The role of higher-order connectivity statistics in shaping neuronal network dynamics.

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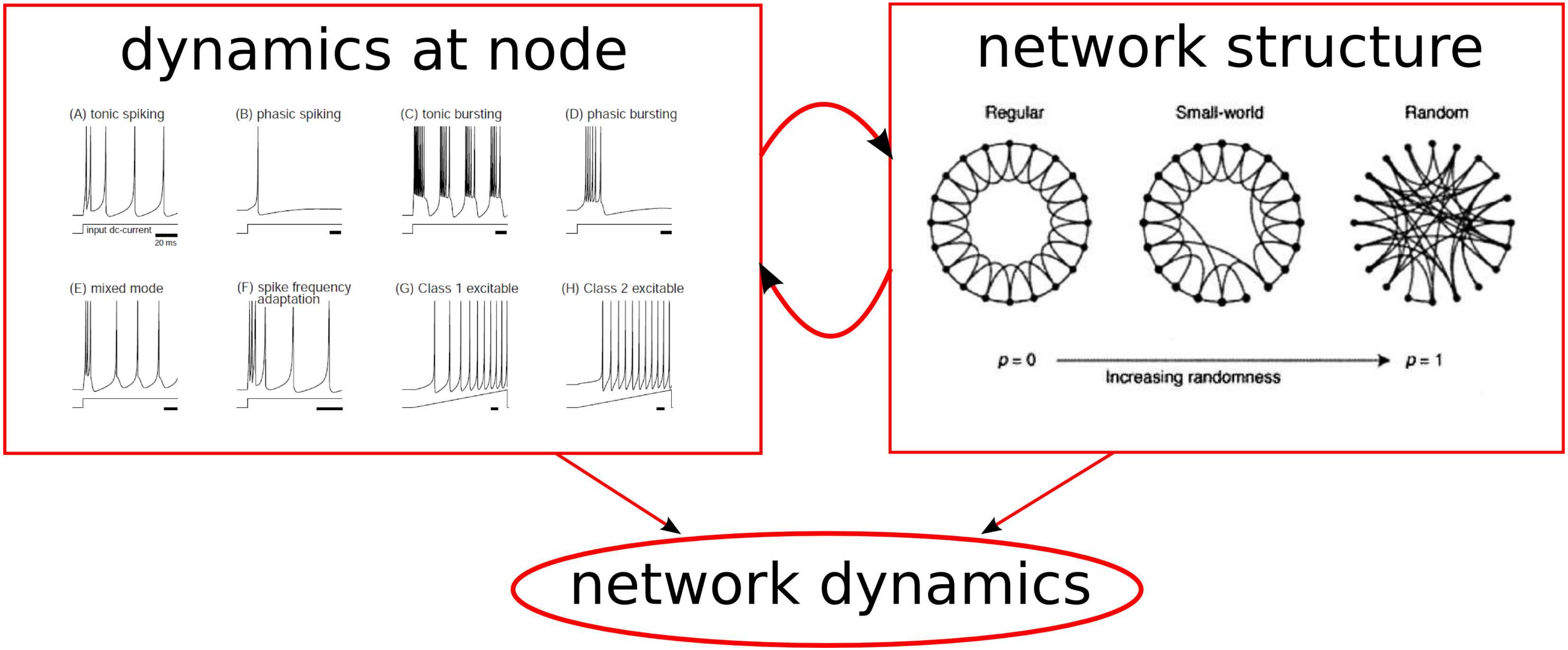
DDays, Benicàssim, October 2012.

Outline

1. What is a neuronal network?

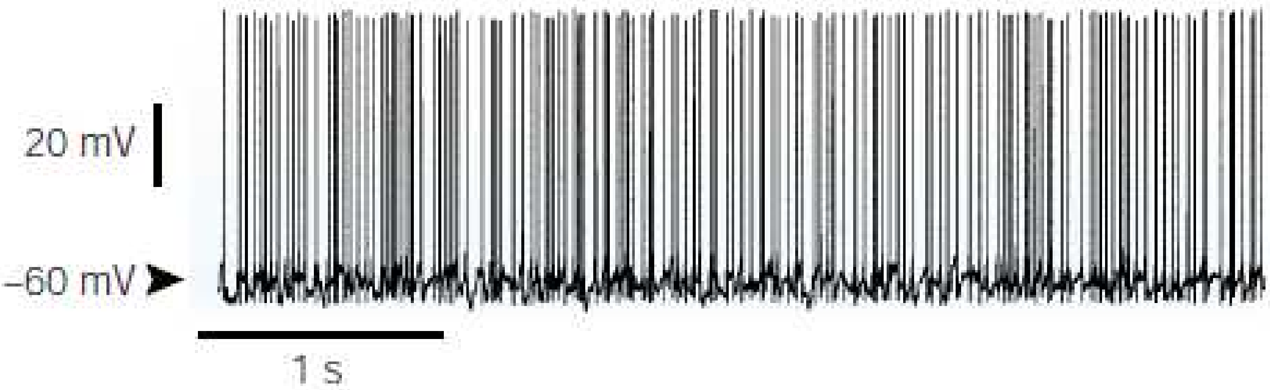
2. How do I capture the statistics of connectivity in a meanfield model?

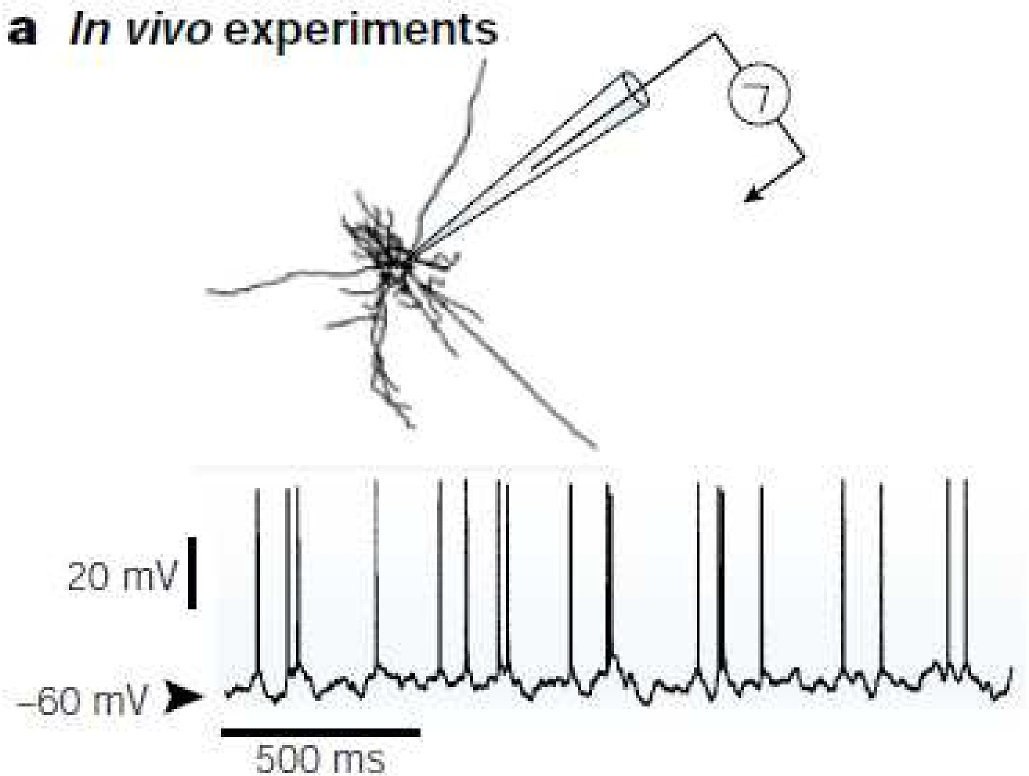

dynamics at node
(A) tonic spiking
input dc-current
20 ms
(B) phasic spiking
(C) tonic bursting
(D) phasic bursting
(E) mixed mode
(F) spike frequency adaptation
(G) Class 1 excitable
(H) Class 2 excitable
network structure
Regular
Small-world
Random
p = 0
p = 1
Increasing randomness
network dynamics

Not all possible node dynamics, nor all possible network structures may be relevant for understanding *neuronal network dynamics*.

So what do neuronal dynamics and network structure look like in the cerebral cortex?

Spiking dynamics of a single cell intracellular



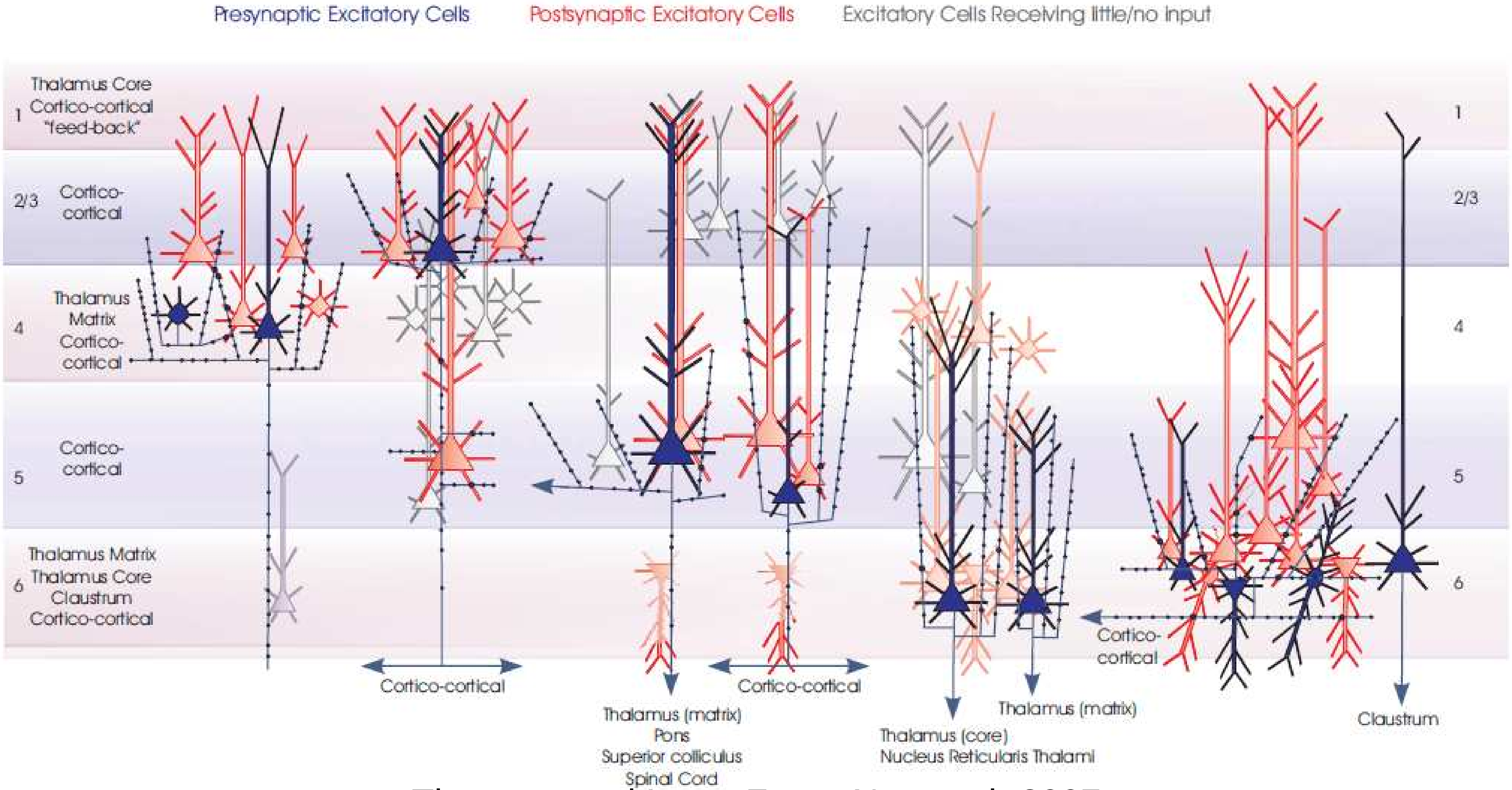


Destexhe et al., Nat. Rev. Neurosci. 2003

Summary of single-neuron dynamics

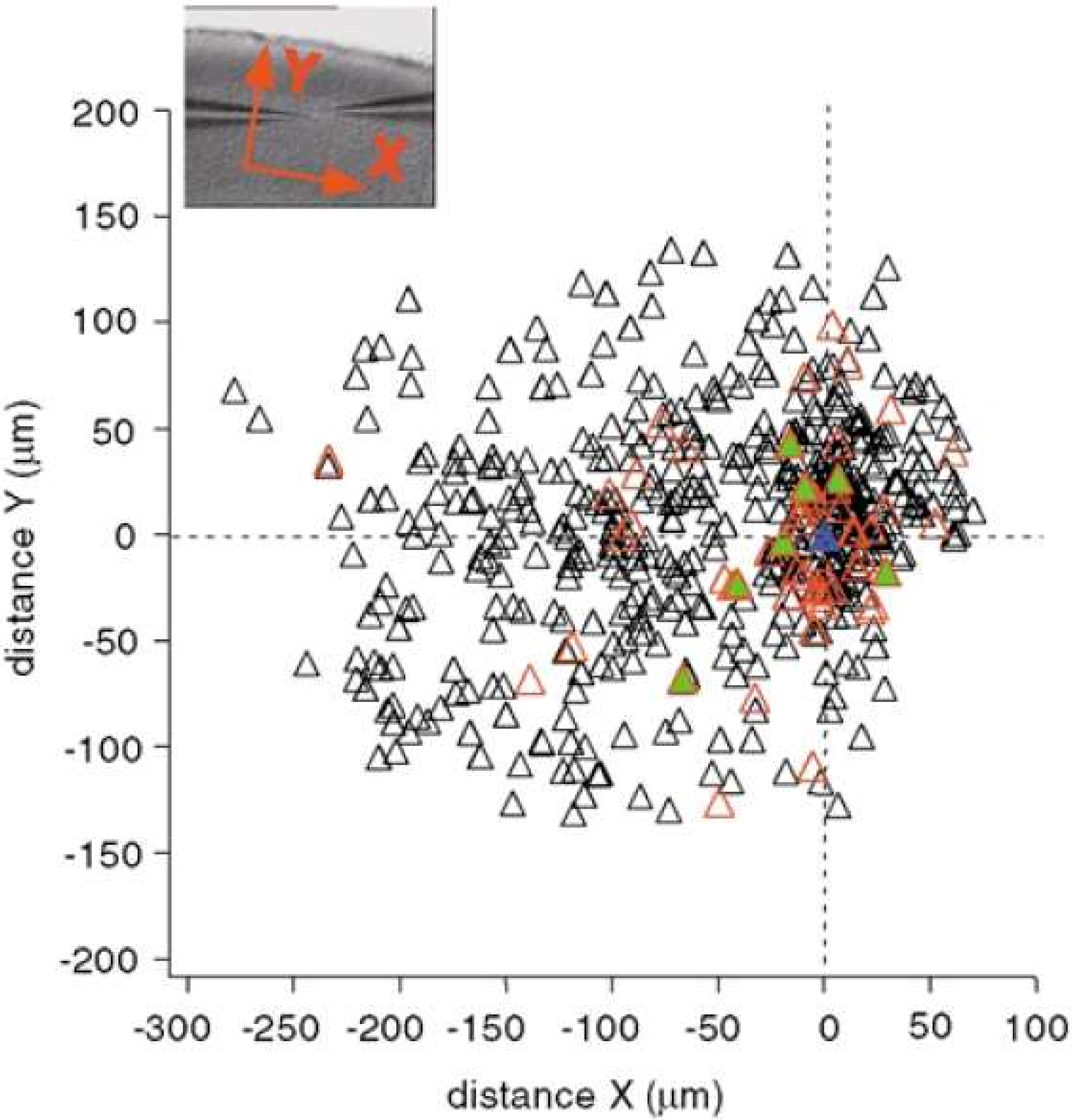
- Cortical spiking acivity is highly irregular (like Poisson process)
- Irregular firing is due to sub-threshold fluctuations (not oscillators)

Cortical network structure



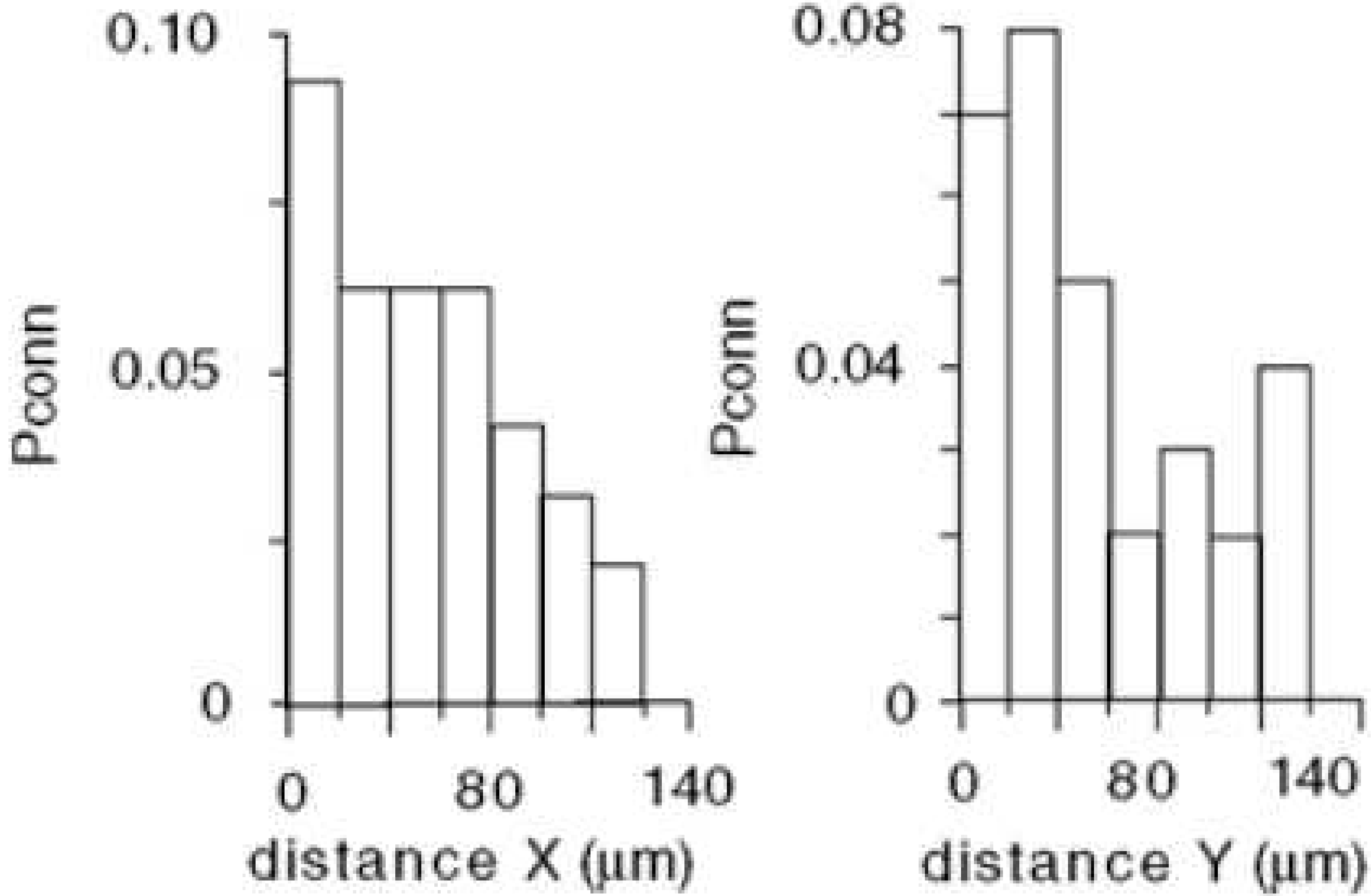
Thomson and Lamy Front. Neurosci. 2007

Cortical network structure



Holmgren et al. J. Neurophysiol. 2003 (rat layer 2/3)

Cortical network structure



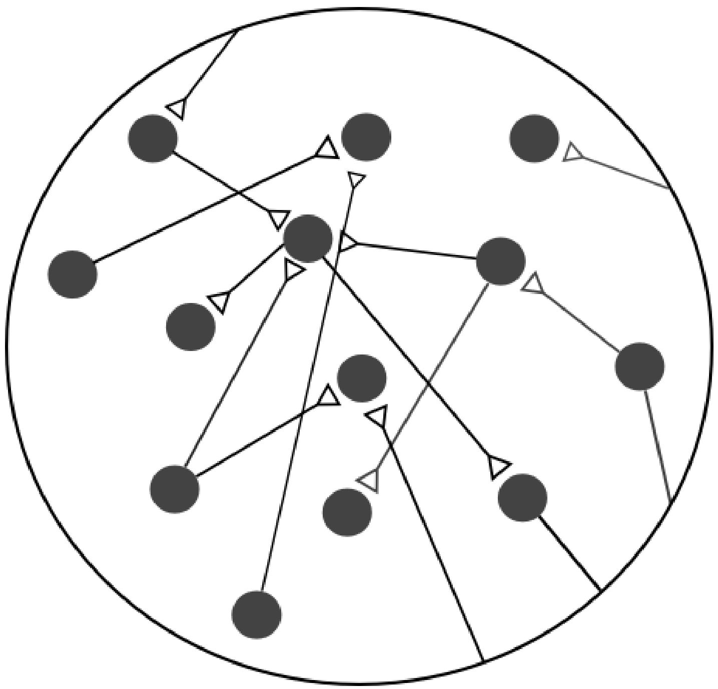
Holmgren et al. J. Neurophysiol. 2003 (rat layer 2/3)

Summary of cortical network structure

- highly specific layered structure
- locally sparse (unspecific?) connectivity

Cortical network models

- irregular spiking activity
- sparse (random?) connectivity



Amit and Brunel, Cereb. Cortex 1997

Amit and Brunel, Network 1997

Brunel and Hakim, Neural Comp. 1999

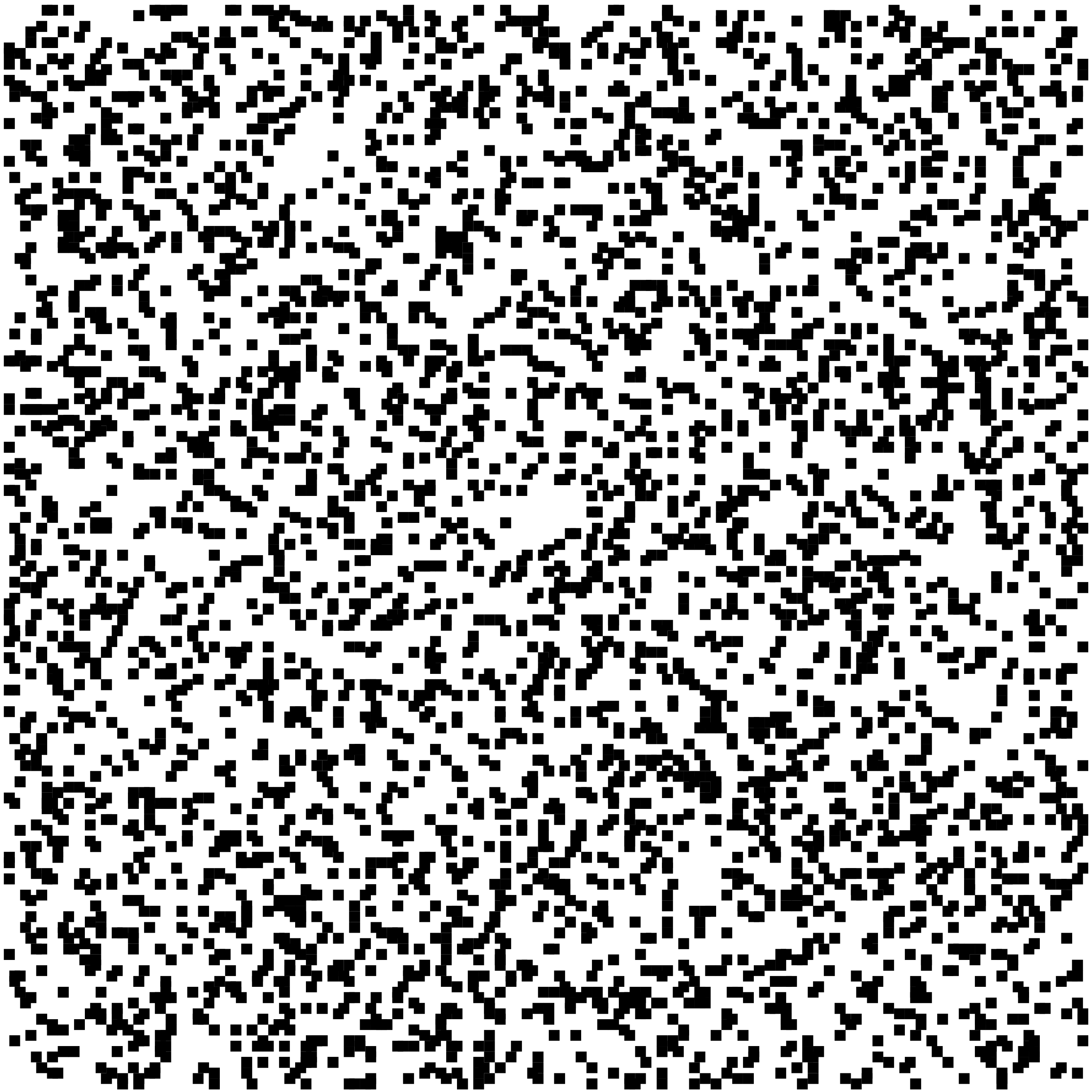
Brunel, J. Comp. Neurosci., 2000

Input to a neuron i

$$
\begin{aligned}
I_i(t) &= J \sum_{j=1}^{N} W_{ij} \sum_{k} s(t - t_j^k) + I_{ext}, \\
&= \bar{I} + \Delta I_i + \delta I_i(t)
\end{aligned}
$$

$W_{ij} \equiv$ connection from neuron j to i
$s(t) \equiv$ post-synaptic current
$\bar{I} \equiv$ population mean input = $J \cdot p \cdot N \cdot r(t) + I_{ext}$
$\Delta I_i \equiv$ quenched variability in inputs
$\delta I_i(t) \equiv$ temporal variability in inputs

typical W for Erdos-Renyi network



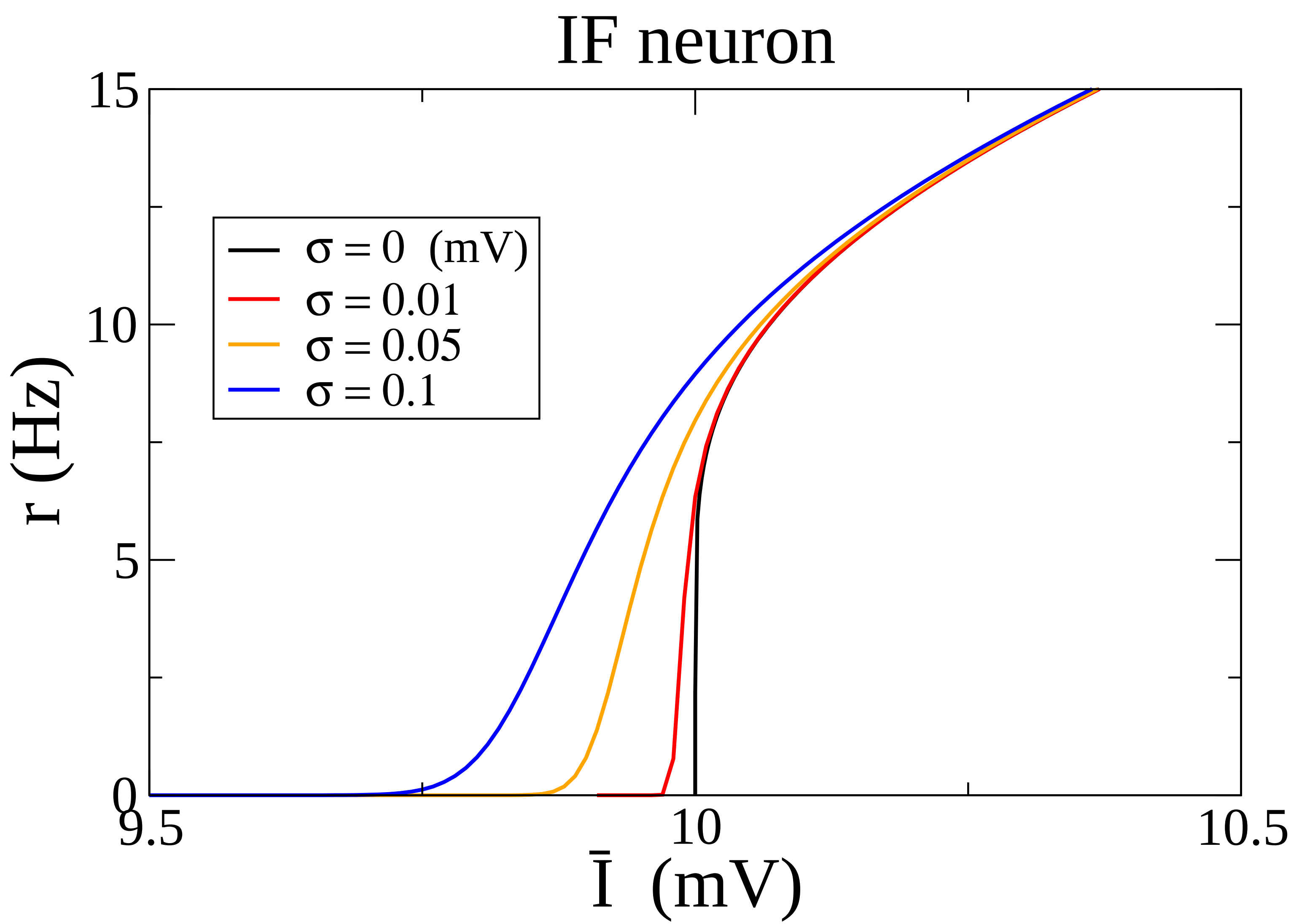
Input to a neuron i

$$
\begin{aligned}
I_i(t) &= J \sum_{j=1}^{N} W_{ij} \sum_k s(t - t_j^k) + I_{ext}, \\
&= \bar{I} + \Delta I_i + \delta I_i(t)
\end{aligned}
$$

$W_{ij} \equiv$ connection from neuron j to i $s(t) \equiv$ post-synaptic current
$\bar{I} \equiv$ population mean input = $J \cdot p \cdot N \cdot r(t) + I_{ext}$ $\Delta I_i \equiv$ quenched variability in inputs $\delta I_i(t) \equiv$ temporal variability in inputs

Self-consistent condition for mean rate

$$
\begin{aligned}
r &= \phi(\bar{I}), \\
&= \phi(JpNr + I_{ext})
\end{aligned}
$$



Self-consistent firing rate: $r = \phi(\bar{I}; \sigma)$

$\delta I_i(t) = \sigma \xi(t), \langle \xi(t) \xi(t-t') \rangle = \delta(t-t')$

Heuristic dynamics for mean rate

$$\tau \dot{r} \; = \; -r + \phi(JpNr + I_{ext})$$

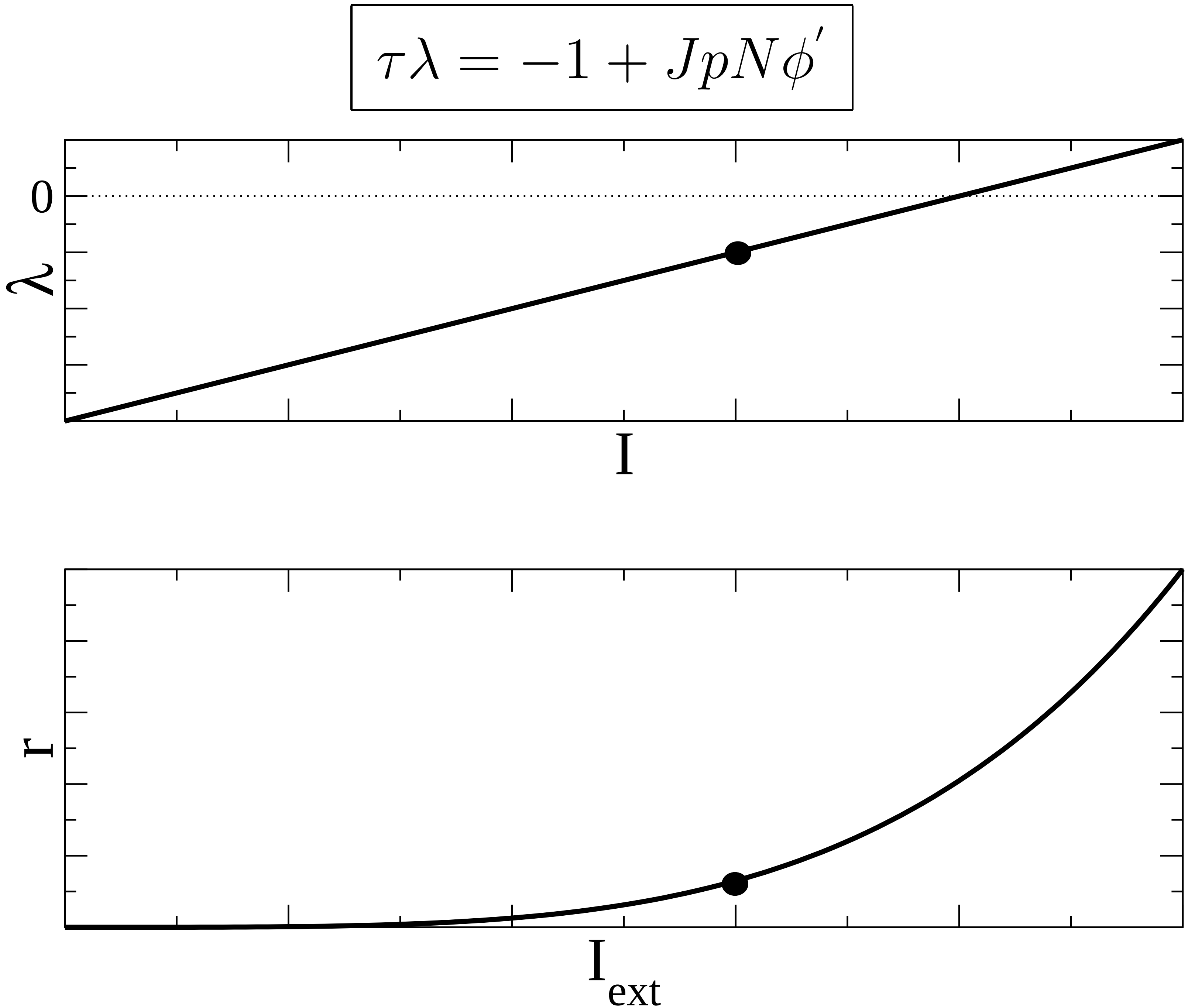
Linear stability of mean rate

$$\boxed{\tau \dot{r} = -r + \phi(JpNr + I_{ext})}$$

$$r(t) = R + \delta r e^{\lambda t}$$

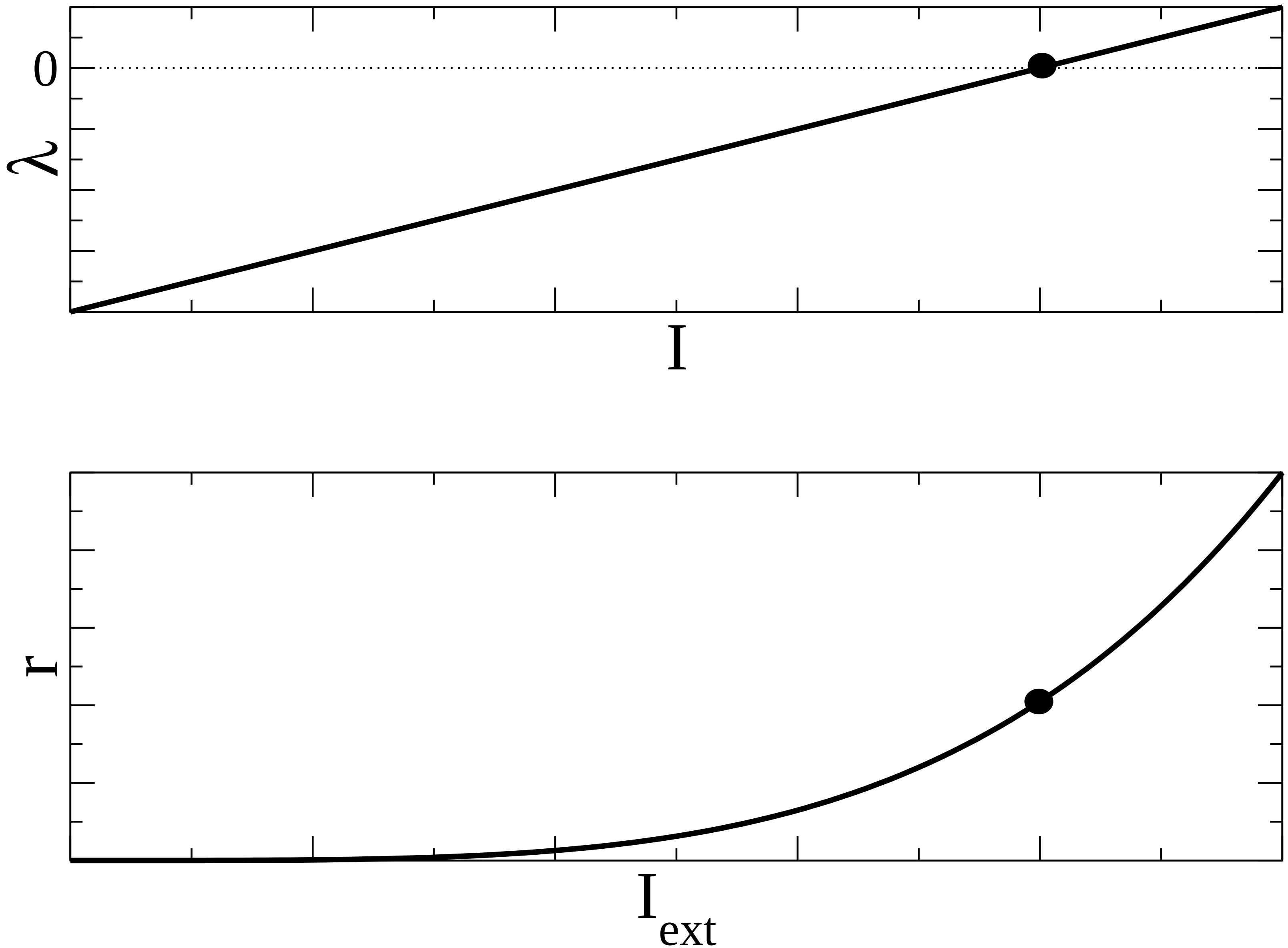
$$\boxed{\tau \lambda = -1 + JpN\phi'}$$

stability depends on the gain of the transfer function ϕ'



$\tau\lambda = -1 + JpN\phi'$
0
λ
I
r
I_{ext}

$$\tau\lambda = -1 + JpN\phi'$$



standard meanfield equations

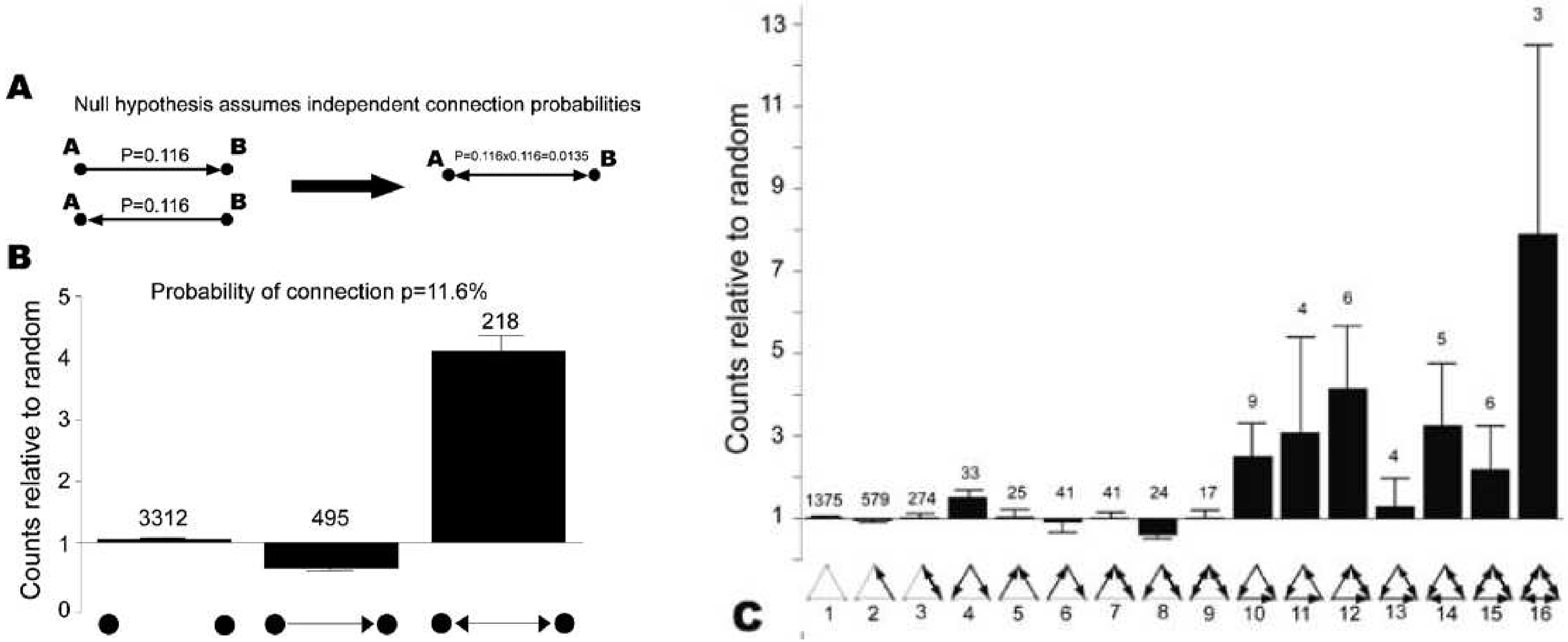
1. fast oscillations in inhibitory network

$$\dot{r} = -r + \phi\Big(- Jr(t-D) + I \Big)$$

2. E-I network (Wilson and Cowan, Biophys. J. 1972)

$$\begin{aligned} \tau_e \dot{r}_e &= -r_e + \phi_e\Big(J_{ee} r_e - J_{ei} r_i + I_e \Big) \\ \tau_i \dot{r}_i &= -r_i + \phi_i\Big(J_{ie} r_e - J_{ii} r_i + I_i \Big) \end{aligned}$$

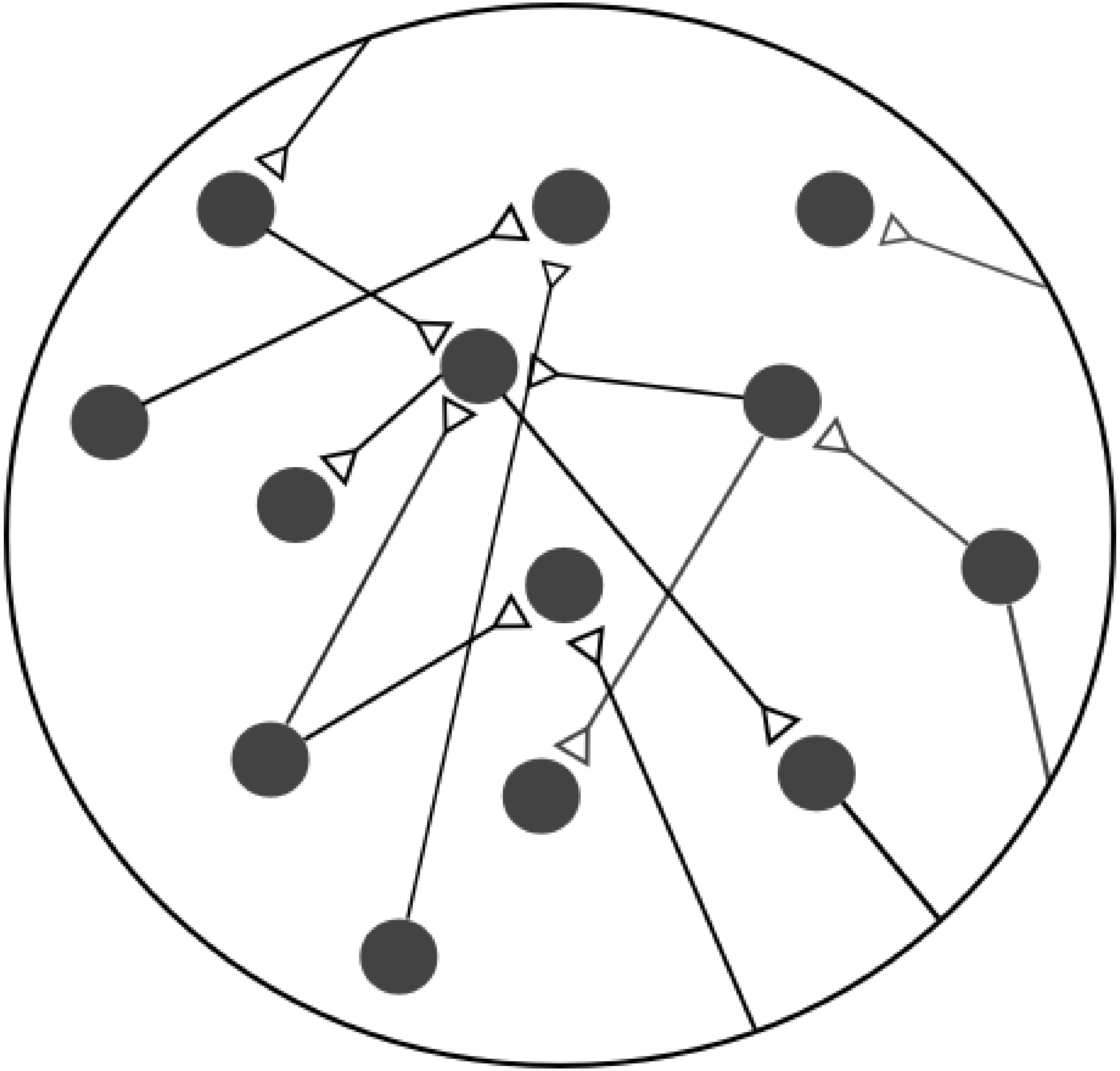
Heuristic meanfield models describe the qualitative dynamics in random network models, but there is growing evidence that cortical microcircuits are not standard random networks...



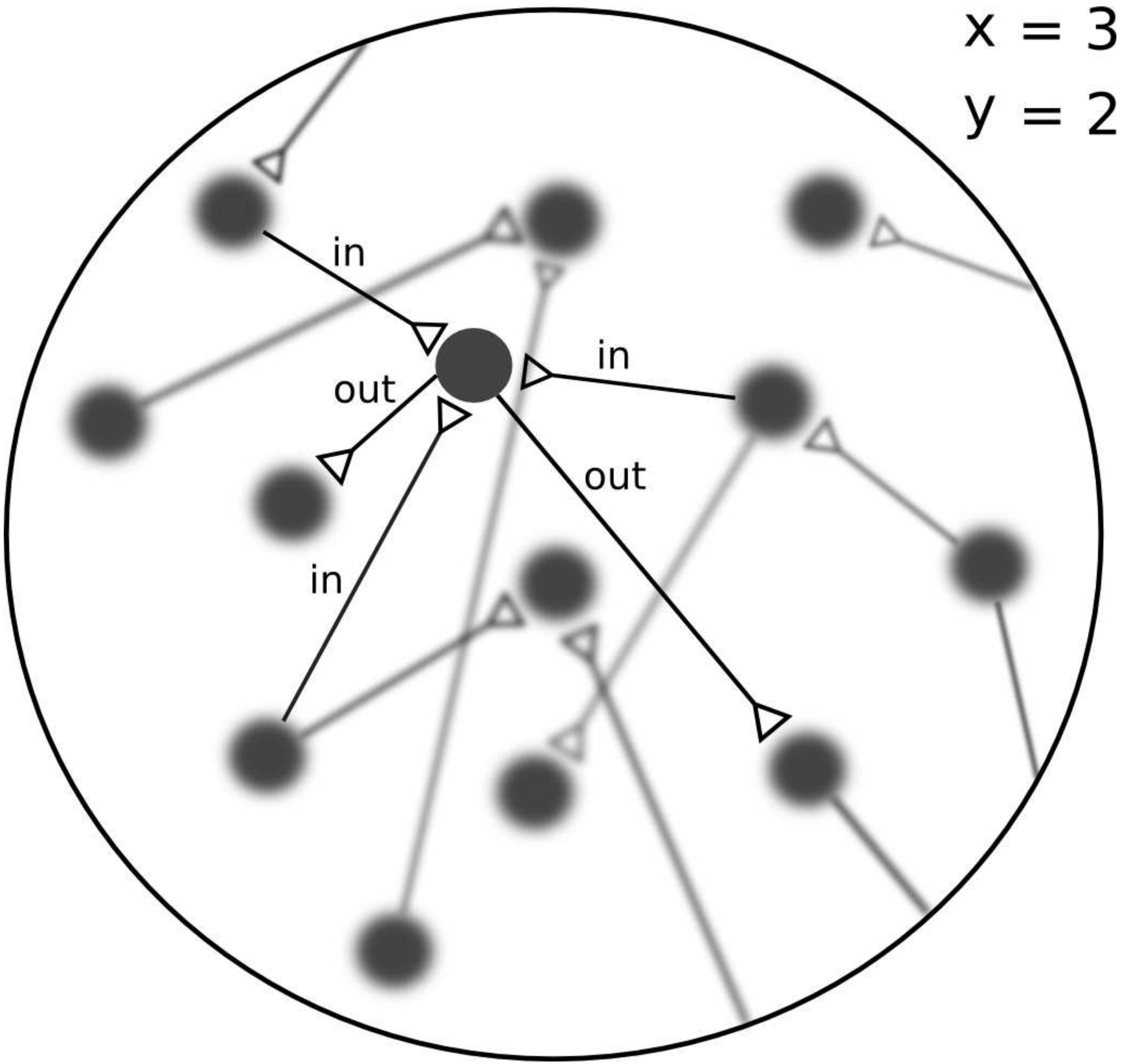
Song et al. PLoS Biol. 2005.

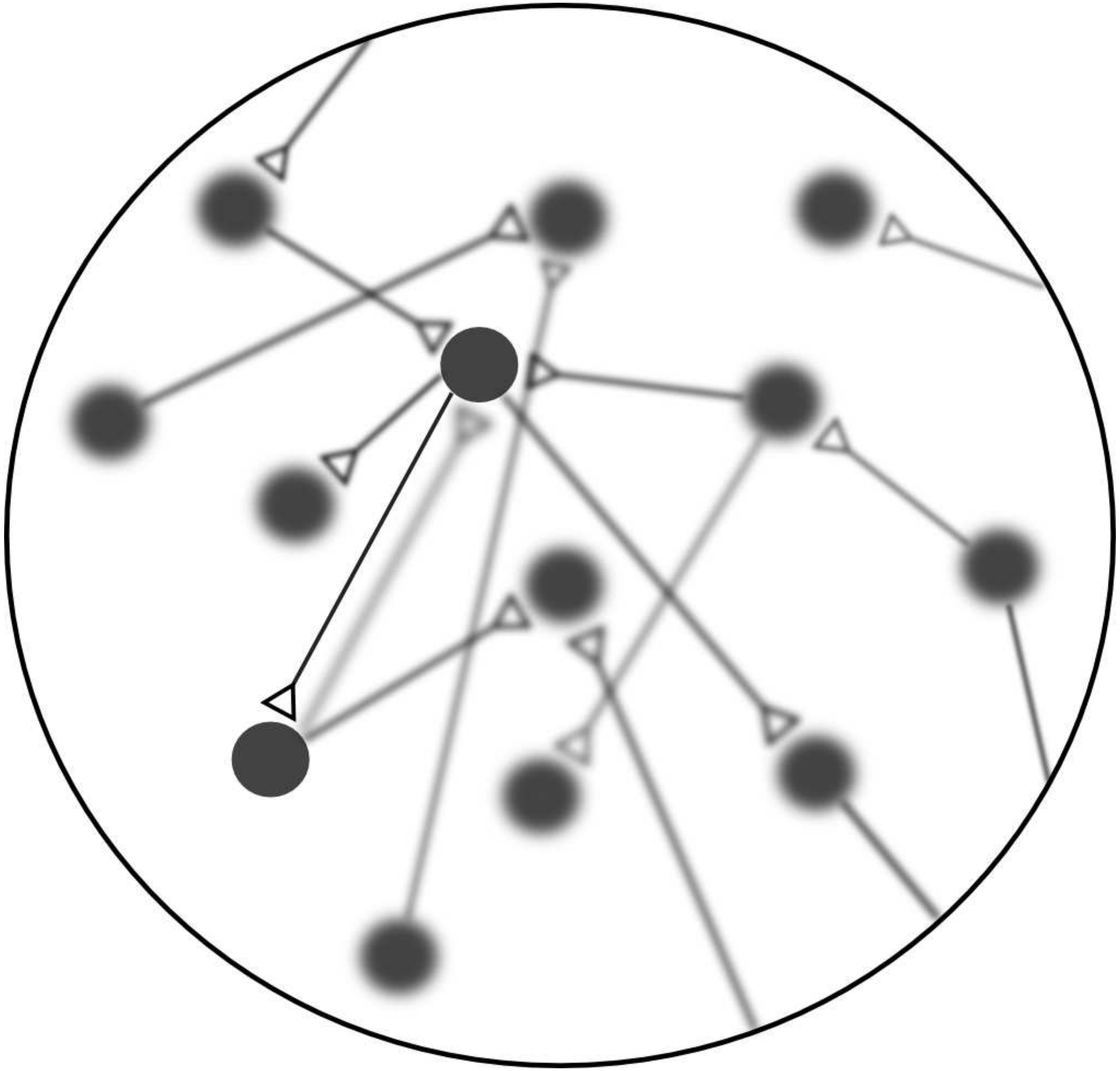
Higher-order statistics in random networks

Network connectivity. How to parameterize it?

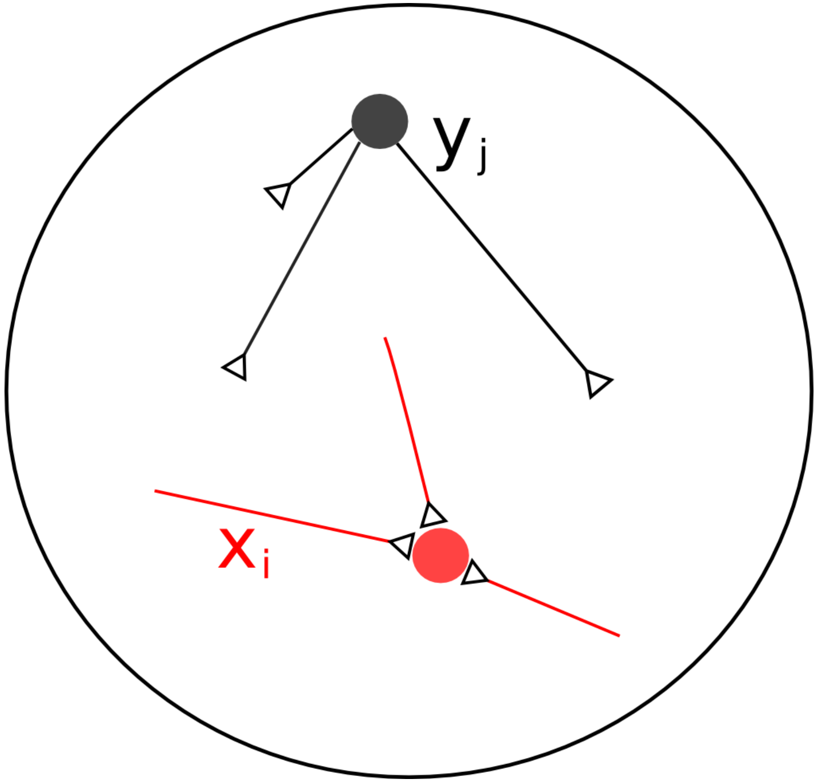


Consider degree distributions.



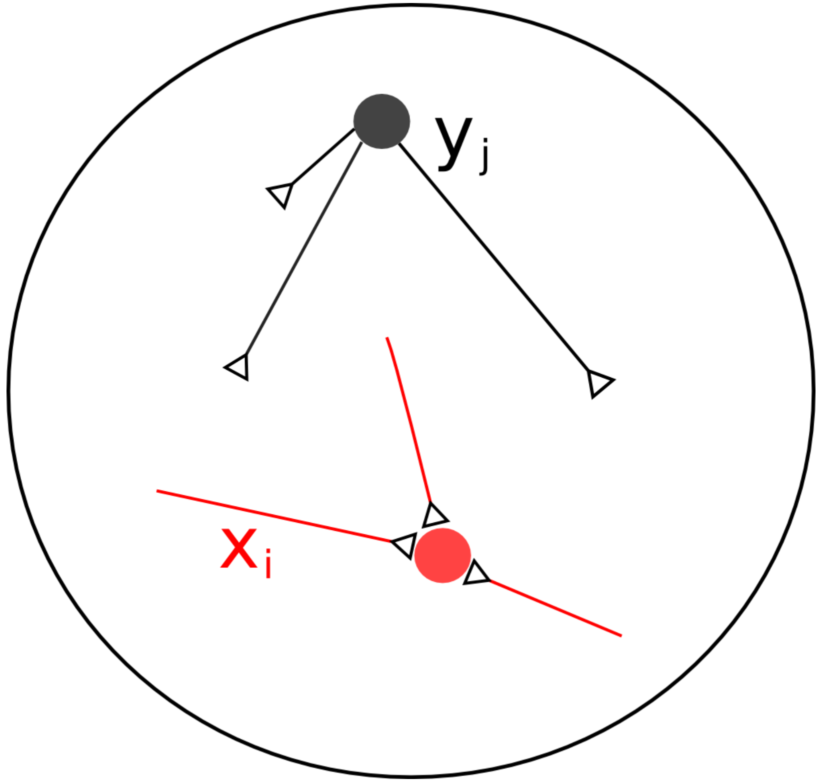


$p(w_{ij} = 1) \equiv$ sparseness

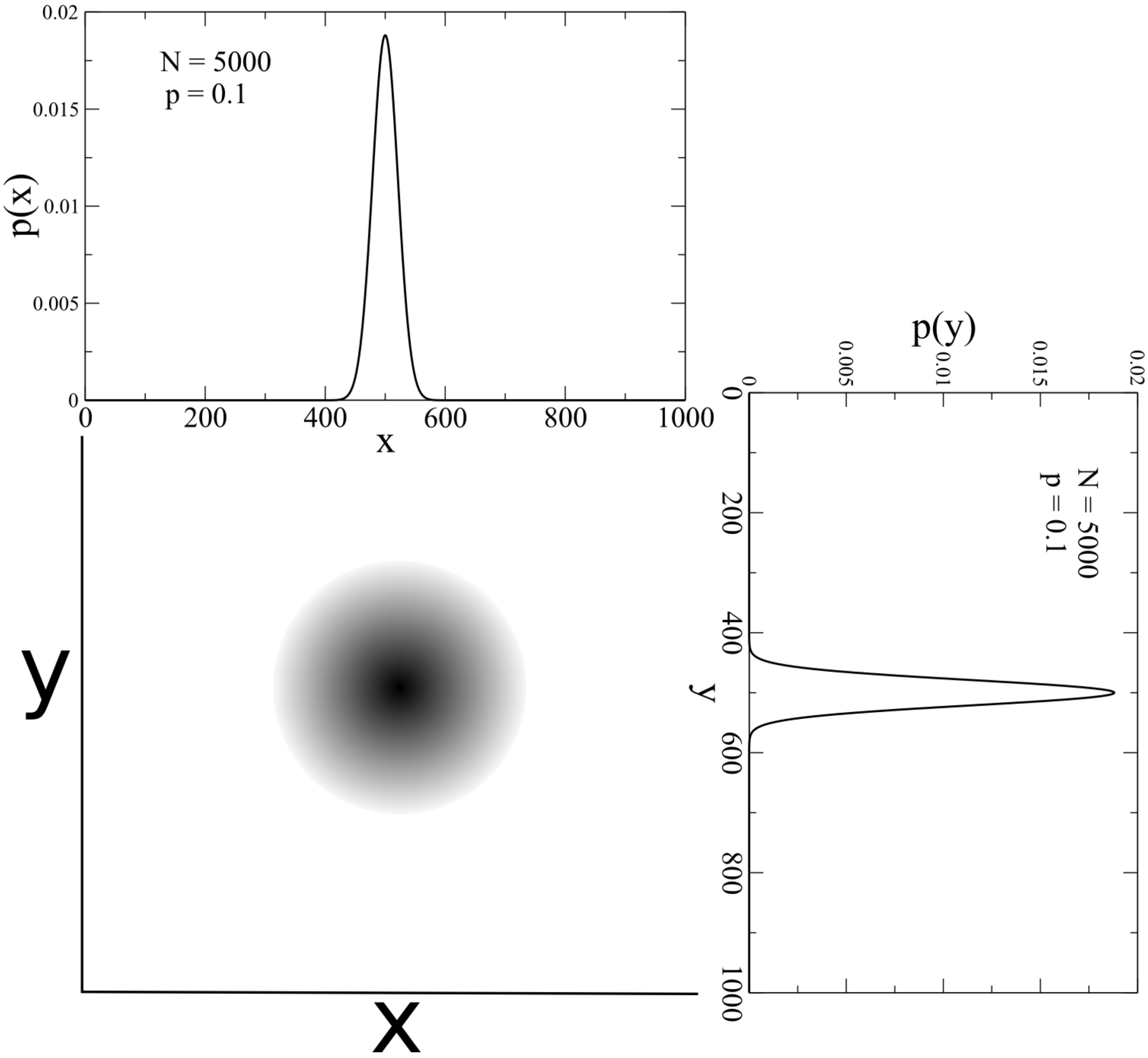


$$p(w_{ij} = 1) = \frac{x_i y_j}{\sum_{l \neq i} y_l}$$

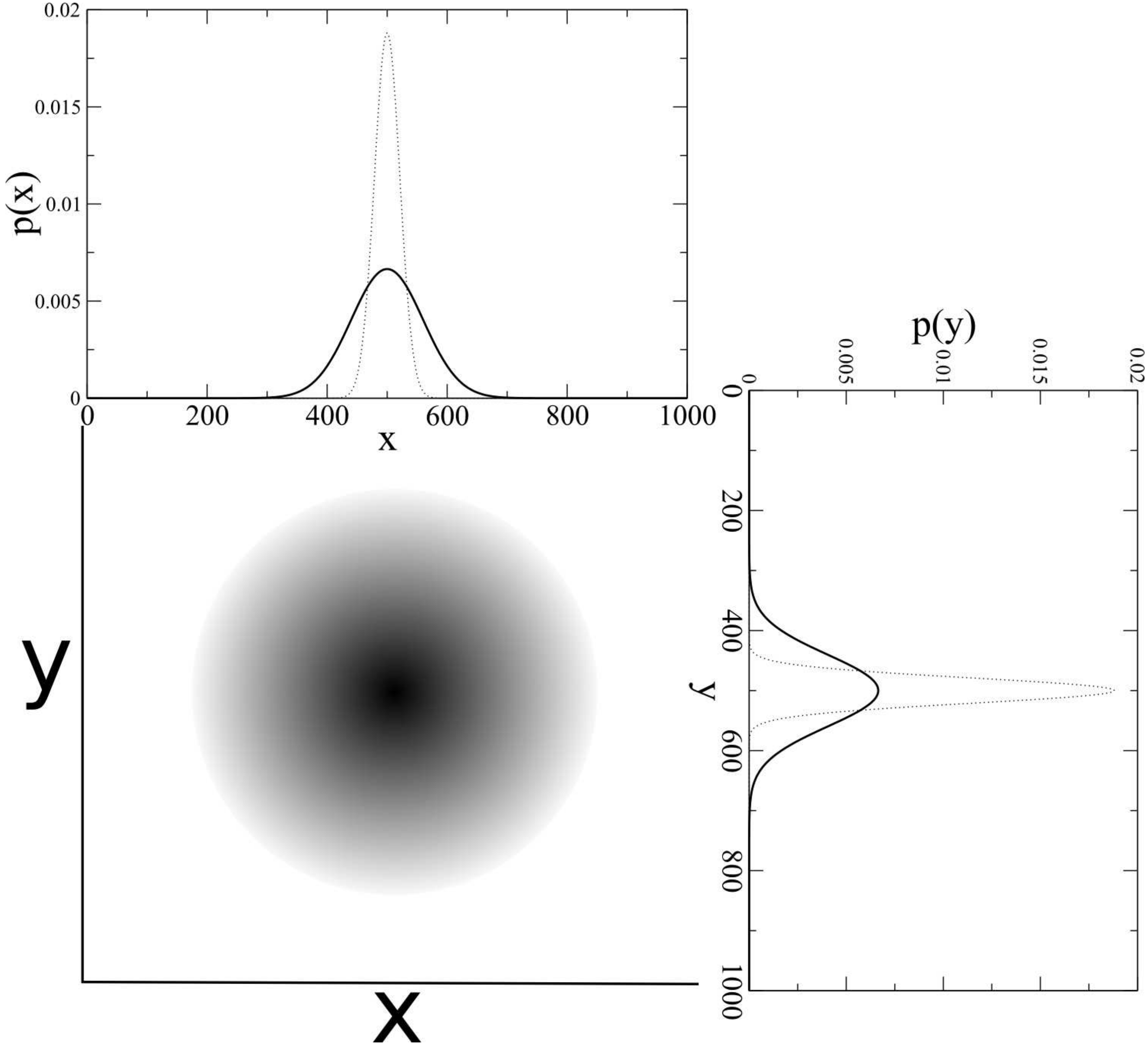
$$\sim \frac{x_i y_j}{d \cdot (N-1)}$$



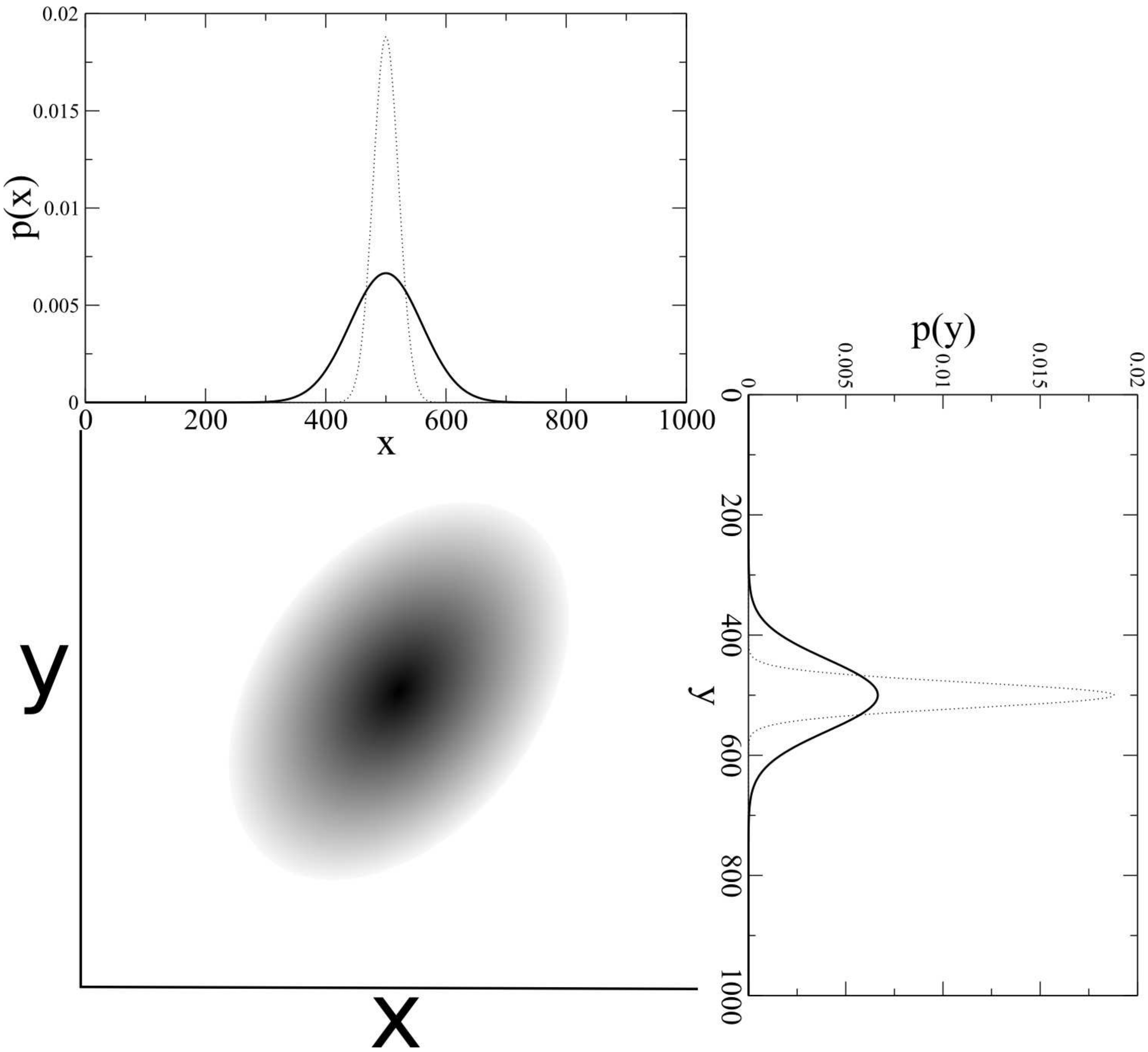
$$
\begin{aligned}
E(p(w_{ij}=1)) &= \frac{E(x_i)E(y_j)}{d\cdot(N-1)} \\
&= \frac{d}{N-1} \\
&= \bar{p}.
\end{aligned}
$$

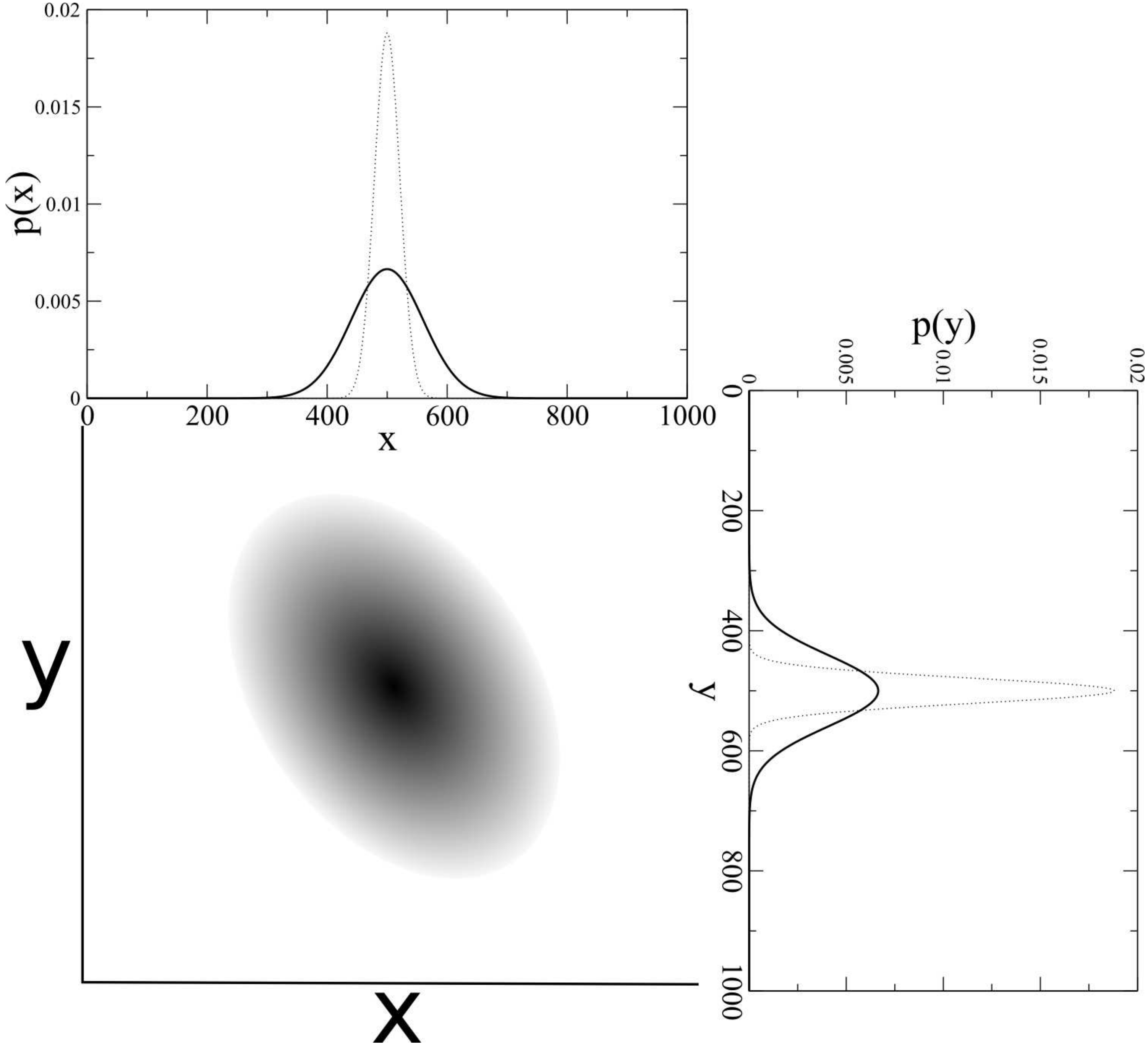


N = 5000
p = 0.1
p(x)
x
p(y)
y
N = 5000
p = 0.1
y
x



p(x)
x
p(y)
y
y
x


p(x)
x
p(y)
y


p(x)
0.02
0.015
0.01
0.005
0
0
200
400
600
800
1000
x
p(y)
y
X
Y

$$\dot{r}(x,y) \quad = \quad -r(x,y) + \phi\Big(J \int\int dx^{'} dy^{'} xy^{'} \rho(x^{'}, y^{'}) r(x^{'}, y^{'}) + I_{ext}\Big),$$

$$\dot{r}(x,y) = -r(x,y) + \phi\Big(J \int\int dx' dy' \, x y' \rho(x',y') r(x',y') + I_{ext}\Big),$$
$$\dot{r}(x) = -r(x) + \phi\Big(J x \int\int dx' dy' \, y' \rho(x',y') r(x') + I_{ext}\Big),$$

$$
\begin{aligned}
\dot{r}(x,y) &= -r(x,y) + \phi\Big(J \int\int dx^{'} dy^{'} x y^{'} \rho(x^{'}, y^{'}) r(x^{'}, y^{'}) + I_{ext}\Big), \\
\dot{r}(x) &= -r(x) + \phi\Big(Jx \int\int dx^{'} dy^{'} y^{'} \rho(x^{'}, y^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(x^{'}, y^{'}) r(x^{'}) + I_{ext}\Big),
\end{aligned}
$$

$$
\begin{aligned}
\dot{r}(x,y) &= -r(x,y) + \phi\Big(J\int\int dx^{'}dy^{'}xy^{'}\rho(x^{'},y^{'})r(x^{'},y^{'}) + I_{ext}\Big), \\
\dot{r}(x) &= -r(x) + \phi\Big(Jx\int\int dx^{'}dy^{'}y^{'}\rho(x^{'},y^{'})r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx\int dx^{'}\int dy^{'}y^{'}\rho(x^{'},y^{'})r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx\int dx^{'}\int dy^{'}y^{'}\rho(y^{'}|x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big),
\end{aligned}
$$

$$
\begin{aligned}
\dot{r}(x,y) &= -r(x,y) + \phi\Big(J \int\int dx^{'} dy^{'} x y^{'} \rho(x^{'}, y^{'}) r(x^{'}, y^{'}) + I_{ext}\Big), \\
\dot{r}(x) &= -r(x) + \phi\Big(Jx \int\int dx^{'} dy^{'} y^{'} \rho(x^{'}, y^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(x^{'}, y^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(y^{'}|x^{'}) \rho(x^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(y^{'}|x^{'}) \rho(x^{'}) r(x^{'}) + I_{ext}\Big),
\end{aligned}
$$

$$
\begin{aligned}
\dot{r}(x,y) &= -r(x,y) + \phi\Big(J \int\int dx^{'} dy^{'} x y^{'} \rho(x^{'}, y^{'}) r(x^{'}, y^{'}) + I_{ext}\Big), \\
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&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(x^{'}, y^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(y^{'}|x^{'}) \rho(x^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \int dy^{'} y^{'} \rho(y^{'}|x^{'}) \rho(x^{'}) r(x^{'}) + I_{ext}\Big), \\
&= -r(x) + \phi\Big(Jx \int dx^{'} \mu(x^{'}) \rho(x^{'}) r(x^{'}) + I_{ext}\Big).
\end{aligned}
$$

$$\dot{r}(x) = -r(x) + \phi\Big(Jx\int dx^{'}\mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big),$$

$$\begin{aligned}\dot{r}(x) &= -r(x) + \phi\Big(Jx\int dx' \mu(x')\rho(x')r(x') + I_{ext}\Big),\\ \dot{S} &= -S + \int dx \mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).\end{aligned}$$

$$
\begin{aligned}
\dot{r}(x) &= -r(x) + \phi\Big(Jx\int dx^{'}\mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big), \\
\dot{S} &= -S + \int dx\mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).
\end{aligned}
$$

linear stability: $S(t) = S_0 + \delta S e^{\lambda t}$

$$
\begin{aligned}
\dot{r}(x) &= -r(x) + \phi\Big(Jx \int dx^{'} \mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big), \\
\dot{S} &= -S + \int dx \mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).
\end{aligned}
$$

linear stability: $S(t) = S_0 + \delta S e^{\lambda t}$

$$
\lambda = -1 + \frac{\bar{J}}{d} \int \int dx dy \cdot x \cdot y \rho(x, y) \phi^{'}(x),
$$

$$
\begin{aligned}
\dot{r}(x) &= -r(x) + \phi\Big(Jx \int dx^{'} \mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big), \\
\dot{S} &= -S + \int dx \mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).
\end{aligned}
$$

linear stability: $S(t) = S_0 + \delta S e^{\lambda t}$

$$
\begin{aligned}
\lambda &= -1 + \frac{\bar{J}}{d} \int\int dxdy \cdot x \cdot y\rho(x,y)\phi^{'}(x), \\
&= -1 + \frac{\bar{J}}{d} \int\int dxdy \cdot x \cdot y\rho(x,y) \frac{\int\int dxdy \cdot x \cdot y\rho(x,y)\phi^{'}(x)}{\int\int dxdy \cdot x \cdot y\rho(x,y)}
\end{aligned}
$$

$$\begin{aligned}\dot{r}(x) &= -r(x) + \phi\Big(Jx\int dx^{'}\mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big),\\ \dot{S} &= -S + \int dx\mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).\end{aligned}$$

linear stability: $S(t) = S_0 + \delta S e^{\lambda t}$

$$\begin{aligned}\lambda &= -1 + \frac{\bar{J}}{d}\int\int dxdy\cdot x\cdot y\rho(x,y)\phi^{'}(x),\\ &= -1 + \frac{\bar{J}}{d}\int\int dxdy\cdot x\cdot y\rho(x,y)\frac{\int\int dxdy\cdot x\cdot y\rho(x,y)\phi^{'}(x)}{\int\int dxdy\cdot x\cdot y\rho(x,y)}\\ &= -1 + \bar{J}d(1+\mathrm{cov(x,y)})\frac{\int\int dxdy\cdot x\cdot y\rho(x,y)\phi^{'}(x)}{\int\int dxdy\cdot x\cdot y\rho(x,y)}\end{aligned}$$

$$
\begin{aligned}
\dot{r}(x) &= -r(x) + \phi\Big(Jx \int dx^{'} \mu(x^{'})\rho(x^{'})r(x^{'}) + I_{ext}\Big), \\
\dot{S} &= -S + \int dx \mu(x)\rho(x)\phi\Big(JxS + I_{ext}\Big).
\end{aligned}
$$

linear stability: $S(t) = S_0 + \delta S e^{\lambda t}$

$$
\begin{aligned}
\lambda &= -1 + \frac{\bar{J}}{d} \int \int dx dy \cdot x \cdot y \rho(x, y) \phi^{'}(x), \\
&= -1 + \frac{\bar{J}}{d} \int \int dx dy \cdot x \cdot y \rho(x, y) \frac{\int \int dx dy \cdot x \cdot y \rho(x, y) \phi^{'}(x)}{\int \int dx dy \cdot x \cdot y \rho(x, y)} \\
&= -1 + Jd(1 + \mathrm{cov}(\mathrm{x}, \mathrm{y})) \frac{\int \int dx dy \cdot x \cdot y \rho(x, y) \phi^{'}(x)}{\int \int dx dy \cdot x \cdot y \rho(x, y)} \\
&= -1 + Jd(1 + \mathrm{cov}(\mathrm{x}, \mathrm{y})) \langle \phi^{'} \rangle_w
\end{aligned}
$$

Linear stability in naive meanfield model:

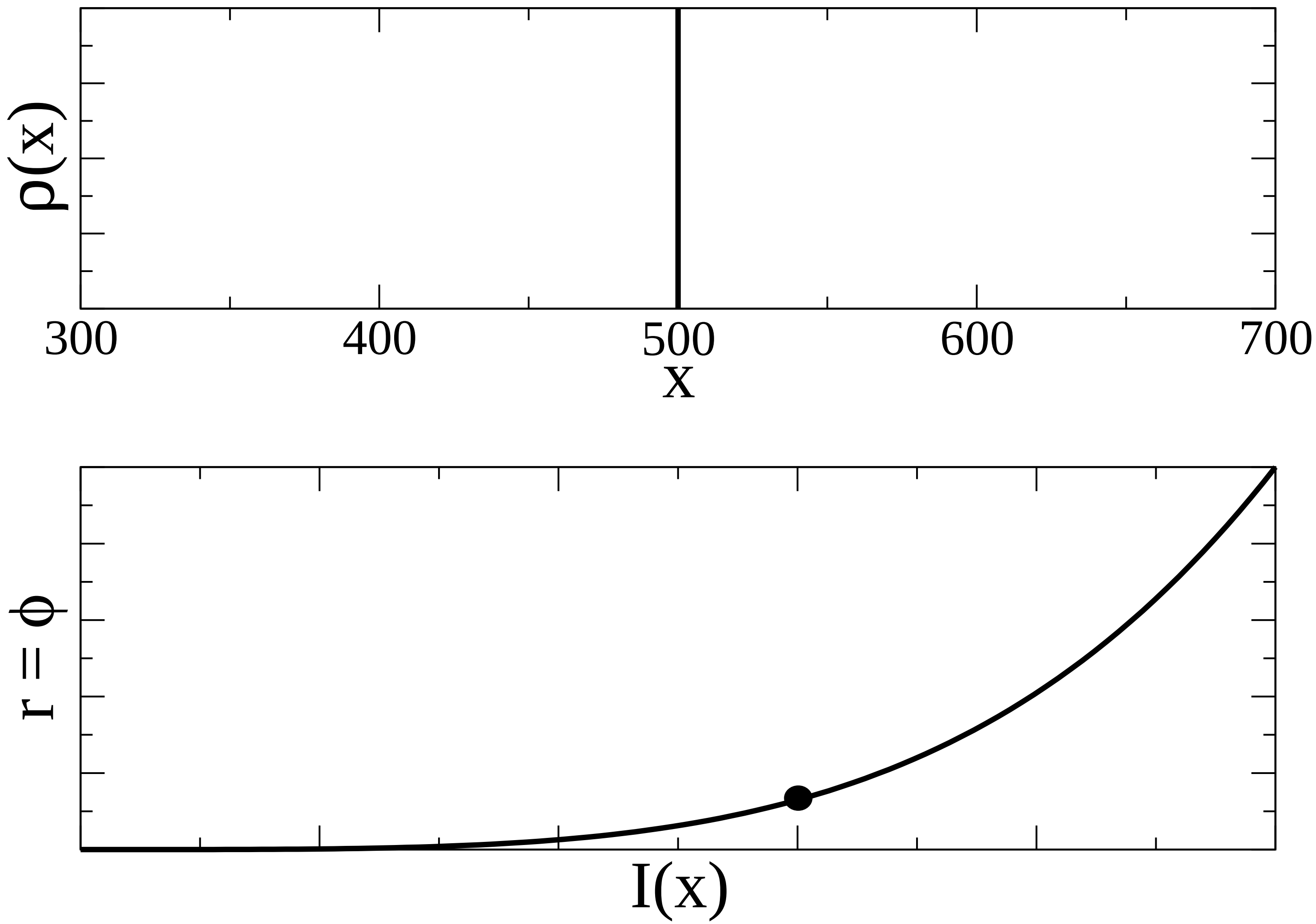
$$\lambda = -1 + JpN\phi^{'}.$$

Linear stability taking degree into account:

$$\lambda = -1 + JpN(1 + \mathrm{cov}(\mathrm{x}, \mathrm{y}))\langle\phi^{'}\rangle_w$$

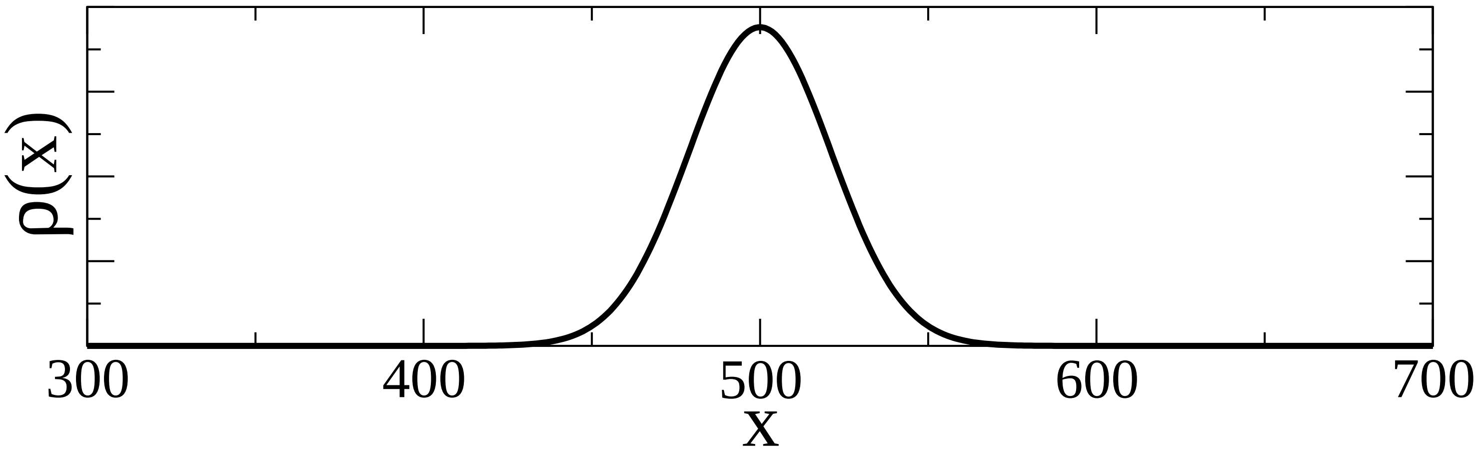
Uncorrelated degrees

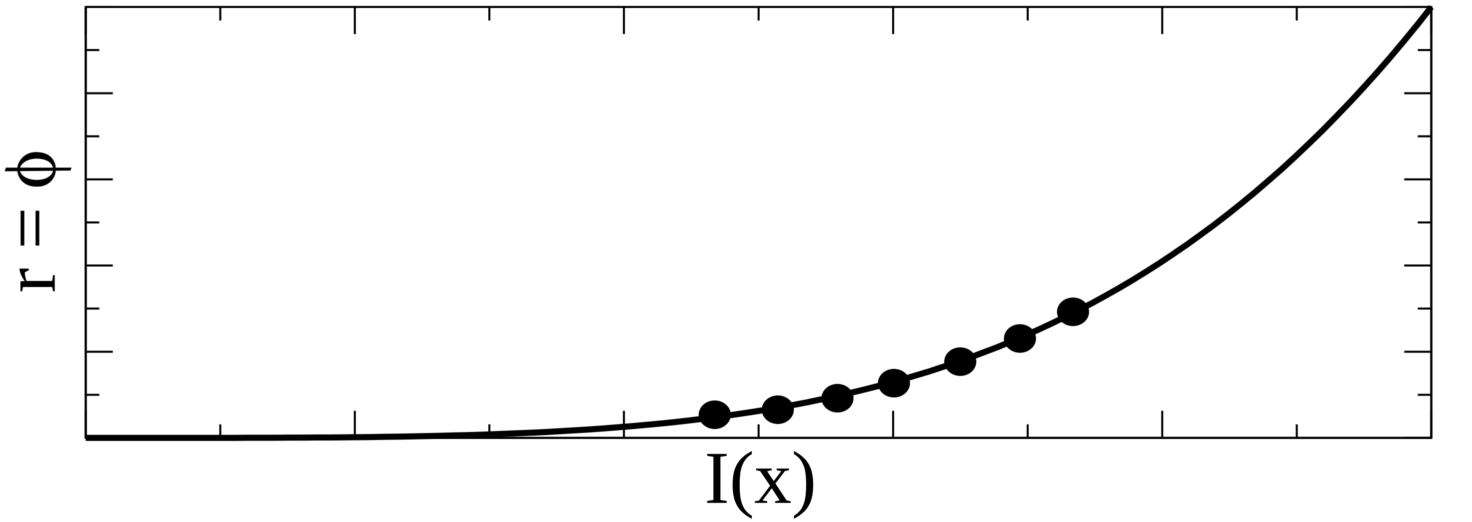
$$\lambda = -1 + JpN\langle \phi' \rangle_w$$



Uncorrelated degrees

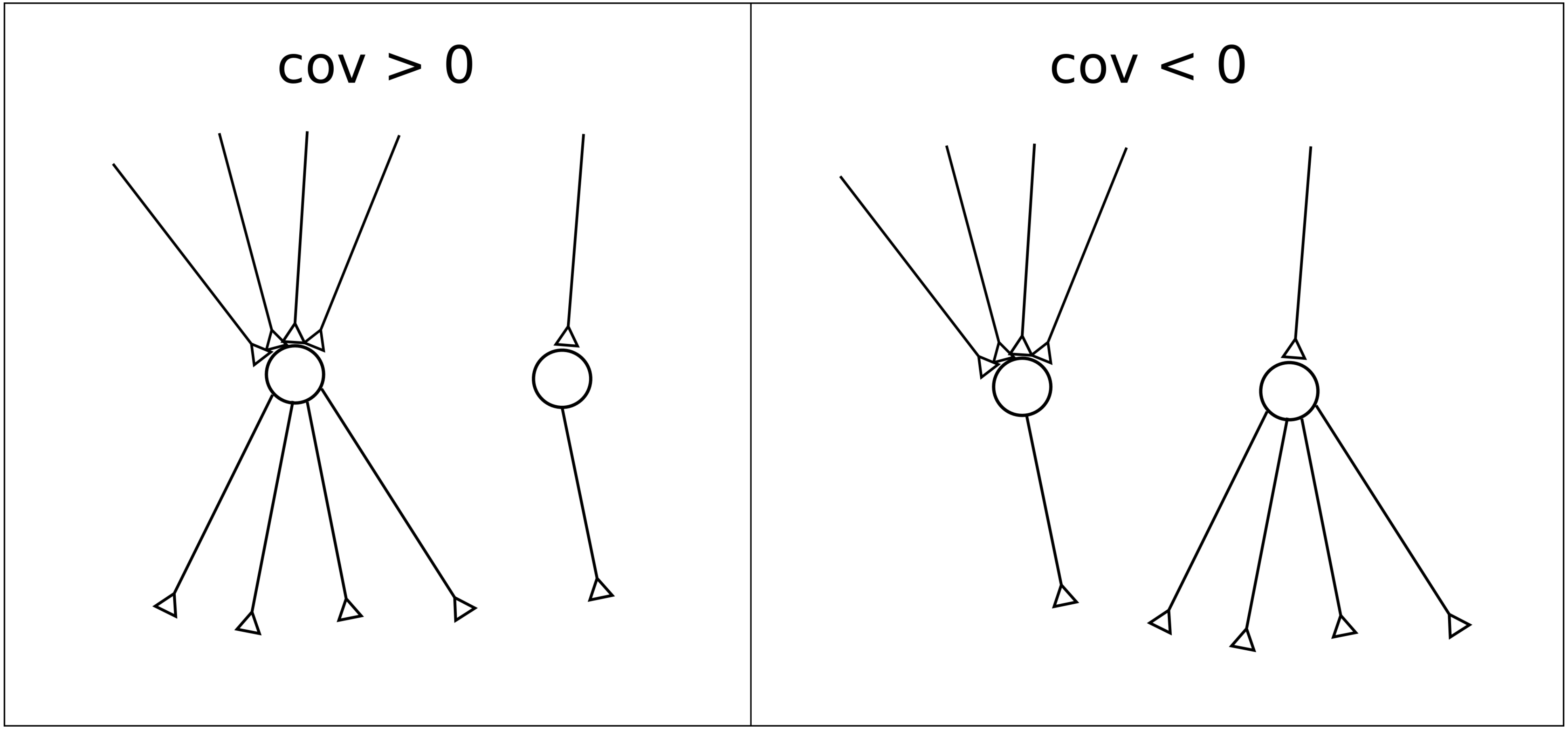
$$\lambda = -1 + JpN\langle\phi'\rangle_w$$



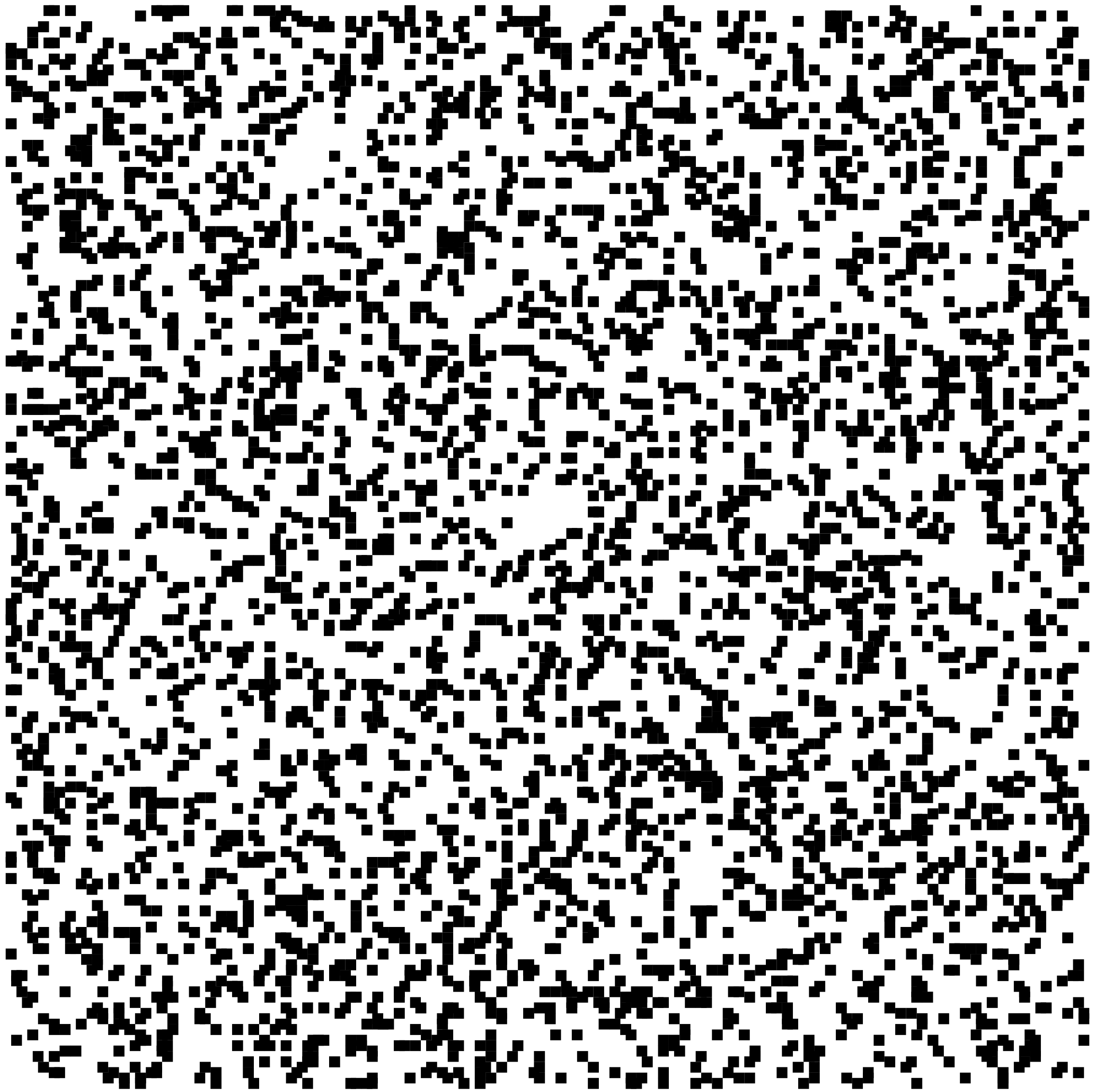


Correlated degrees

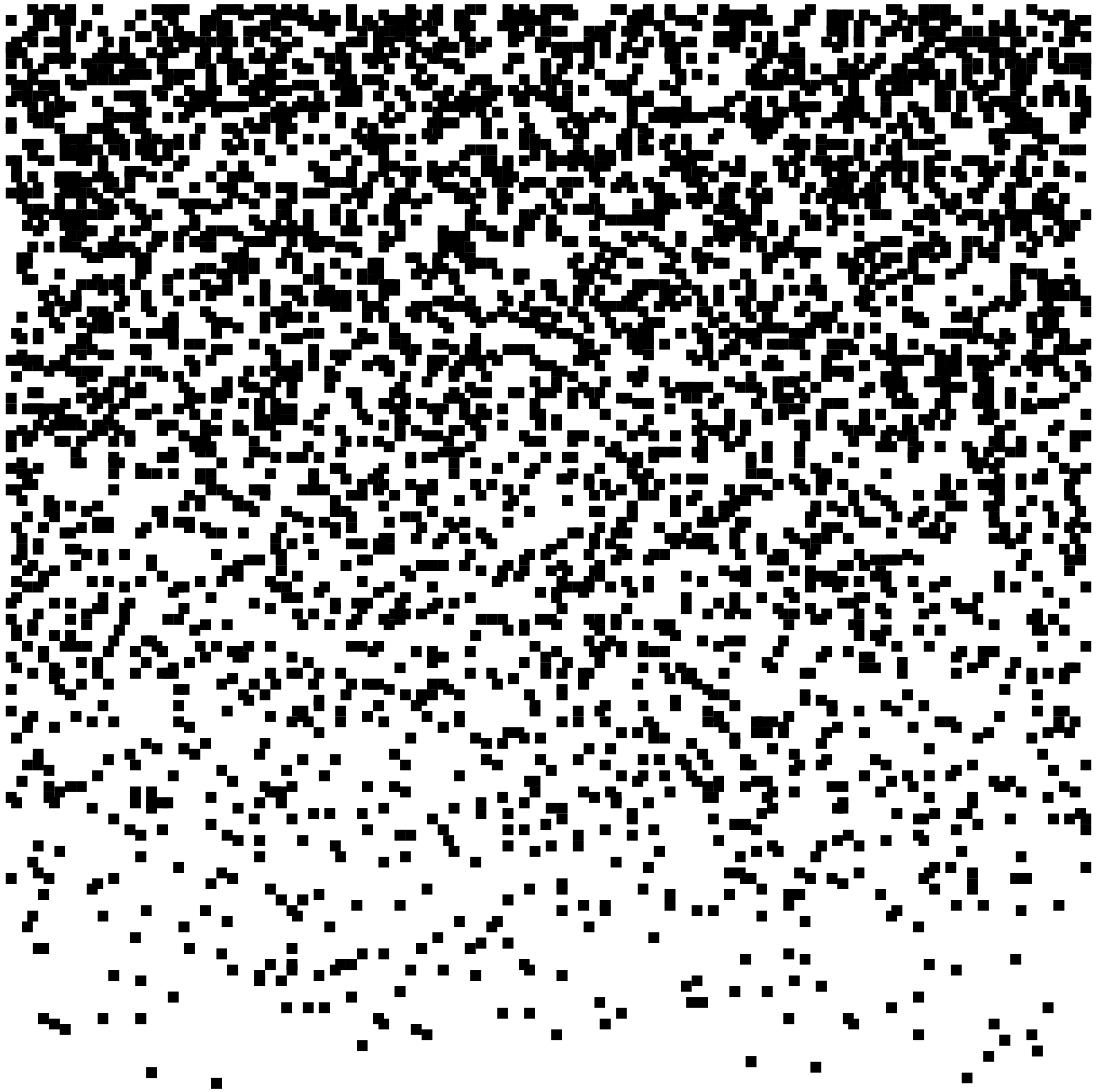
$$\lambda = -1 + JpN(1 + \mathrm{cov}(\mathrm{x},\mathrm{y}))\langle\phi'\rangle_w$$



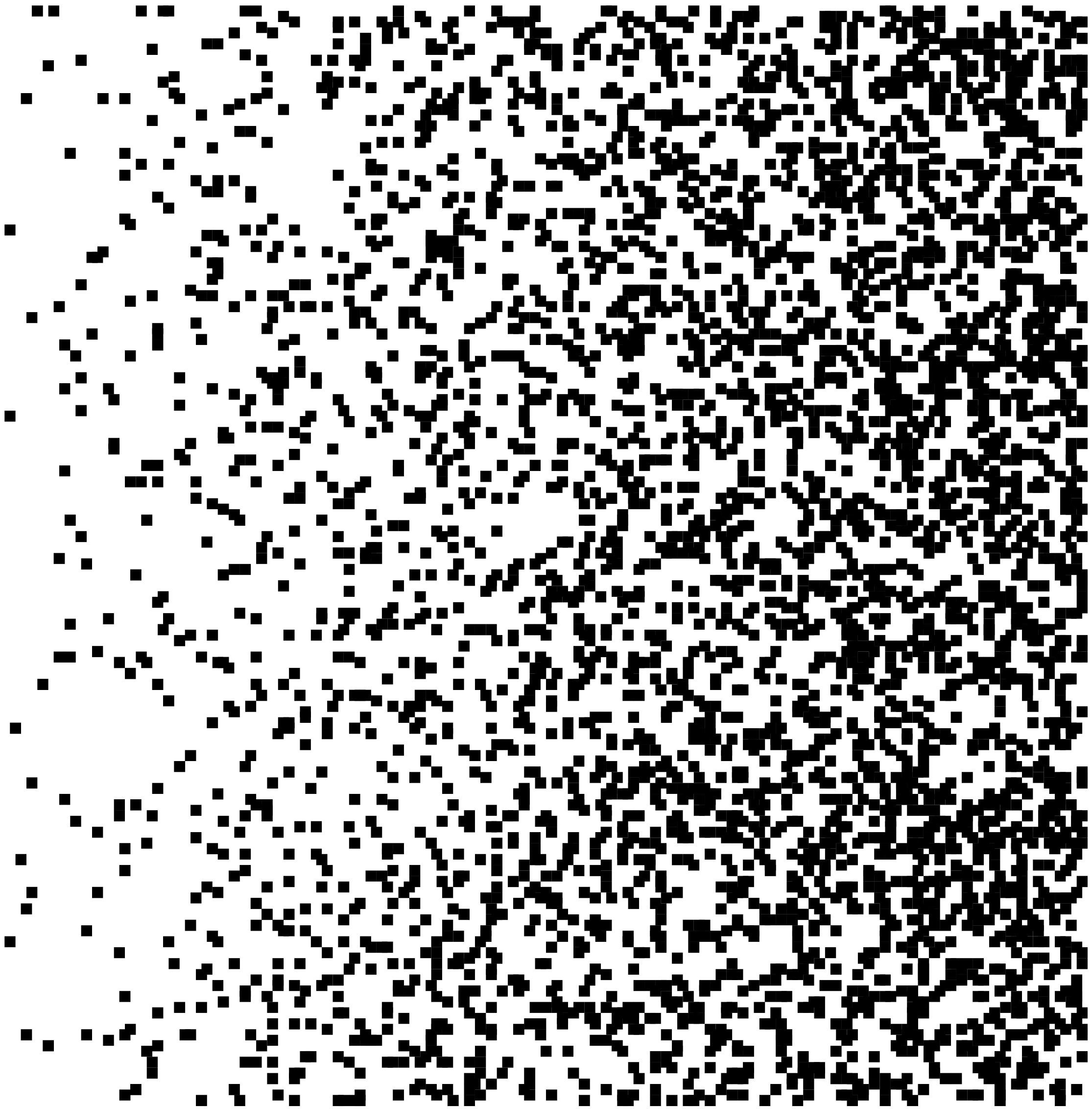
Erdös-Renyi Network



Broad In-degree

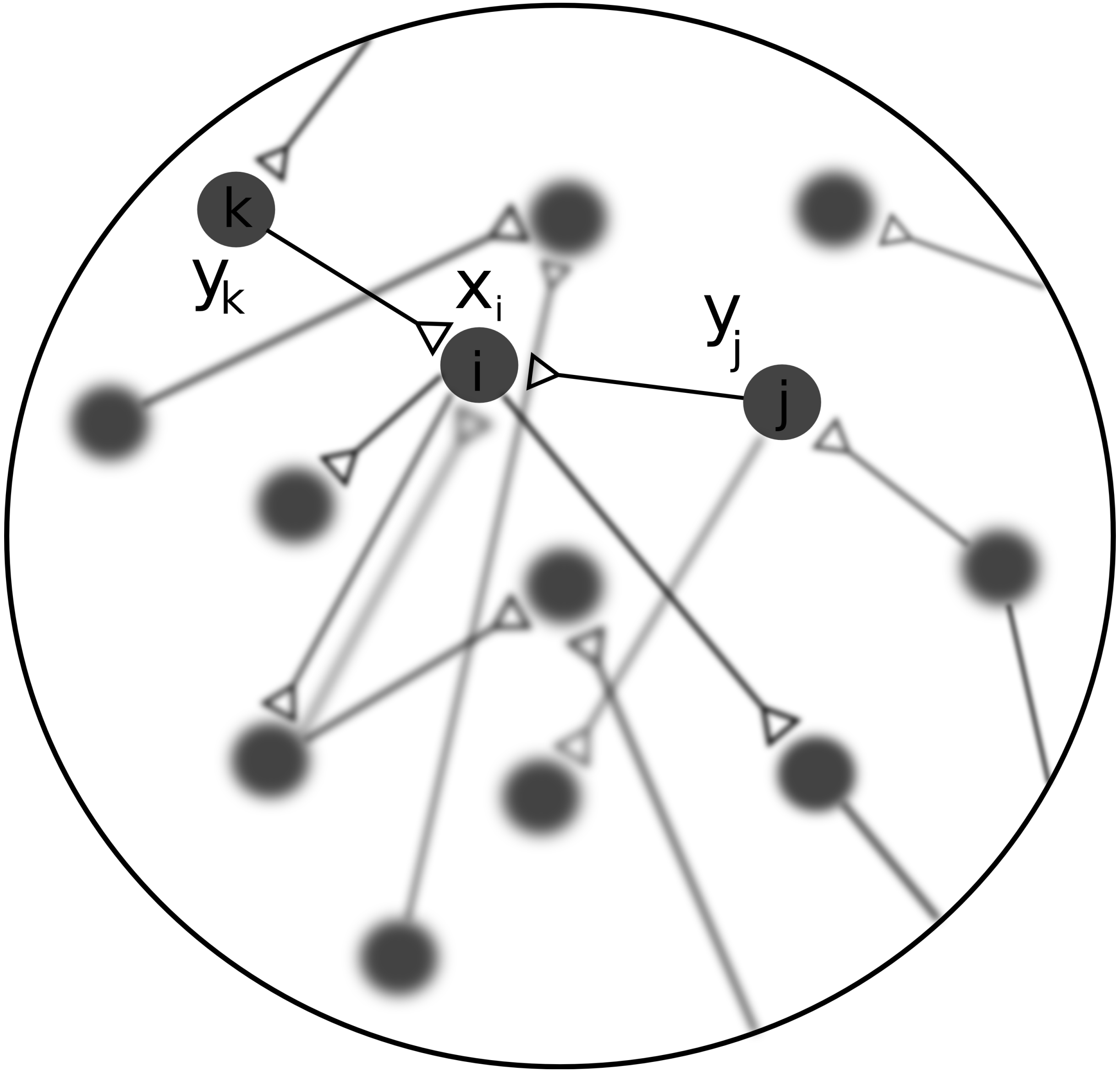


Broad Out-degree

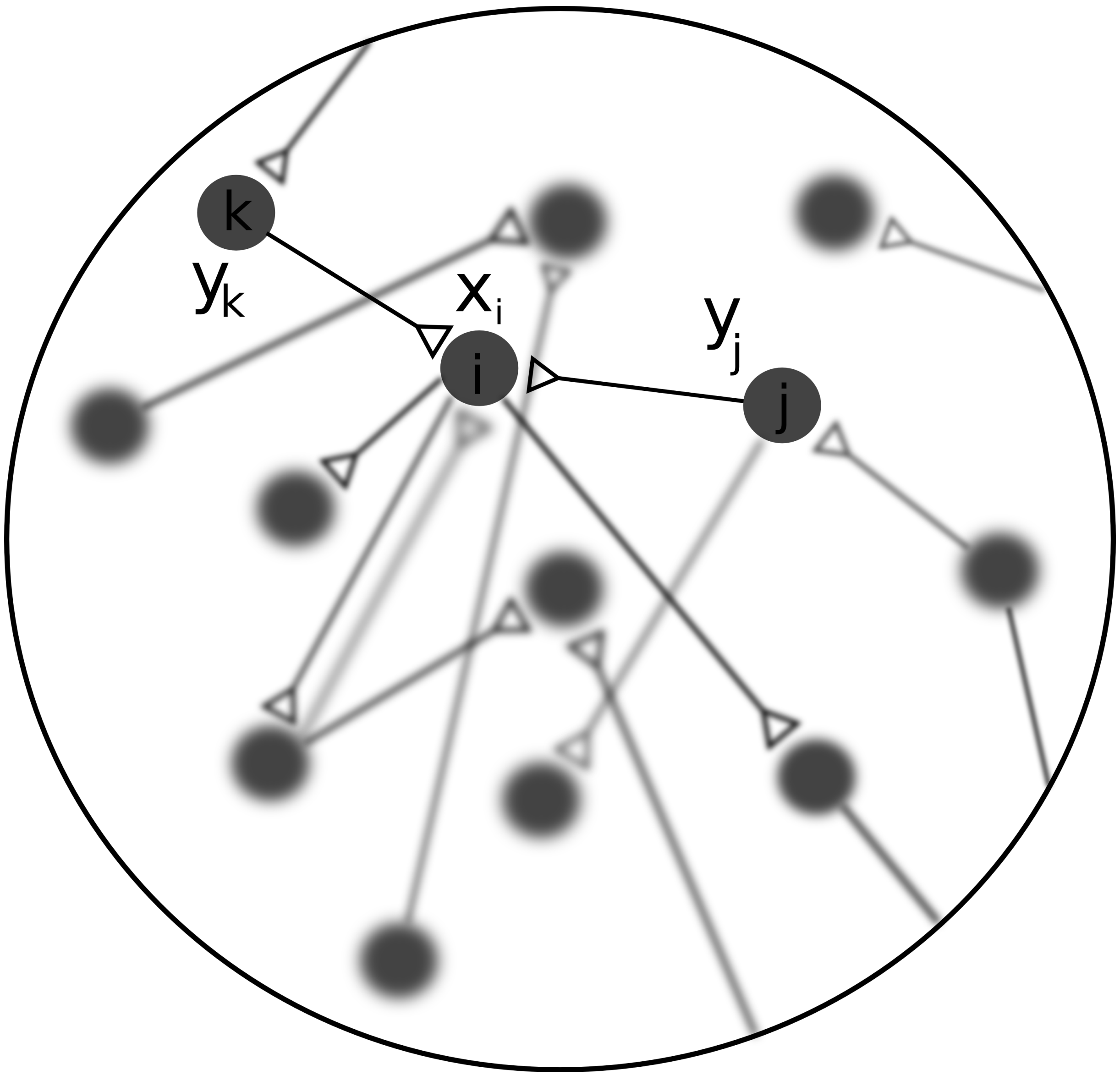


Broad, Pos. Correlated In and Out-degree

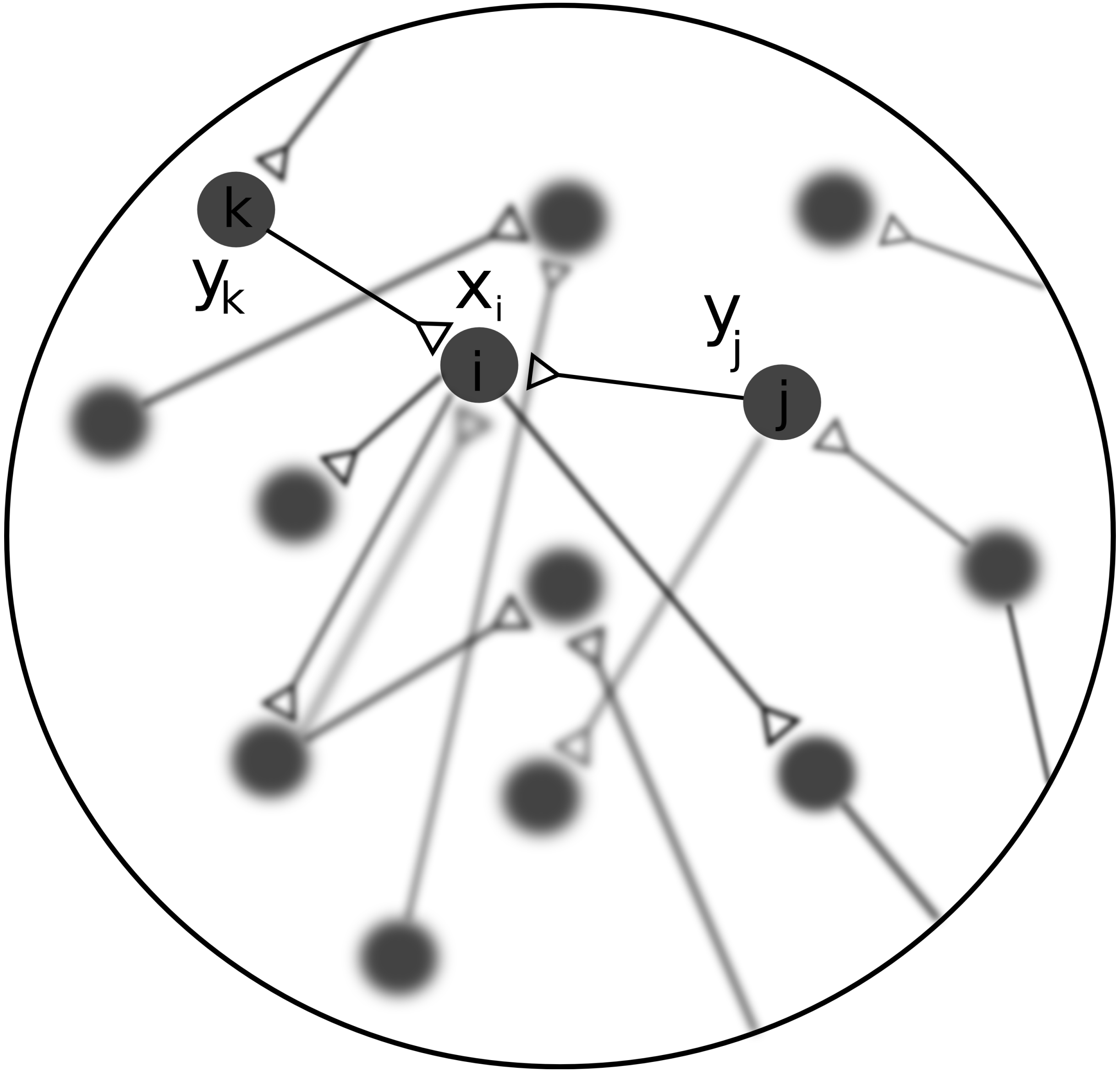
Fine, but how can I relate all this to network motifs?



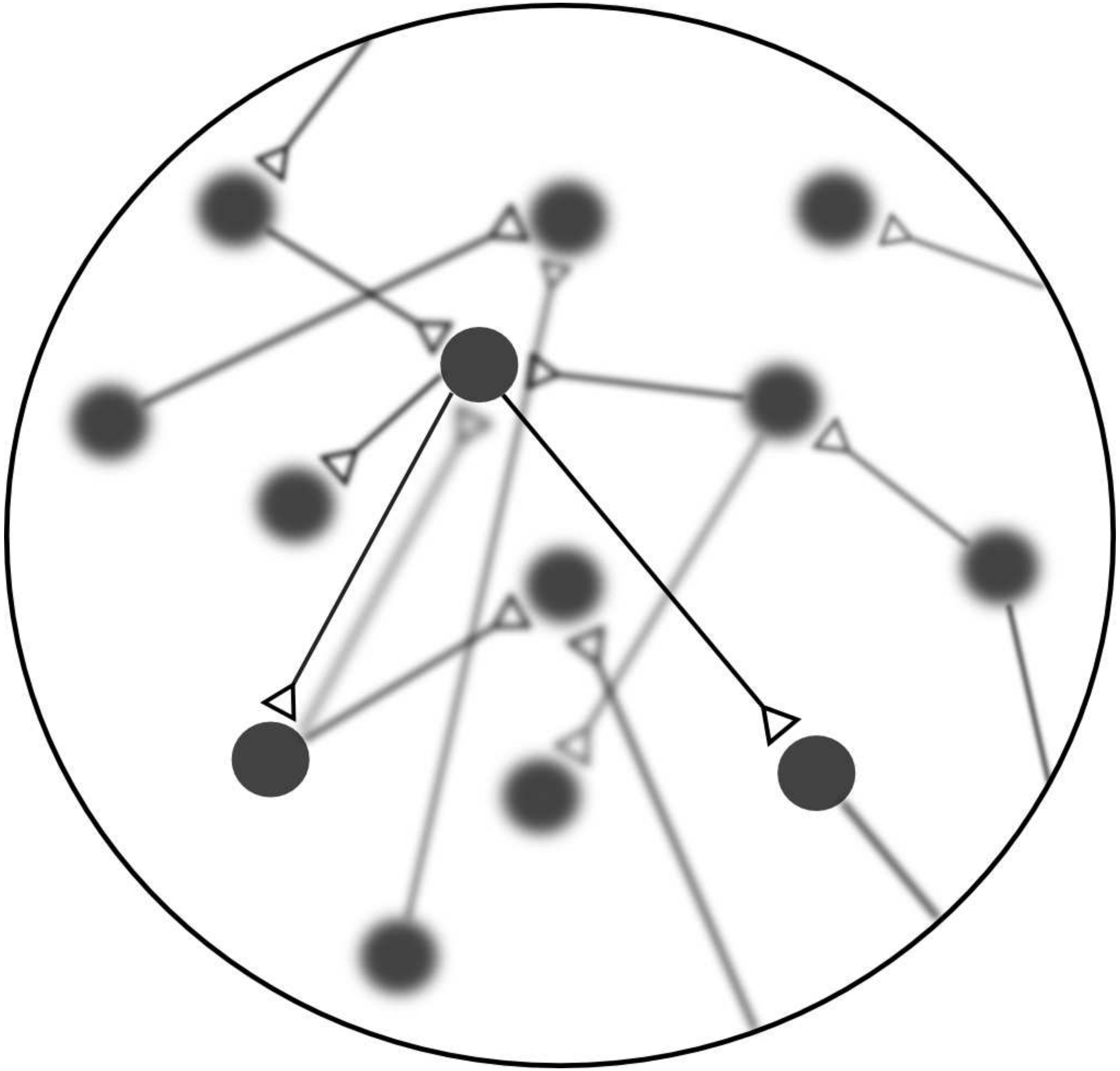
$$p(w_{ij} = 1, w_{ik} = 1) \propto x_i y_j \cdot (x_i - 1) y_k$$



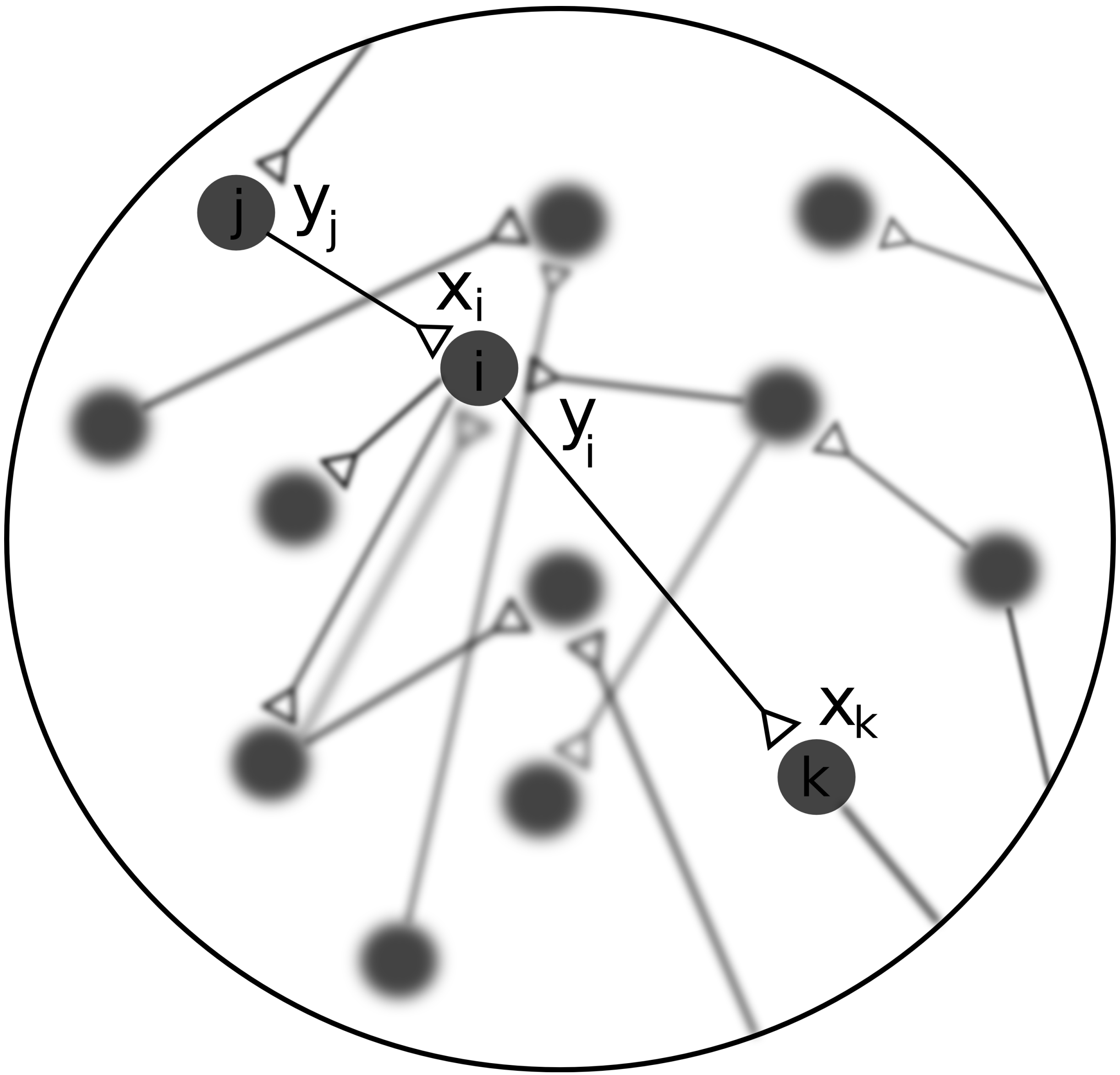
$$E(p(w_{ij} = 1, w_{ik} = 1)) \propto E(x_i(x_i - 1))$$



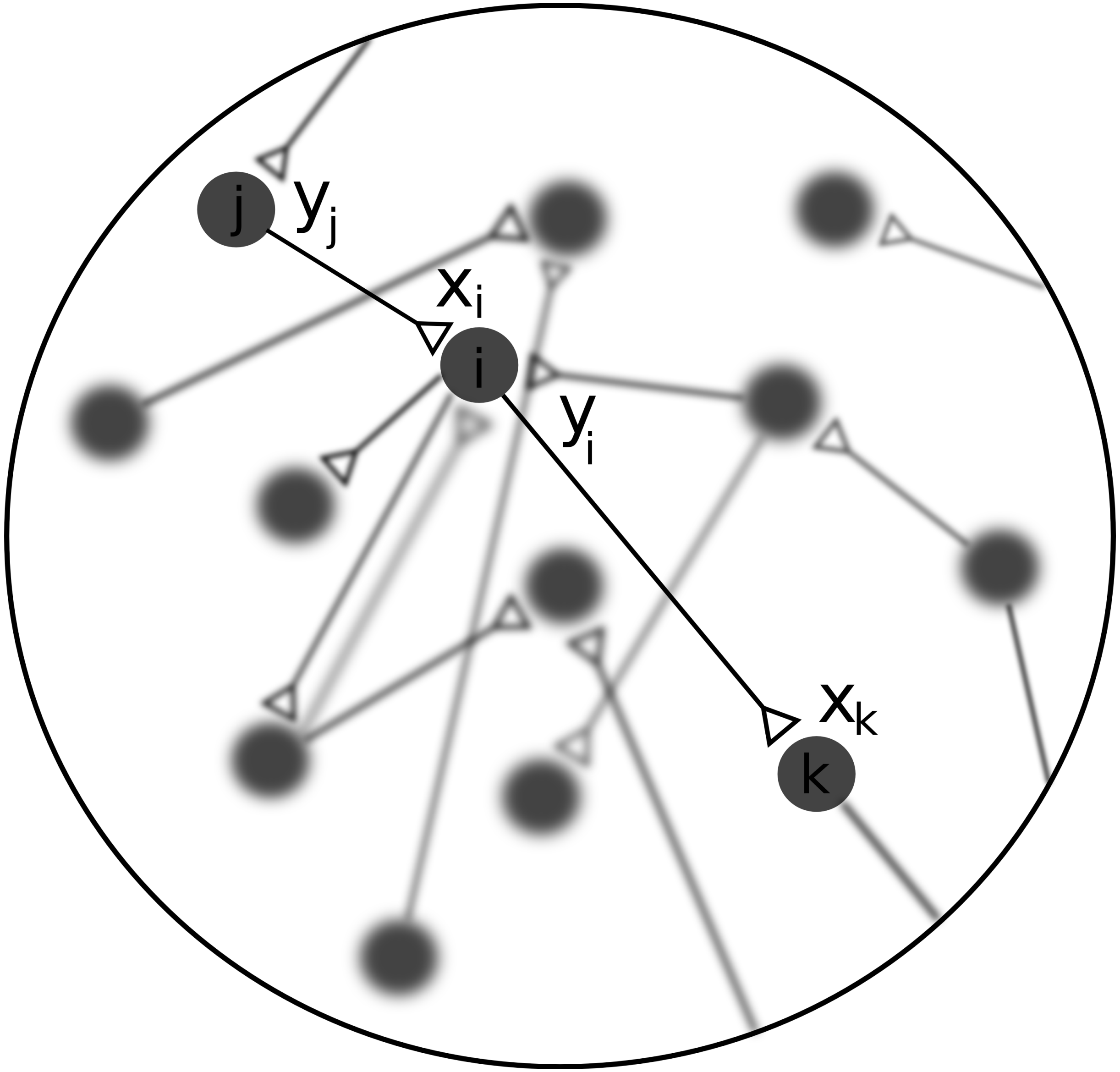
$$E(p(w_{ij} = 1, w_{ik} = 1)) \propto d^2 + var(x) - d$$



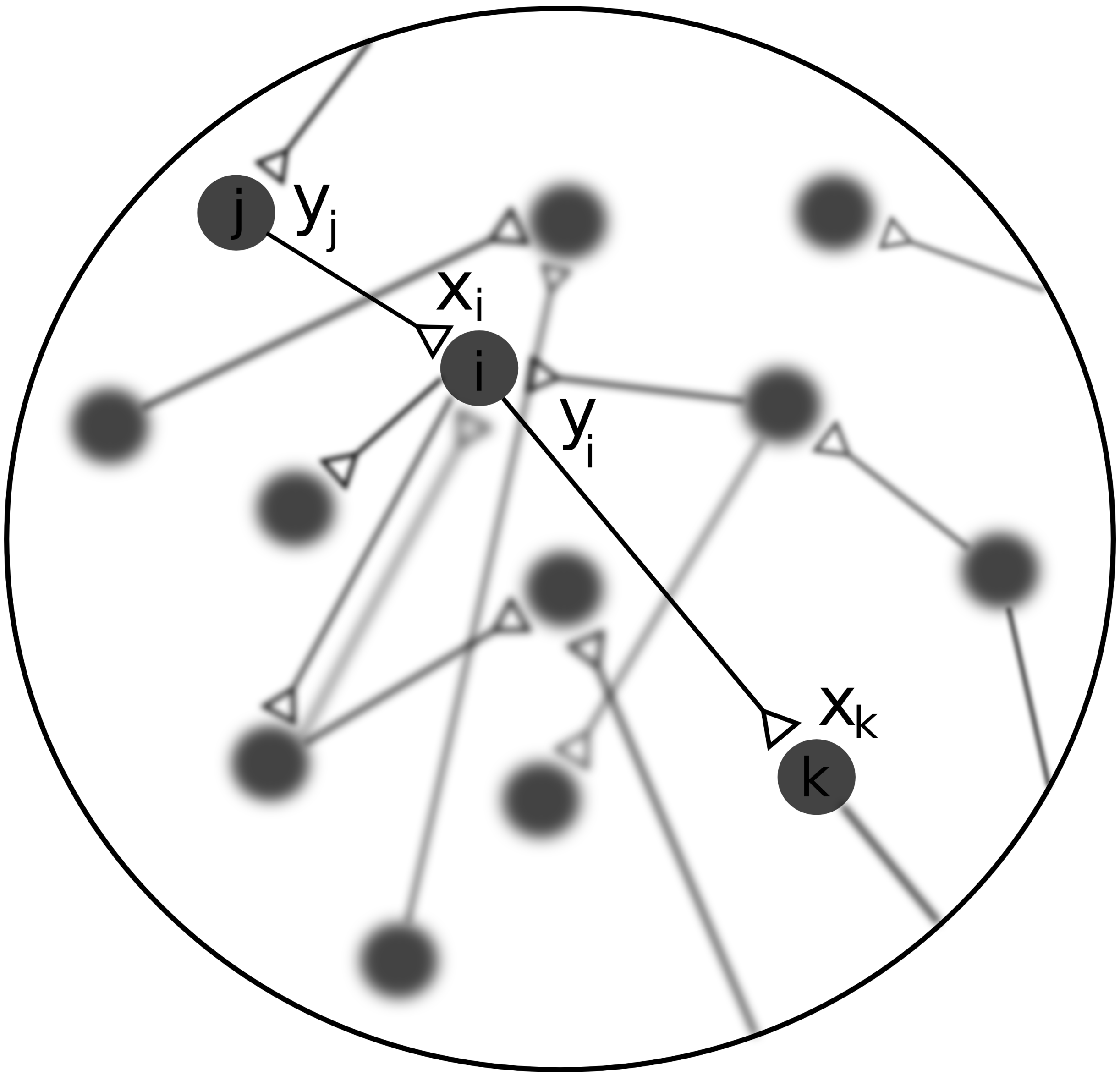
$$E(p(w_{ji} = 1, w_{ki} = 1)) \propto d^2 + var(y) - d$$



$$p(w_{ij} = 1, w_{ki} = 1) \propto x_i y_j \cdot x_k y_i$$



$$E(p(w_{ij} = 1, w_{ki} = 1)) \propto E(x_i y_i)$$



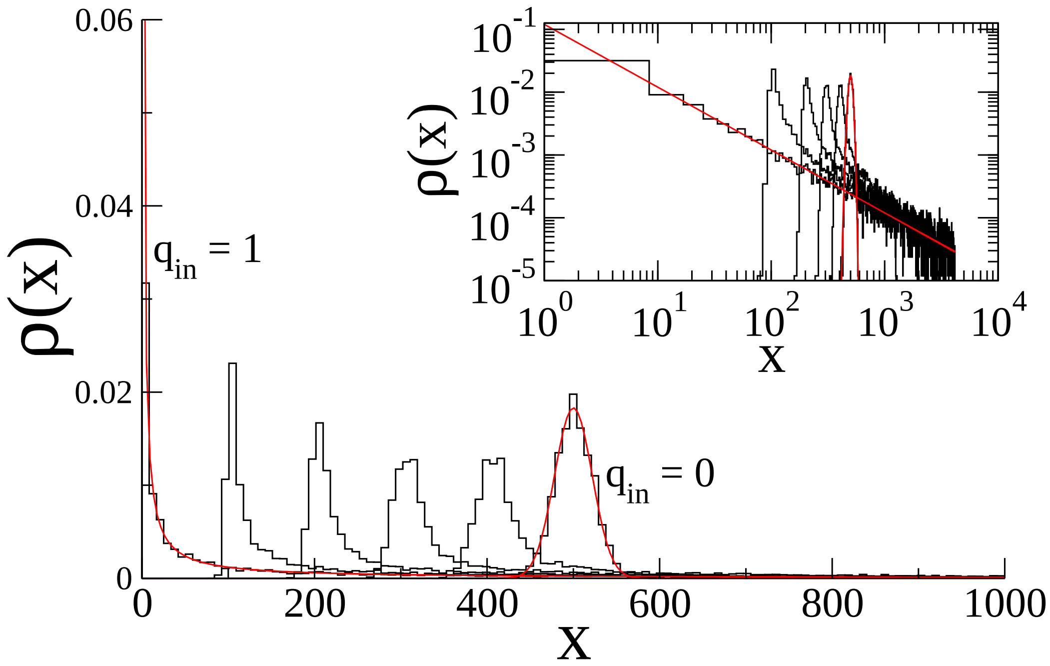
$$E(p(w_{ij} = 1, w_{ki} = 1)) \propto d^2 + cov(x, y)$$

Network simulations:

The degrees for a neuron i are chosen according to

$$\begin{aligned} x_i &= (1-q_{\text{in}})x_i^B + q_{\text{in}}x_i^P, \\ y_i &= (1-q_{\text{out}})y_i^B + q_{\text{out}}y_i^P \end{aligned}$$

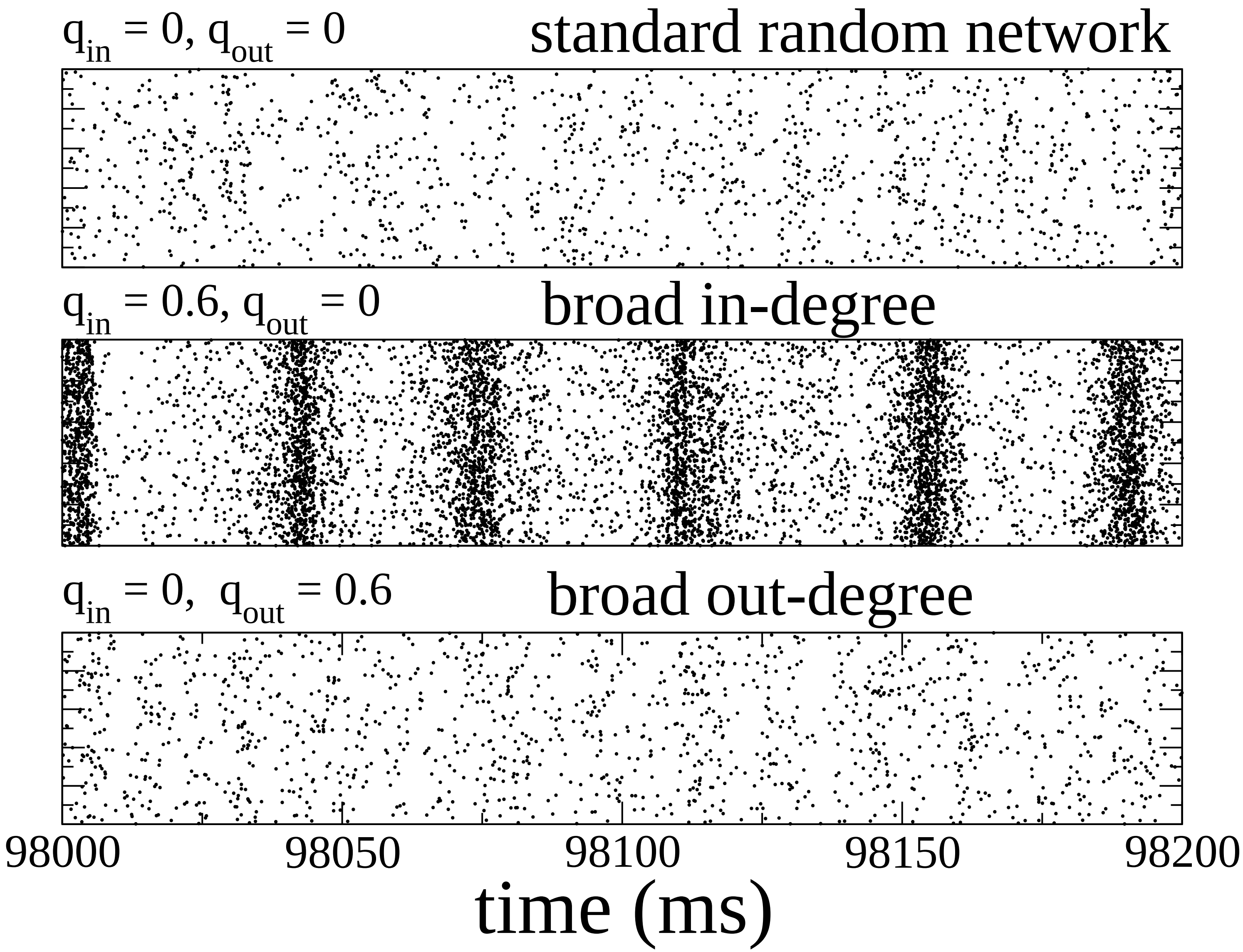
where x^B and x^P are drawn from Binomial and Powerlaw dists with same mean.



EI network. (Brunel 2000).

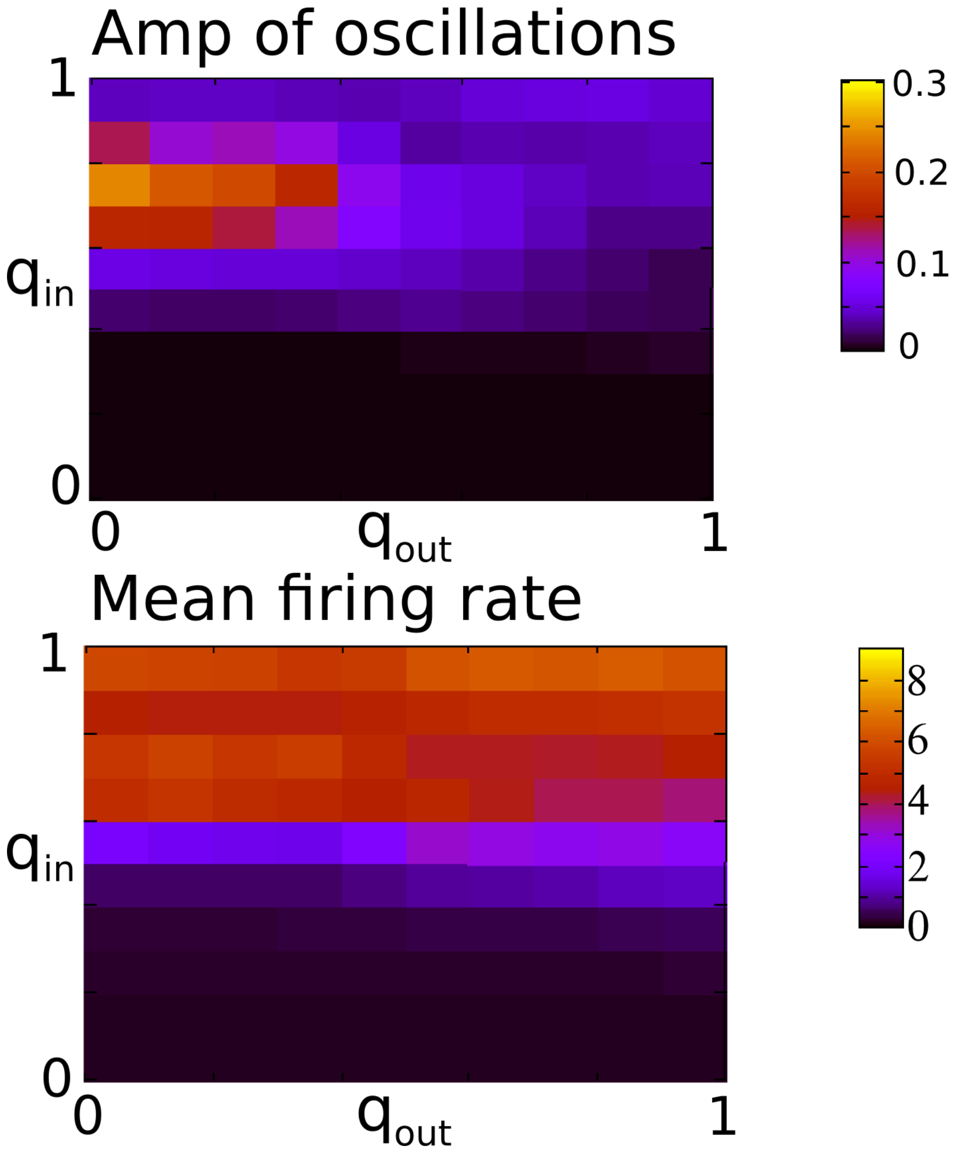
- 10,000 excitatory and 2,500 inhibitory neurons.
- PSCs are delta functions with delay.
- E-to-E coupling is hybrid degree-distribution (5% sparseness).
- All other coupling is standard random with p=0.1.

EI network.



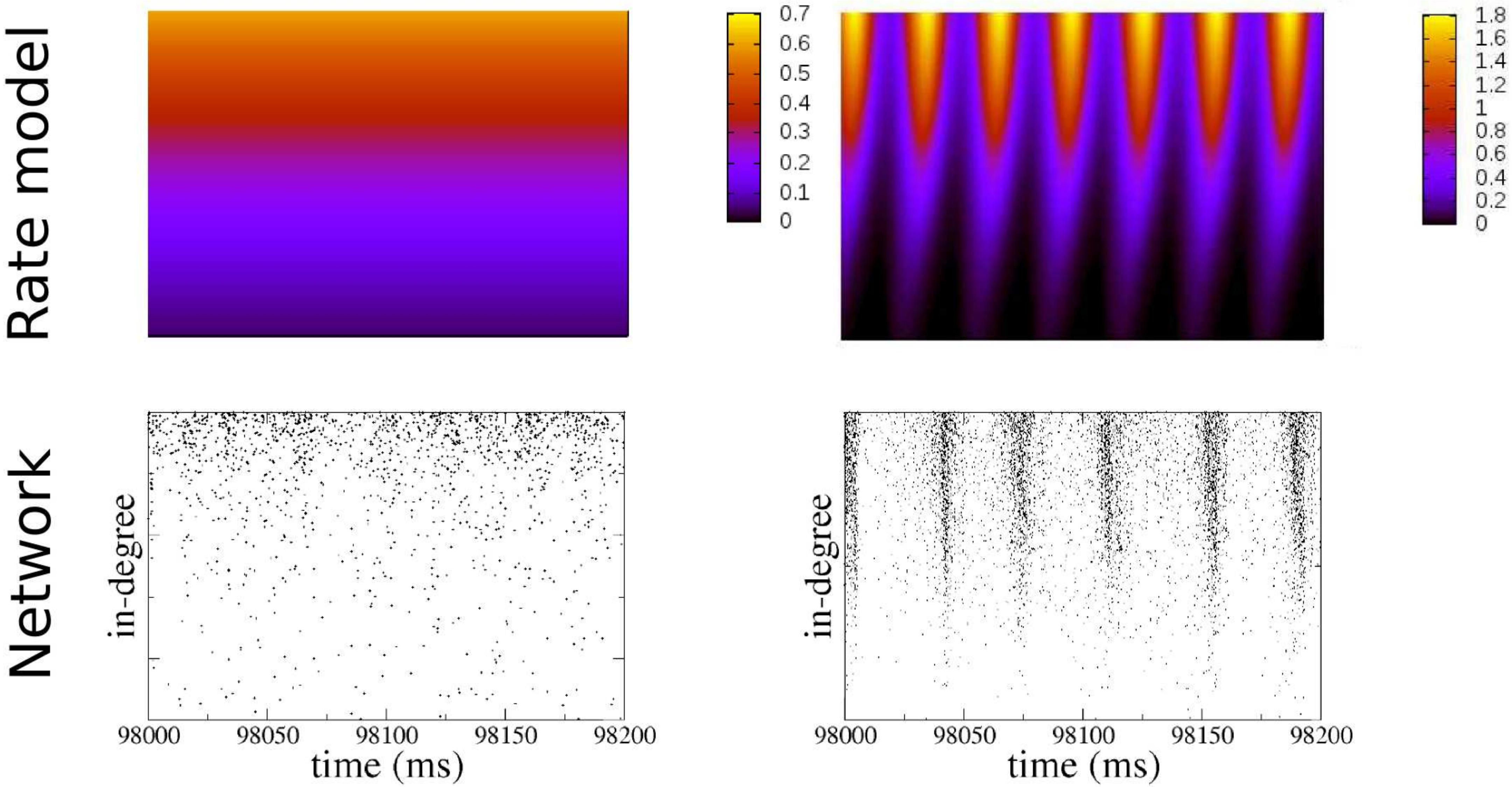
q_{in} = 0, q_{out} = 0
standard random network
q_{in} = 0.6, q_{out} = 0
broad in-degree
q_{in} = 0, q_{out} = 0.6
broad out-degree
98000
98050
98100
98150
98200
time (ms)

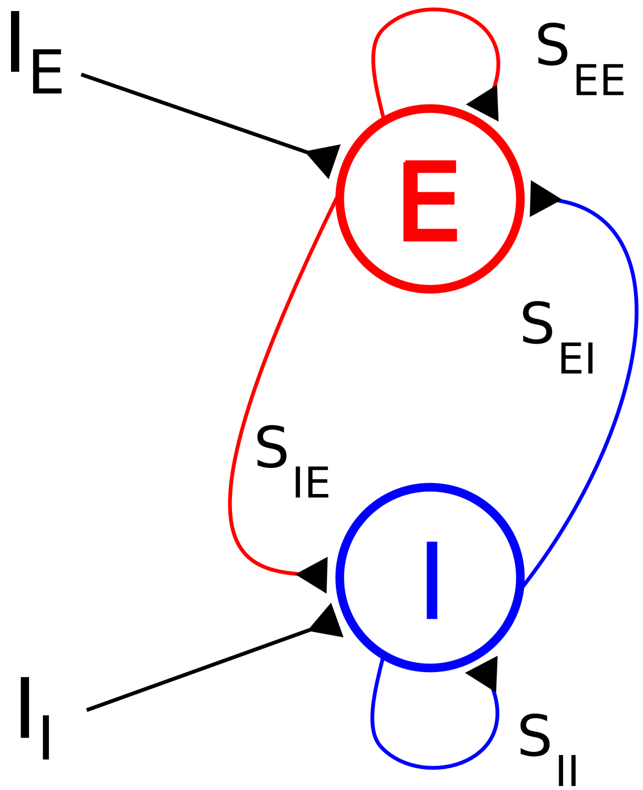
EI network.



Amp of oscillations
1
0
q_{in}
0
q_{out}
1
0.3
0.2
0.1
0
Mean firing rate
1
0
q_{in}
0
q_{out}
1
8
6
4
2
0

comparing rate model and network (EI network)





For EI network have: $\rho_{EE}, \rho_{EI}, \rho_{IE}, \rho_{II}$

This leads to four coupled equations for synaptic ouputs:

$S_{EE}, S_{EI}, S_{IE}, S_{II}$

Conclusions

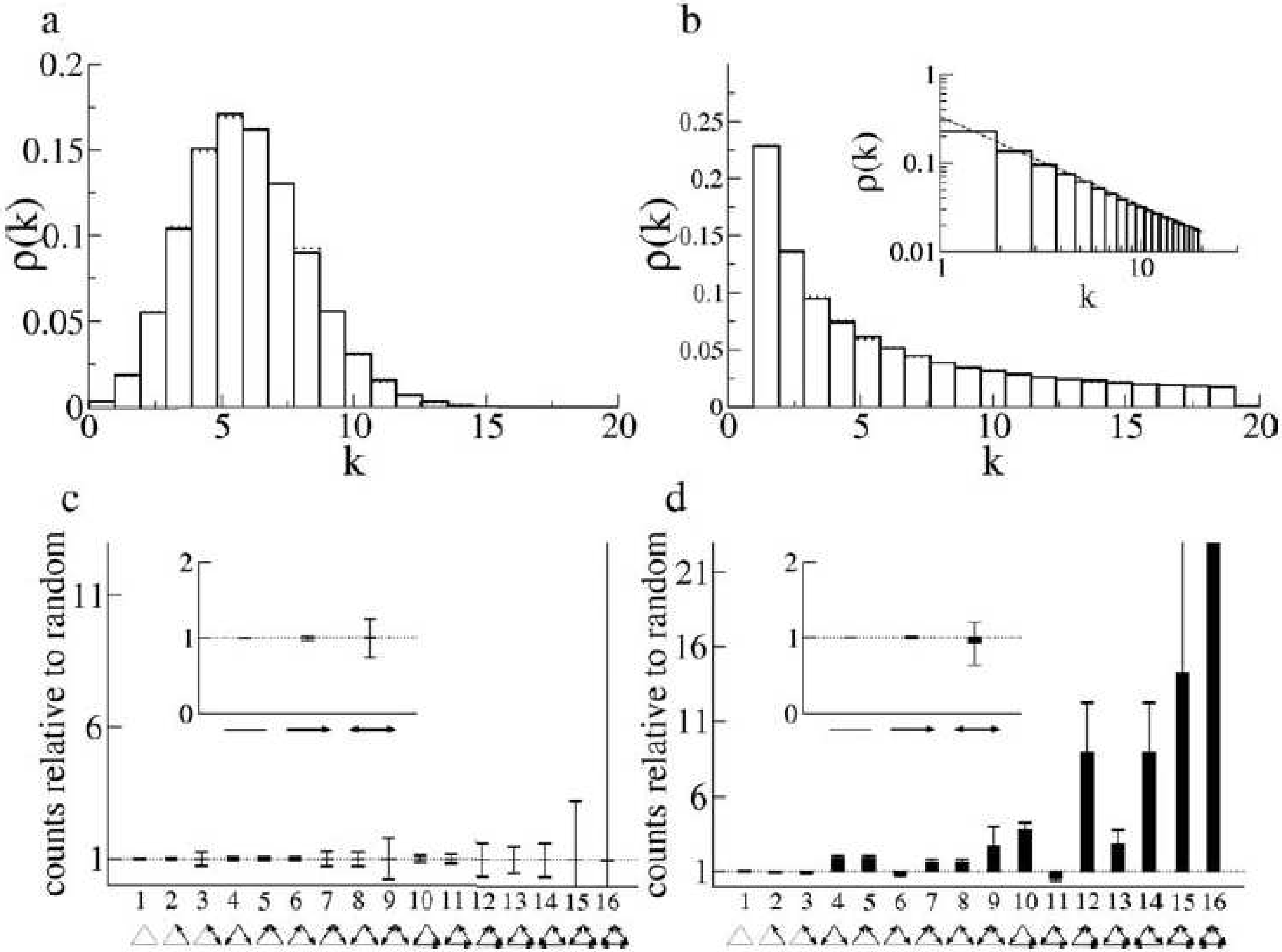
1. Second-order statistics of connectivity in meanfield model

2. Convergent motif (variance of in-degree) changes gain

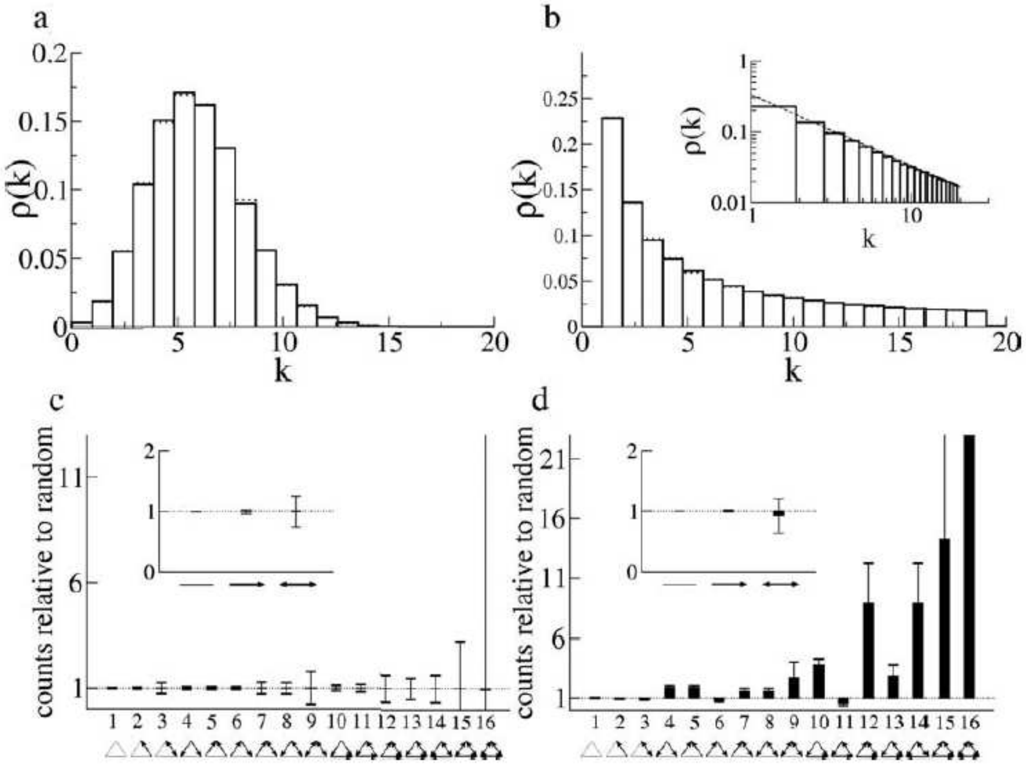
3. Chain motif (covariance of degrees) changes effective coupling

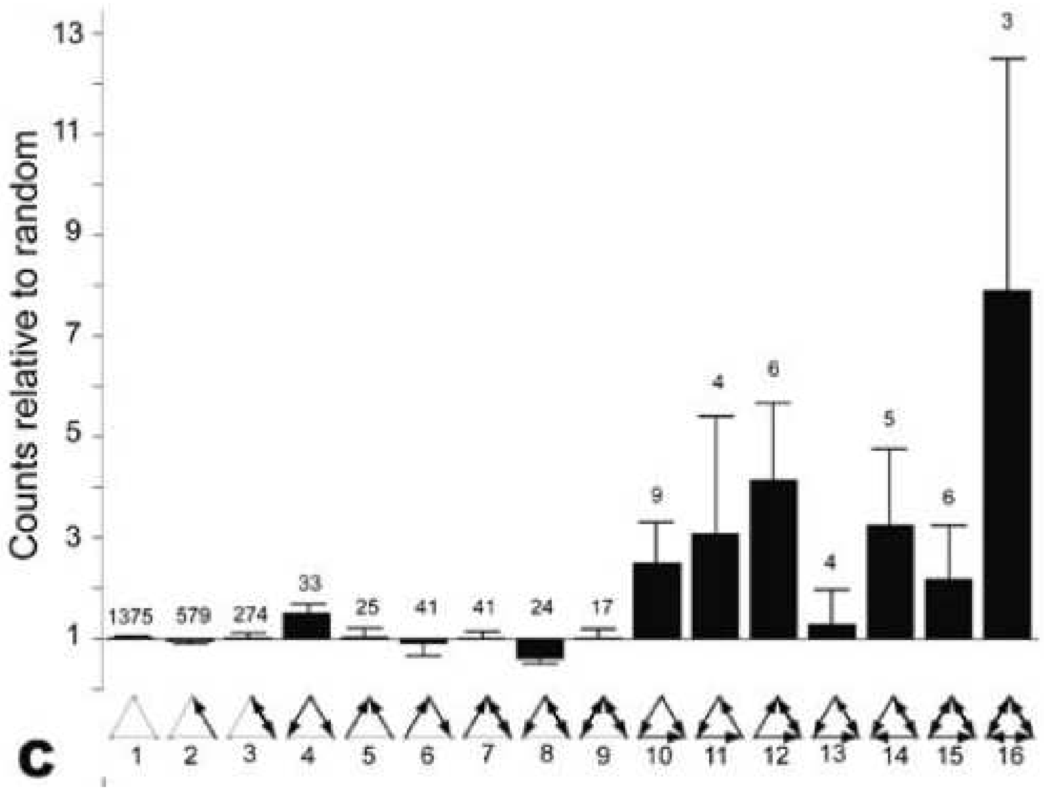
4. Meanfield dynamics of 2 populations (EI) really 4D.

5. Divergent motif? (correlations)

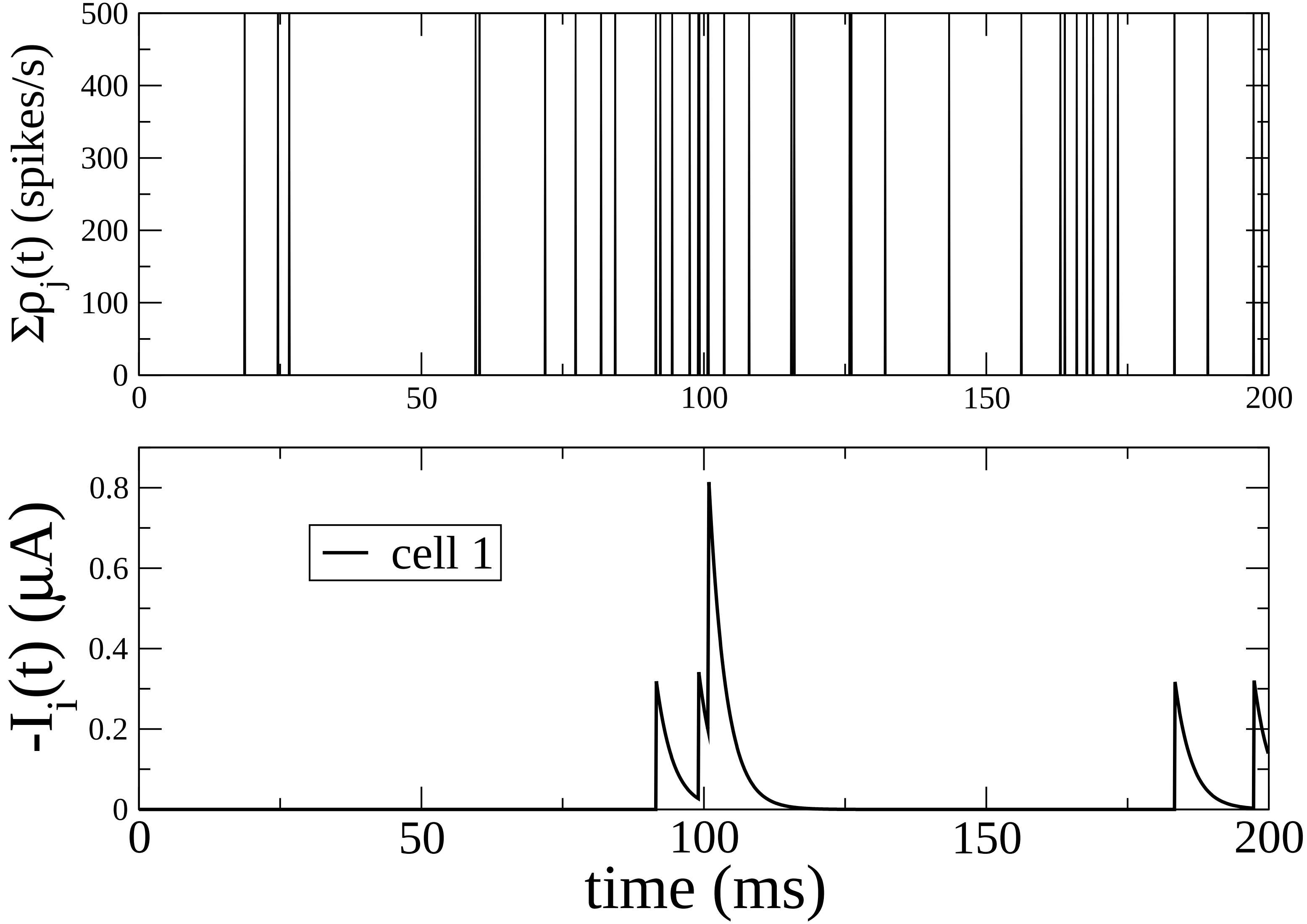


Roxin et al. J. Neurosci. 2008.

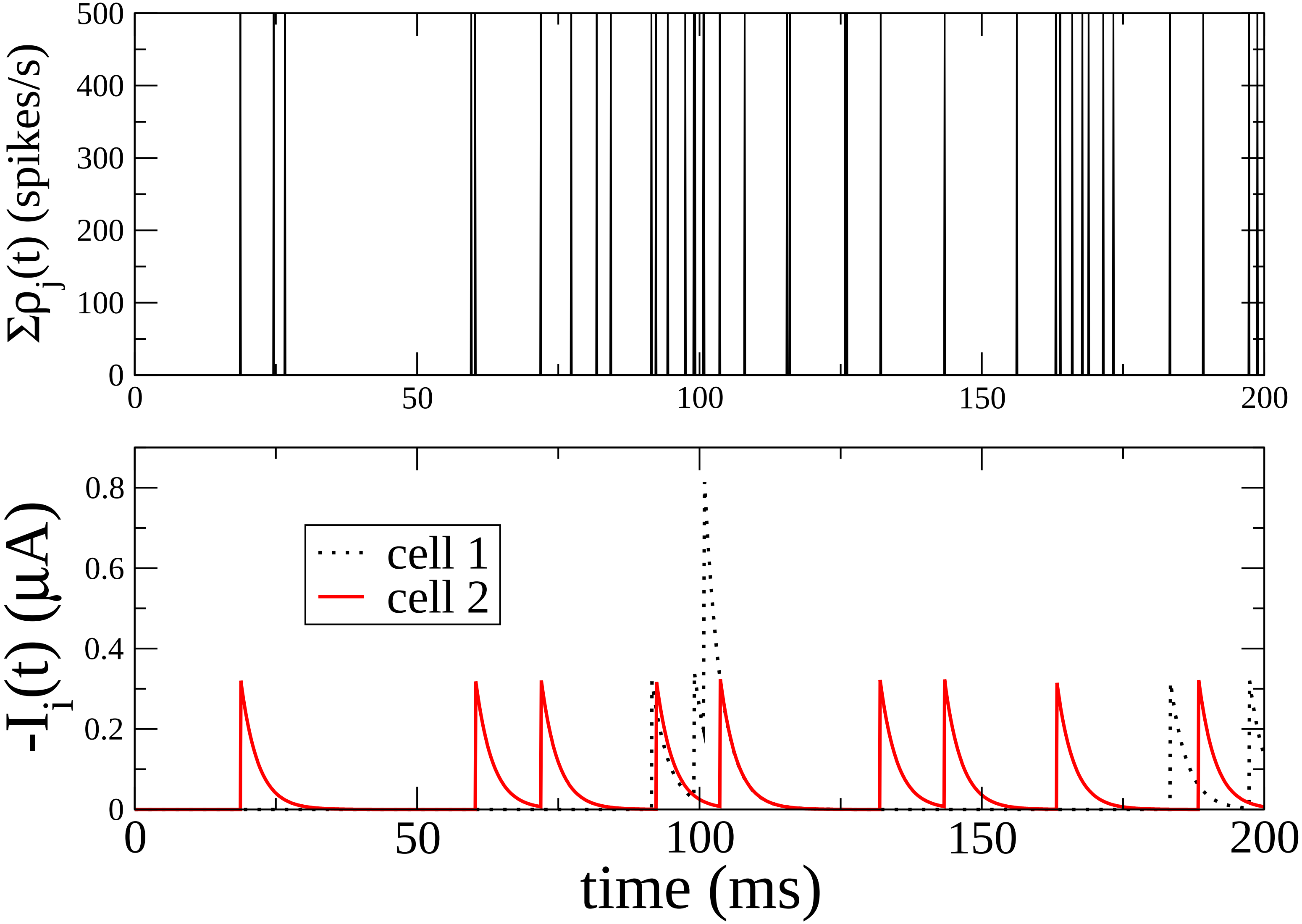

a
ρ(k)
k
b
ρ(k)
k
c
counts relative to random
d
counts relative to random


Counts relative to random
1375
579
274
33
25
41
41
24
17
9
4
6
4
5
6
3
c

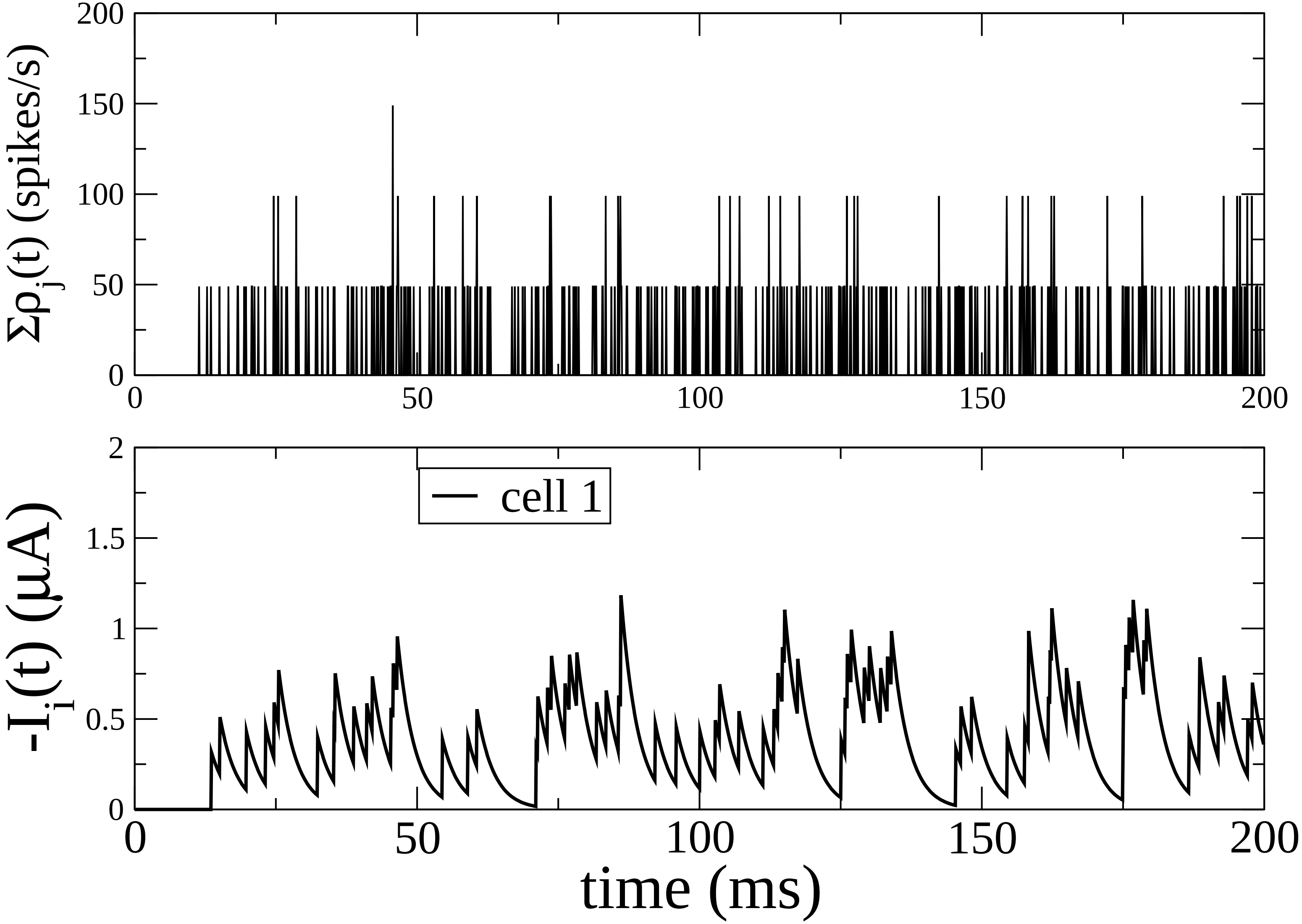
Network of 20 inhibitory cells and $p = 0.2$

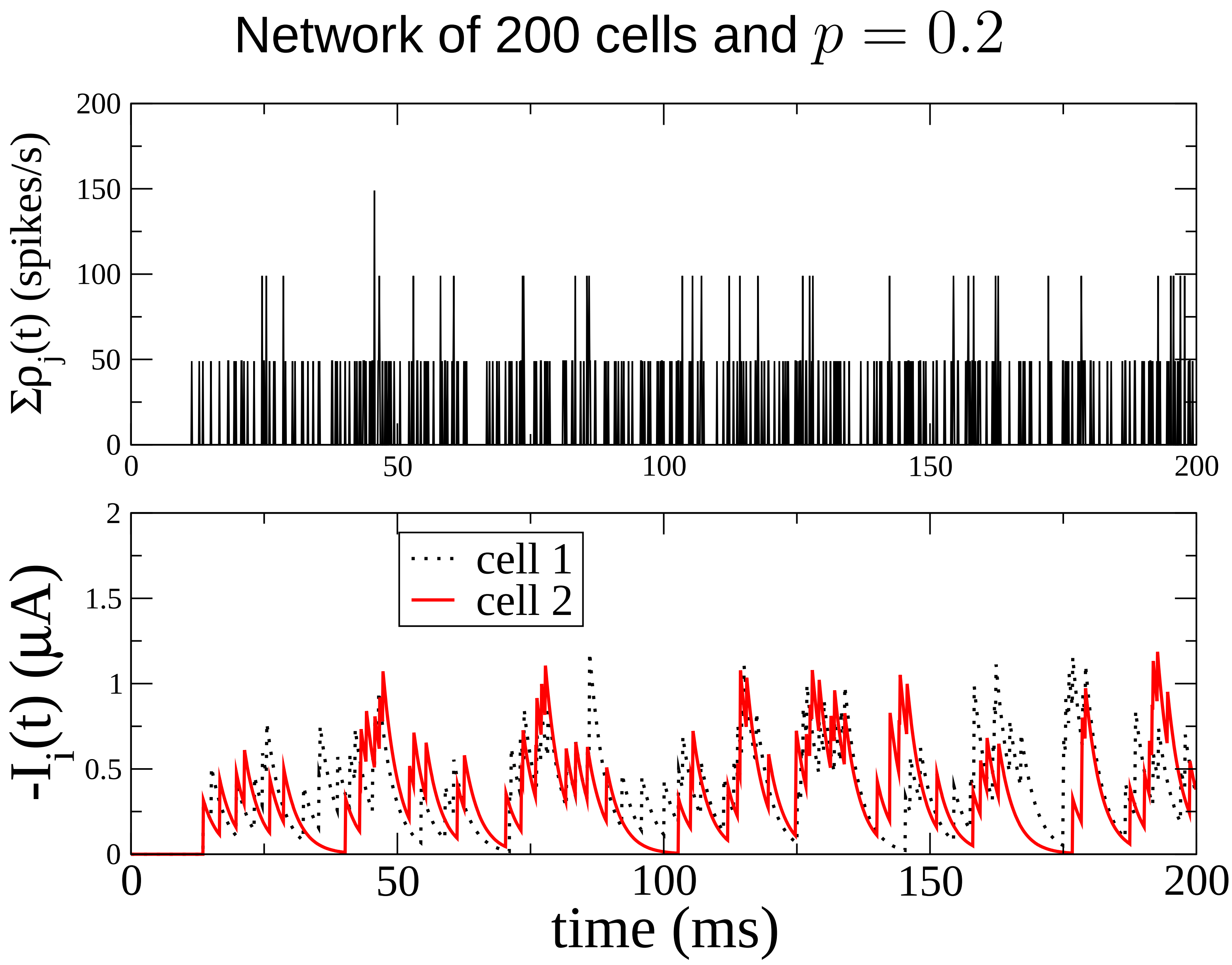


Network of 20 cells and $p = 0.2$



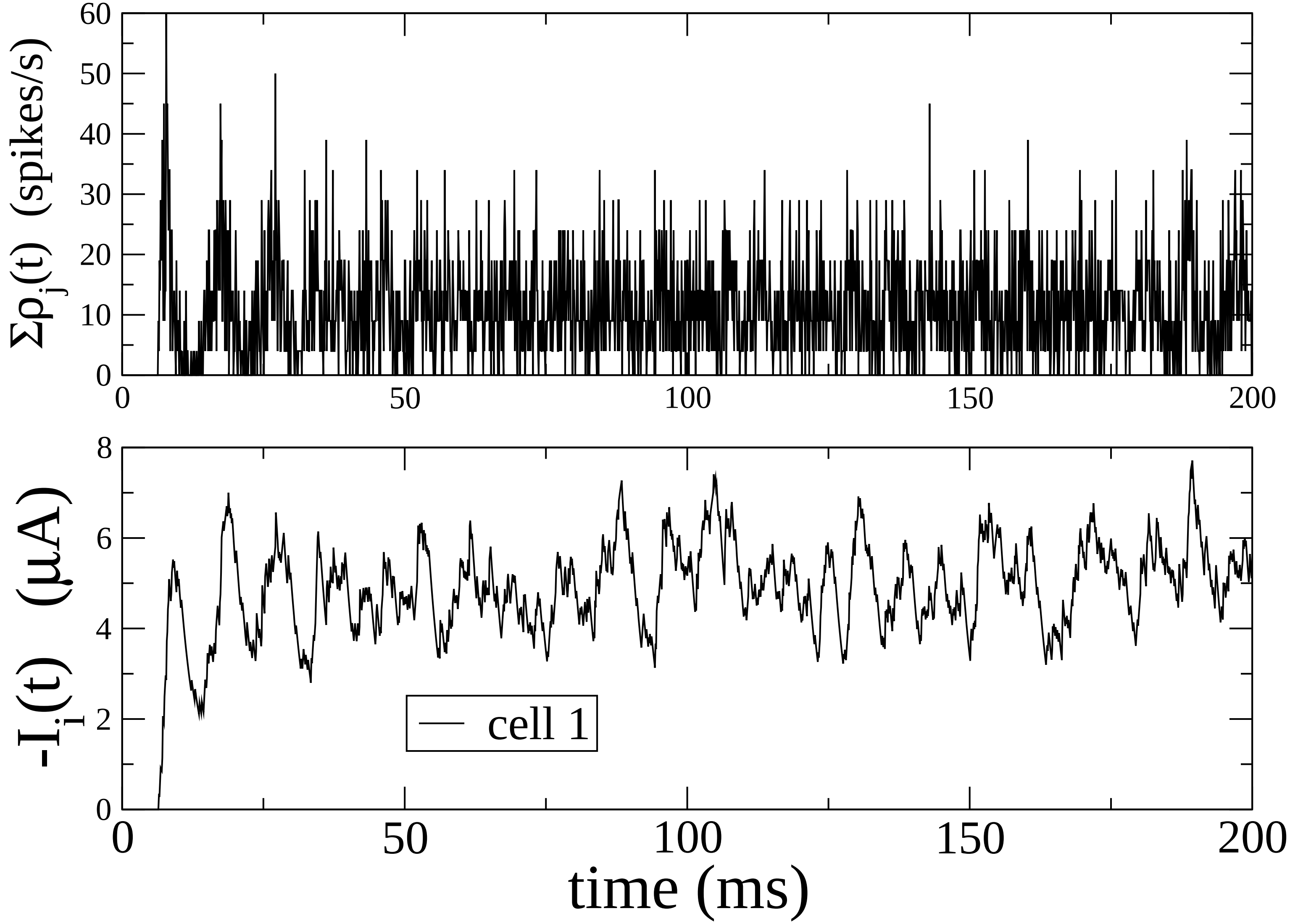
Network of 200 cells and $p = 0.2$



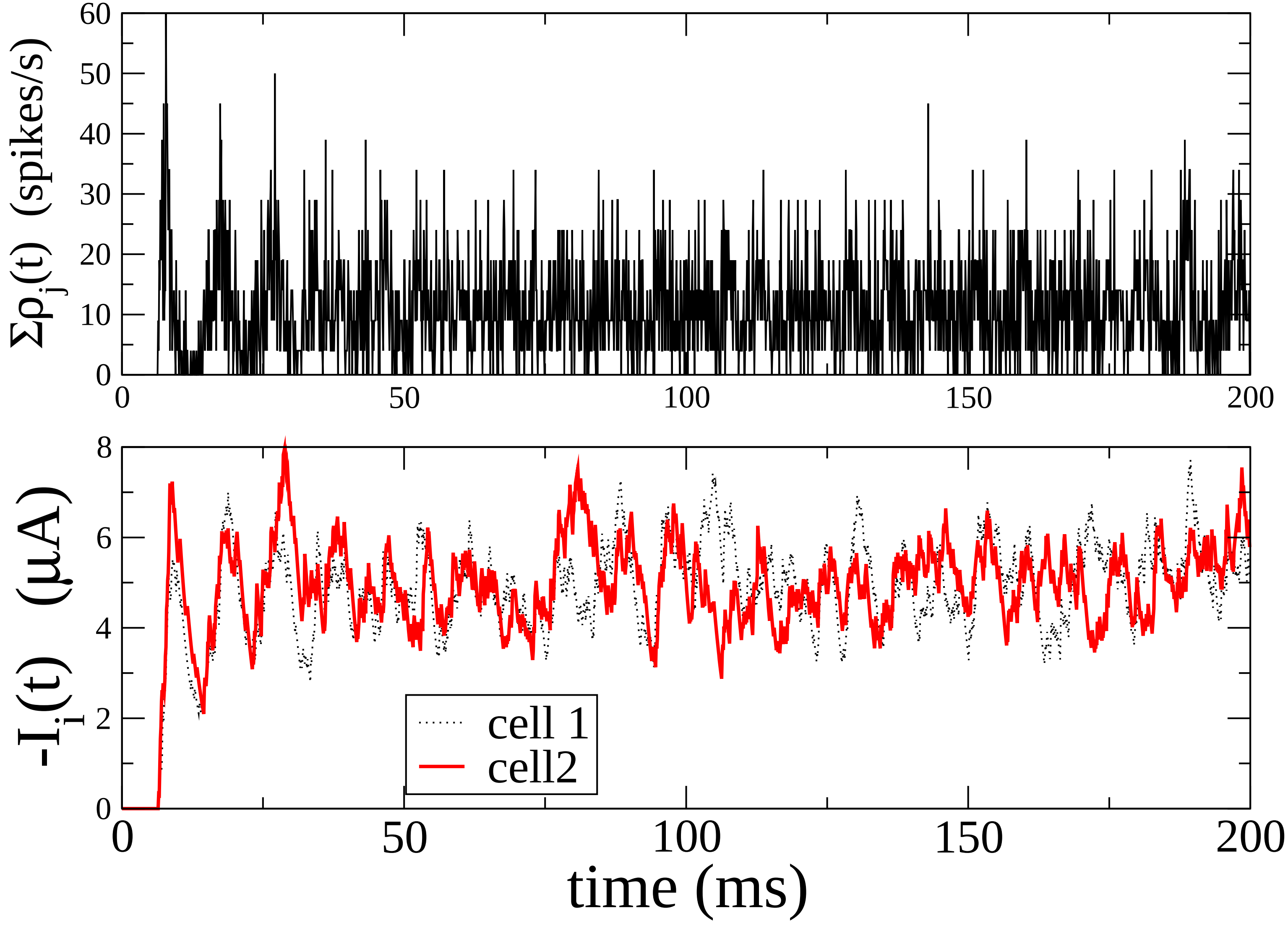


Network of 200 cells and $p = 0.2$
$\Sigma\rho_j(t)$ (spikes/s)
200
150
100
50
0
0
50
100
150
200
$-I_j(t)$ (μA)
2
1.5
1
0.5
0
cell 1
cell 2
0
50
100
150
200
time (ms)

Network of 2000 cells and $p = 0.2$ sparseness.



Network of 2000 cells and $p = 0.2$ sparseness.



What I will show:

1. If in-degree and out-degree are uncorrelated, only in-degree affects firing rates.

2. If they are correlated, both affect rates.

3. These rate effects can be captured in a rate model.

4. Out-degree determines current cross-correlations.