



20th winter school in Dynamical Systems of the DANCE Spanish network

(Badajoz, 29 January 2024 - 02 February 2024)

Triangulation of High-Dimensional Toroidal Manifolds from Quasi-Periodic Three-Body Problem Dynamics

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Abstract.- This work presents an extension of a genus-1 point cloud triangulation method to high-dimensional (4D and 6D) point clouds sampled along quasi-periodic orbital trajectories in the circular-restricted three-body problem. Specifically, it allows triangulation of the underlying 2-torus manifolds on which solution points lie in the 4D-6D state space. The resulting triangulations provide novel surface-based representations of high-dimensional topologies which could previously only be represented as point clouds. These results lay the foundation for new computational models and visualizations of high-dimensional solution spaces for dynamical systems. Such representations promise to enhance the utility of the three-body problem for the modelling, visualization, and design of space mission trajectories.



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Quartic rigid systems in the plane and in the Poincaré sphere.

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Abstract.- We consider the planar family of rigid systems of the form $x' = -y + xP(x, y)$, $y' = x + yP(x, y)$, where P is any polynomial with monomials of degree one and three. This is the simplest non-trivial family of rigid systems with no rotatory parameters. The family can be compactified to the Poincaré sphere such that the vector field along the equator is not identically null. We study the centers, singular points and limit cycles of that family on the plane and on the sphere.



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The dynamics of coupled logistic maps

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Abstract.- We consider the system of difference equations

$$(1) \quad \begin{cases} x_{n+1} = (1 - \varepsilon)f_{\mu}(x_n) + \varepsilon f_{\mu}(y_n), \\ y_{n+1} = \varepsilon f_{\mu}(x_n) + (1 - \varepsilon)f_{\mu}(y_n), \end{cases}$$

where $\varepsilon \in [0, 1]$ is a coupling parameter and $f_{\mu}(x) = \mu x(1 - x)$, $\mu \in (0, 4]$, $x \in [0, 1]$, is the classical logistic family widely studied [5]. This model was proposed and studied in [3, 4] and it is motivated by difference equations associated to the Belousov-Zhabotinsky chemical reaction [1]. This model is included in the family of the so-called dynamics of coupled map lattices (see e.g. [2]). We study the synchronization properties of this family of maps and solve an open question posed in [3] by proving that the dynamics of the model need not be lying on the diagonal set $\Delta = \{(x, x) : x \in \mathbb{R}\}$.

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Transition equations with nonautonomous asymptotic dynamics

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Abstract.- We consider a scalar transition equation that asymptotically approximates as time increases and as time decreases to nonautonomous equations with concave derivative and robust hyperbolic dynamics. The skewproduct formalism allows us to understand the transition as a heteroclinic orbit between past and future and to classify the global dynamics. Some population models that fit this description are shown.



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Neural bursting chaos can be found far from modal transitions

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Abstract.- We report the presence of widespread chaotic dynamics in a modified version of the Plant locomotor model. The model exhibits three characteristic types of voltage activity: tonic-spiking, bursting, and hyperpolarized quiescence. Two bifurcation parameters are added to the original conductance based model to control the slow dynamics, which are responsible for modulating spiking activity. A bifurcation diagram of the model with these parameters is presented. A transition route from tonic-spiking to bursting activity begins with a saddle-saddle bifurcation and is followed by an adjacent region of transitional chaos associated with a codimension-two Belyakov bifurcation point. Transitioning from a stable equilibrium state, representing neural quiescence, to bursting passes through a Andronov-Hopf bifurcation curve with a codimension-two Bautin point. On the left of the Bautin point, the bifurcation is subcritical causing an onset of complex bursting with chaotic subthreshold oscillations or with unpredictably varying trains of spikes and coincides with a Shilnikov-Hopf saddle-focus bifurcation, while on the right it is supercritical giving rise to small subthreshold oscillations morphing into a large bursting pattern. Finally, in the bursting region we find spike-adding chaos at transitions between periodic attractors associated with successive homoclinic bifurcations.



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Elliptic Restricted Three-Body Problem: Some results about the ejection-collision orbits.

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Abstract.- In this work we have studied the Elliptical Restricted Three-Body Problem (Elliptical RTBP), from a theoretical and numerical point of view. In the first part, we will see first the Circular Restricted Three-Body Problem, its formulation and some properties, and then we will see the same with the Elliptical RTBP, to then be able to compare both. In the second part, we will introduce a regularization method, the so-called Levi-Civita method, and then we will deduce the equations of motion in the new variables and also some properties for, again, the two problems: Circular and Elliptical RTBP. In the last part we will perform a numerical analysis in order to find a particular kind of orbits, the ejection-collision orbits. In order to do this, we will present a definition and a characterization that will allow us to compute these orbits, following an algorithm that we will explain step by step. Finally, we will see some numerical simulations regarding the number of these orbits, varying the different parameters that we have. By doing this, we will see that for a certain range of the variables, some discontinuities will appear, so we will also study why these phenomena take place.



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Sectorial decomposition of analytic diffeomorphisms in dimension 2

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Abstract.- We propose a sectorial decomposition of non center-focus analytic diffeomorphism tangent to the identity that can have curves of fixed points. There is a previous work of Dumortier, Roussarie and Rodrigues in which they treat this problem, using different technics, for $\mathcal{C}^\infty$ diffeomorphism which fulfill a Lojasiewicz condition, in particular, avoiding the existence of curves of fixed points.



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Geometrical perspectives of the valuative Nagata conjecture

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Abstract.- The Nagata conjecture arises as a result of the negative answer to the 14th Hilbert problem [5]. Despite many efforts this conjecture remains open after more than 60 years. There exist several reformulations using interesting objects in Algebraic Geometry as Seshadri constants, Mori cone and irrational nef divisors. Recently, a valuative Nagata conjecture has been stated in [2, 4]. This conjecture involves a (real) plane valuation ν of the projective plane and a constant named Seshadri-type constant, denoted $\hat{\mu}(\nu)$ and introduced in [1]. In addition, the valuative Nagata conjecture implies the Nagata conjecture and asymptotic evidences of its truthness in some particular cases have been proved in [4]. In this talk we introduce a natural Seshadri constant for a smooth projective surface S , a nef divisor on S and a divisorial plane valuation of S and we show some of their properties. These results will help us to give several equivalent geometrical statements to the valuative Nagata conjecture for divisorial plane valuations.

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Splitting of 2D invariant manifolds of a periodically forced 3D volume preserving flow

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Abstract.- We consider a 3D volume-preserving flow with two saddle-foci equilibria and we study the splitting of their 2-dimensional stable and unstable invariant manifolds. The unperturbed system is obtained by truncating the normal form of a generic unfolding of a Hopf-Zero bifurcation and has a heteroclinic connection to the saddle-foci. We describe the effect of a non-autonomous periodic perturbation on the unperturbed system which introduces an additional frequency. The interaction of this frequency with the one already present in the unperturbed system causes a quasi-periodic asymptotic behaviour of the splitting of the 2D stable and unstable invariant manifolds.



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Computation of rational first integrals of polynomial foliations by means of algebraic geometry

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Abstract.- The algebraic integrability problem asks for the existence of a rational first integral of a given foliation defined on the complex plane and for its computation in the affirmative case. We extend polynomial foliations to foliations on the projective plane or on Hirzebruch surfaces and present an algorithm which gives an answer to the algebraic integrability problem knowing of the genus of the expected rational first integral. We use algebraic geometry tools to calculate a divisor on the surface obtained after blowing up the dicritical singularities of the foliation that provides us with information about the algebraic integrability of the foliation. This is a joint work with Carlos Galindo and Francisco Monserrat.



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Jet transport for General Linear methods

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Abstract.- In this project, we study how the computational technique called “jet transport” can be applied to the family of numerical integrators known as General Linear methods (GLM), which are a natural generalization of the well-known Linear Multistep (LMM) and Runge-Kutta (RK) methods. Jet transport can be defined as the application of automatic differentiation to a numerical integrator in order to obtain the solution of the variational equations (VE); that is, the linear differential equations that are satisfied by the derivatives of the solution of an initial value problem (IVP). In this context, we prove that using a GLM, the numerical integration of an IVP with jet transport is equivalent to the numerical integration of the VE. We also derive the expressions that the high-order coefficients of jets must satisfy to be a solution of an implicit system, allowing the implementation of implicit General Linear methods with jet transport for the numerical integration of the so-called “stiff” problems. Finally, we apply this implementation to study some scenarios in the field of dynamical systems, such as the computation of periodic orbits and the power expansion of Poincaré maps.



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Robust Estimation for Multivariate Linear Model based on τ -scale, with Full Rank and Reduced Rank

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Abstract.- In this study, we focus on τ -scale robust estimators for multivariate linear models, simultaneously estimating regression coefficients and the error covariance matrix. We minimize the determinant of the covariance matrix with a constraint on a robust scale of Mahalanobis norms of residuals, employing a τ -scale as a robust measure. We address cases of both full and reduced rank in the joint parameter estimation, developing weighted equations and analyzing consistency and asymptotic behavior. Comparative simulations with standard estimators are included, along with an illustration of the asymptotic distribution for Gaussian errors. This approach contributes to a detailed understanding of joint parameter estimation in multivariate linear models.



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On the number of limit cycles for piecewise holomorphic systems

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Abstract.- In this work we are concerned with determining the number of limit cycles for piecewise holomorphic systems with a straight line of discontinuity. For that, we give the general formulas to calculate the Lyapunov quantities for these types of systems and we use these quantities to generate limit cycles for the focus-focus case. Also, the averaging method is employed to study bounds of limit cycles in the center-center case. In addition, we provide sufficient conditions to ensure that piecewise linear holomorphic systems have at least 3 limit cycles bifurcating from the infinity. Finally, we give families of this class of systems that possess algebraic limit cycles.



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Asymptotic motions for time-dependent perturbations of Hamiltonian systems having an invariant torus

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Abstract.- Dynamical systems subject to perturbations that decay over time are relevant in the description of many physical models, e.g. when considering the effect of a laser pulse on a molecule, in epidemiological studies, as well as in celestial mechanics. For this reason, in the present talk, we discuss two cases. First, we consider a time-dependent perturbation of a Hamiltonian dynamical system having an invariant torus supporting quasiperiodic solutions. Next, we analyze the case where the orbits associated with the unperturbed system are arbitrary. Assuming the perturbation decays polynomially fast in time in the first case and exponentially fast in the second one, we prove the existence of orbits converging as time tends to infinity to the dynamics associated with the corresponding unperturbed system. The present work is motivated by the example of the planar three-body problem perturbed by a given comet coming from and going back to infinity asymptotically along a hyperbolic Keplerian orbit, modeled as a time-dependent perturbation.



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Arnolg diffusion in the elliptic Hill restricted four body problem

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Abstract.- We will speak about an elliptic restricted four body problem (ER4BP). This problem describes the movement of a particle of infinitesimal mass, that moves under the gravitational force produced by the three point masses, called primaries with masses m_1, m_2 and m_3 . The primary bodies are in a homographic solution of the general three body problem, which corresponds to the well known equilateral central configuration. The primaries are moving in elliptical orbits with small enough eccentricity. If we consider that the mass m_3 is infinitesimal with respect to m_1 and m_2 , then we can assume that $m_3 \rightarrow 0$ and thus we generate the elliptic Hill restricted four body problem (EHR4BP), in a similar way as in the classical Hill's problem, and precisely in this EHR4BP we prove Arnold diffusion. This EHR4BP was motivated by the Sun - Jupiter - 624 Hektor (Jupiter's Trojan asteroid) - Skamandrios (Hektor's moon) system. Here, Sun - Jupiter - 624 Hektor are the primary bodies and the particle of infinitesimal mass is Scamandrios, which is a moon of the 624 Hektor Trojan asteroid. Evidently the mass of 624 Hektor is infinitesimal compared to the mass of the Sun and Jupiter, so we can formulate the aforementioned EHR4BP.

The EHR4BP can be written as the circular Hill restricted four body problem (CHR4BP) plus a time - dependent perturbation, where the small perturbation parameter is the eccentricity. Consecutively, to prove Arnold diffusion is necessary to study some dynamic aspects of the unperturbed system. We show that there exists a normally hyperbolic invariant manifold (NHIM) foliated by invariant tori, and whose stable and unstable manifolds intersect transversally. When we choose the NHIM, an inner and outer dynamic is generated, determined by the Hill region of the CHR4PB.

The main ingredient to prove Arnold diffusion is to define the scattering map. Given a transverse intersection of the stable and unstable manifolds, we can associate a mapping from a subset of the NHIM to the NHIM called the scattering map. Given a point in the intersection of stable and unstable manifolds, the scattering map associates to the asymptotic orbit in the past, the asymptotic orbit in the future. Finally, another important ingredient is verifying the properties of the Melnikov integrals on the NHIM. Roughly, the diffusion mechanism consists of verifying that the Melnikov-type integrals do not vanish and that some manifolds are transversal.

This is a joint work with **Marian Gidea**, Yeshiva University, and **Jaime Burgos**, Universidad Autónoma de Coahuila.

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Sufficient conditions for the Schur stability of polynomials

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Abstract.- One of the fundamental problems in the field of dynamical systems is examining the local stability of steady-state solutions. This issue entails examining the linearized system's eigenvalues or the characteristic polynomial's roots. In continuous dynamical systems, local stability is achieved when the eigenvalues are within the left half-plane. On the other hand, local stability is attained in a discrete-time dynamical system when the eigenvalues are positioned within the unit disk. A Mobius transformation is capable of establishing the parallels between the two cases. In high-dimensional spaces or systems with parameters, the problem of ascertaining the precise locations of the zeros is known to be a formidable challenge. The Jury stability algorithm is widely recognized and regarded as the most advanced approach for addressing this particular task; however, it has some drawbacks. In this presentation, the authors employ a global stability technique and provide sufficient conditions that ensure the zeros are within the unit disk.