Bibliography of the course:
THEORY OF LIMIT CYCLES

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Abstract: We introduce several tools to study the number of limit cycles for families of planar ordinary differential equations, paying special attention on the polynomial case.

Lesson 1: Poincaré map and local study of periodic orbits: Poincaré formula [1,2]. A geometric interpretation of the formula in terms of the curvature of the orthogonal vector field [3]. An application: quadratic systems with two invariant curves [4]. Codimension 1 bifurcations involving limit cycles [5,6,7].

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Lesson 2: A miscellanea of useful tools: Rational parameterizations [8,9]: applications to Fermat's equation [10] and to a function involving polynomials and square roots [8]. Resultants [11,12]. Poincaré-Miranda theorem: applications to find a counterexample to Kouchnirenko conjecture and to find 3 limit cycles for a linear piecewise planar system [13,14]. Chebyshev systems [15,16].

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Lesson 3: Lower bounds for the number of limit cycles: Abelian integrals [17]. Application to an integrable system [9], to semi-homogeneous systems [14], and to a cubic Hamiltonian: Picard-Fuchs equations [17,18,19].

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Lesson 4: Upper bounds for the number of limit cycles: Bendixson-Dulac Theorem [20,21,22,23]. Van der Pol equation [22], a generalized linear system [24], a Liénard system with 3 limit cycles [21] and rigid systems [14,23].

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