \circ \mathcal{S}$, to any map, $\tilde{\Lambda} = \tilde{A} \circ \mathcal{S}$, with

$$\mathcal{S}(u,v) = \begin{cases} (u,v) & \text{if } (u,v) \in T_0 \\ (2-u,v) & \text{if } (u,v) \in T_1 \end{cases}, \quad \tilde{A} = \begin{pmatrix} a & b \\ c & d \end{pmatrix},$$

whenever

$$\tilde{A}(T_0) \subset (T_0 \bigcup T_1) = T$$



A initial choice could be $\Lambda_t = \tilde{A} \circ \mathcal{S}$, with

$$\tilde{A} = A_t = \begin{pmatrix} t & t \\ t & -t \end{pmatrix},$$

for $0 \leq t \leq 1$, noting that, when, $t = 1$ we recover the bidimensional tent map Λ.

First properties of these last EBM Λ_t

1. The triangle $T = T_0 \cup T_1$ is invariant.

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5. All the periodic orbits with no critical points are repelling if $t > 1/\sqrt{2}$.

Evolution for the attractor of Λ_t:TYPES OF BREADS

As we said before, for $0 \leq t < 1/\sqrt{2}$, $(0,0)$ the origin is a global attractor.

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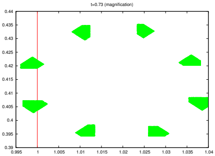
For $1/\sqrt{2} < t < (1/4)^{1/5}$, we have a 8-pieces strange attractor.



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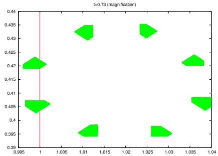

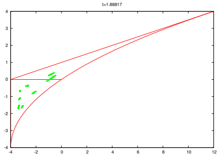

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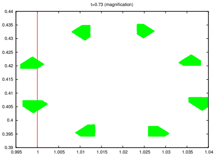

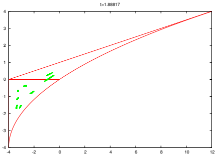

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Fairy Cakes Attractor

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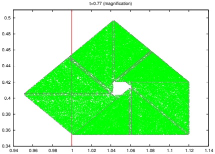

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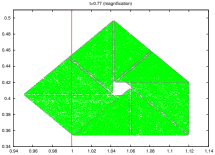

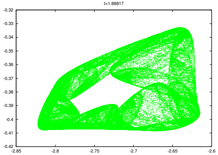

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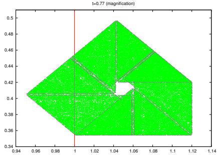

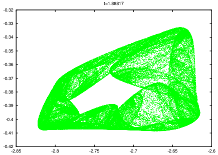

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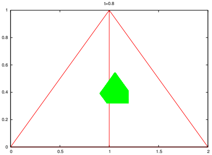
Bread roll's attractor
Typical spanish
"rosca"

Evolution for the attractor of Λ_t:TYPES OF BREADS

For $(1/2)^{1/3} < t < 1$, we have a 1-piece strange attractor without holes:



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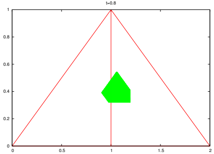

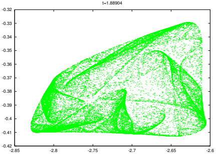

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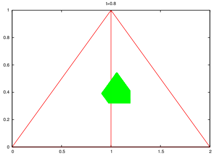

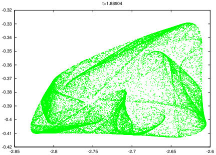

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Hogaza's attractor
None translation
founded.

Final comments

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- For EBM it is enough (as in the one dimensional case) to control the orbits of the critical points in order to capture attractors.
- Using the results of Tsujii (Invent. math. 143 (2001)) it seems possible to prove that the invariant sets that we find have absolutely continuous invariant measures.

More final comments

- Choosing different expanding matrices A in the definition of EBMs it is also possible to produce what seems to be 1-D strange attractors. For instance, if we take
  $\Lambda = A \circ \mathcal{S}$, with
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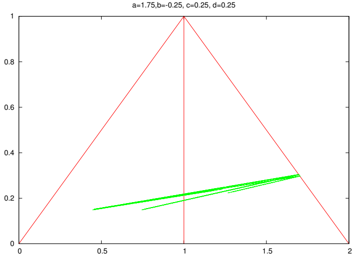

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e) R.: [-2.825,-2.616]x[-0.412,-0.328], t= 1.88762

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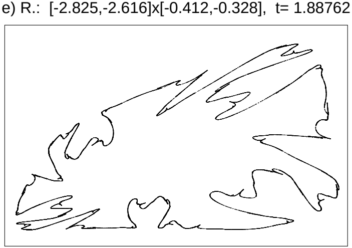

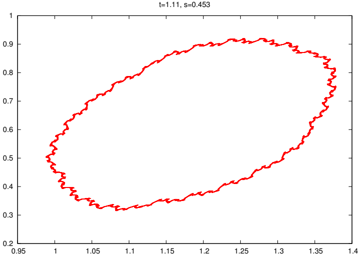

However, if we extend our family of EBMs to a 2D-parametric scenario new types of breads appear.

BREAD IS SERVED......THANK YOU VERY MUCH