

Book of abstracts

RADS 2026

Oviedo, June 1-5, 2026



Contents

1	Plenary sessions	5
1.1	Linear response for skew-product maps (José F. Alves ¹)	5
1.2	Aperiodic classes of C^1 -generic diffeomorphisms (Christian Bonatti ¹ ; Katsutoshi Shinohara)	6
1.3	Flexibility and Ubiquity of Ergodic Nonhyperbolicity (Lorenzo J. Díaz ¹)	7
1.4	Creating limit cycles from canard cycles in dimension 2 (Freddy Dumortier ¹)	8
1.5	Heteroclinic and homoclinic orbits for some delay differential equations (Tibor Krisztin ¹)	9
1.6	Computer-assisted proofs of existence of KAM tori in a planetary dynamical model of the orbital motion of v -And $\mathbf{b}$ (Ugo Locatelli ¹)	10
1.7	No infinite spin for total collisions in the spatial N-body problem (Gabriella Pinzari ¹ , Piotr Zgliczynski)	11
1.8	Localised structures in spatially extended systems (Jens Rademacher ¹)	12
2	Communications	13
2.1	Qualitative Planar Dynamics with a Star Nodes and Homogeneous Nonlinearities: Beyond Hilbert's 16th Problem (<u>Begoña Alarcón Cotillas</u> ¹)	13
2.2	The topology behind the affine symmetries of the Julia set induced by a polynomial (<u>Víctor Álvarez-Aparicio</u> ¹)	14
2.3	Poincaré's stratification for vector fields in $\mathbb{R}^3$ (<u>Clementa Alonso-González</u> ¹)	15
2.4	Dynamical Systems in Sobolev Spaces (Assis Azevedo, <u>Davide Azevedo</u> ¹ , Joana Torres, Mário Bessa)	16
2.5	Geometry of bifurcation sets: exploring the parameter space (<u>Roberto Barrio</u> ¹ , Santiago Ibáñez ² , Lucía Pérez ²)	17
2.6	When does a nonlinear wave survive a perturbation? A spectral approach for integrable systems (<u>Marcos Caso-Huerta</u> ¹ , Antonio Degasperis ² , Sara Lombardo ³ , Matteo Sommacal ⁴)	18
2.7	Asymptotic density of limit cycles (<u>Bartomeu Coll</u> ¹ , Armengol Gasull, Rafael Prohens)	20
2.8	Dynamics of the Nonautonomous Bonifacio–Lugiato Model: A Concave–Convex Equation Framework (<u>Jesús Dueñas</u> ¹ , Carmen Núñez, Rafael Obaya)	21
2.9	On the dynamics of planar continuous piecewise affine maps (<u>Armengol Gasull</u> ¹)	22

2.10	Formal Expansions beyond ODE Perturbation Theory (<u>Joan Gimeno</u> ¹)	24
2.11	A dynamical explanation for the elusive nature of Kordylewsky clouds (Joan Gimeno, <u>Marc Jorba-Cuscó</u> ¹ , Begoña Nicolás, Àngel Jorba)	25
2.12	Singular perturbation theory in a cardiomicite model with three time scales (<u>Jorge A. Jover Galtier</u> ¹ , Roberto Barrio ¹ , Lucía Pérez ²)	26
2.13	Stable periodic motions in a bouncing ball model (<u>Stefano Marò</u> ¹)	27
2.14	Dynamics of the quadratic family: the origin of the two-dimensional strange attractors (<u>Alejandro Marqués-Lobeiras</u> ¹ , Antonio Pumariño ² , José Ángel Rodríguez ² , Enrique Vigil ²)	28
2.15	From statistical properties to young structures in partially hyperbolic systems (<u>João S. Matias</u> ¹)	30
2.16	Unfolding of a double resonance at a normally elliptic invariant curve in 3D volume-preserving maps (<u>A. Murillo</u> ¹ , A. Vieiro ¹)	31
2.17	Lorenz dynamics prevail under compressibility (<u>Juan Pello</u> ¹ , Antonio Pumariño ¹ , Santiago Ibáñez ¹)	32
2.18	Refining the Dynamic Modeling of Rocking Motion (<u>Set Pérez-González</u> ¹ , Jesús Suárez Pérez del Río ¹ , Francisco Prieto ¹)	33
2.19	On the stability chart for reversible Hill's equations (Emilio Freire, Manuel Ordóñez, <u>Enrique Ponce</u> ¹)	34
2.20	The Bosch and Simó conjecture on the Shilnikov-Hopf bifurcation (<u>Alexandre Rodrigues</u> ¹)	35
2.21	Dimension Reduction and Global Attraction in Delayed Wilson-Cowan Neural Networks (<u>Alfonso Ruiz Herrera</u> ¹)	36
2.22	Trajectories of vector fields asymptotic to formal invariant curves (<u>F. Sanz Sánchez</u> ¹)	37
2.23	Nonsmooth bifurcations and fractalization for families of quasiperiodically forced one-dimensional piecewise linear maps (<u>Joan Carles Tatjer</u> ¹)	38
2.24	Non-uniform expansion and diffusive noise imply dense random horseshoes and existence of a random Gibbs-Markov map (<u>Giuseppe Tenaglia</u> ¹ , Dmitry Turaev, Jeroen Lamb)	39
2.25	Discrete averaging for maps (<u>A. Vieiro</u> ¹)	40
3	Poster sessions	41
3.1	Pattern formation in an active-passive lattice gas (<u>Jesús Ait Idir</u> ¹ , Maria Bruna, Jose Carrillo)	41

3.2	Tatjer tangencies in periodic perturbations of generic unfoldings of the three-dimensional nilpotent singularity. (<u>P. Álvarez-Lombardero</u> ¹ , A. Pumariño ¹ , J. A. Rodríguez ¹)	42
3.3	Spectral Measures for Multivariate Extremes in Dynamical Systems (<u>Alessio Corveddu</u> ¹)	43
3.4	Stochastic Properties of Non-Uniformly Hyperbolic Dynamics (<u>Paolo Flavoni</u> ¹) .	44
3.5	Local behavior of Taylor and Padé methods for stiff ODEs (<u>Philip P. Forrier</u> ¹ , Joan Gimeno ¹)	46
3.6	Dynamic methods in non-autonomous differential equations. Stability, bifurcation and critical transitions (<u>Paula Martínez Morais</u> ¹)	48
3.7	Extreme synchronization for product Dynamical Systems (<u>Duarte Sá Pinho</u> ¹)	49
3.8	The averaging principle for ordinary differential equations depending on the slow time (<u>Carmen Núñez</u> ¹ , Rafael Obaya ¹ , <u>Jorge Rodríguez</u> ¹)	50
3.9	Finitude of Limit Cycles of Linear Piecewise ODEs in the Cylinder (<u>J.L. Bravo</u> ¹ , <u>R. Trinidad Forte</u> ¹)	51

1 Plenary sessions

1.1 Linear response for skew-product maps (José F. Alves¹)

(1) *University of Porto, Portugal.*

Abstract

We study linear response for families of skew-product dynamical systems with contracting fibres. Our approach is based on a sectional transfer operator acting on families of probability measures along the fibres. This operator allows us to describe invariant measures of the skew-product in terms of sample measures over the base dynamics, irrespective of whether the base map is invertible. Under general assumptions, we prove their differentiability with respect to system parameters in suitable Banach spaces. This is joint work with Wael Bahsoun.

1.2 Aperiodic classes of C^1 -generic diffeomorphisms (Christian Bonatti¹; Katsutoshi Shinohara)

(1) *C.N.R.S. and Institut de Mathématiques de Bourgogne, France.*

Abstract

The dynamics of (non conservative) homeomorphisms of compact metric set admits a canonical decomposition in chain recurrence classes (Conley). For C^1 -generic diffeomorphisms, the chain recurrence class of a periodic point is its homoclinic class ([B-,Crovisier] based on Hayashi connecting lemma). However there are locally C^r -generic diffeomorphisms having uncountably many *aperiodic classes*, i.e. chain recurrence classes without periodic orbit. Up to recently all such aperiodic classes was *adding machines*. We present C^1 open sets of partially hyperbolic diffeomorphisms in dimension 3 present a wide variety of dynamical behaviors: minimal but non uniquely ergodic, or at the contrary uniquely ergodic but non transitive, etc. This work consists in 4 long papers ending with [Invent. Math. 238 (2024), 63pages.] In this talk, I will present the motivation, the setting of our example and the different steps of the construction, in particular the key notion of flexible periodic which may have many other applications.

Bibliography

- [1] Aperiodic chain recurrence classes of C^1 -generic diffeomorphisms Bonatti, Christian; Shinohara, Katsutoshi Invent. Math. 238 (2024), no. 2, 637–689.
- [2] A mechanism for ejecting a horseshoe from a partially hyperbolic chain recurrence class Bonatti, Christian; Shinohara, Katsutoshi Ergodic Theory Dynam. Systems 44 (2024), no. 8, 2080–2142.
- [3] Volume hyperbolicity and wildness Bonatti, Christian; Shinohara, Katsutoshi Ergodic Theory Dynam. Systems 38 (2018), no. 3, 886–920.
- [4] Flexible periodic points Bonatti, Christian; Shinohara, Katsutoshi Ergodic Theory Dynam. Systems 35 (2015), no. 5, 1394–1422.

1.3 Flexibility and Ubiquity of Ergodic Nonhyperbolicity (Lorenzo J. Díaz¹)

(1) *Pontifícia Universidade Católica do Rio de Janeiro, Brazil.*

Abstract

The context is robustly transitive and nonhyperbolic C^1 -diffeomorphisms, where there is abundance of nonhyperbolic ergodic measures (one Lyapunov exponent is zero).

Our assumptions involve partially hyperbolicity with a one-dimensional center direction, minimal foliations and blender-horseshoes. The huge range of applications includes fibered-by-circles systems, flow-type systems, certain Derived from Anosov diffeomorphisms, and some non-dynamically coherent examples.

We show that every feasible entropy value can be realized by such measures. We also prove that this class has a rich structure within the space of ergodic measures. We also discuss fractal-like properties of the set of points whose center Lyapunov exponent is zero.

This is joint work with K. Gelfert, M Rams, and J Zhang.

1.4 Creating limit cycles from canard cycles in dimension 2 (Freddy Dumortier¹)

(1) *Hasselt University, Belgium.*

Abstract

The canard cycles in the talk are simple closed curves in a slow-fast system, consisting of a succession of fast orbits and curves of singularities, along which one has alternating normally hyperbolic attracting behaviour and normally hyperbolic repelling behaviour. Canard cycles can be perturbed to limit cycles. A theoretical framework has been developed to study these limit cycles.

Besides the possible application to a thorough study of specific models, the theory is very useful in the study of Hilbert's 16th problem, asking for the maximal number of limit cycles a polynomial vector field can have, depending on its degree. Special attention will go to Liénard equations. We will give a survey of the most important techniques that can be used in proving the existence of limit cycles near the canard cycles. The canard cycles contain fast orbits, occurring in layers of fast orbits, and between these layers we restrict, along the curves of singularities, to generic Hopf breaking mechanisms and generic jump breaking mechanisms. As shown in previous talks, there is a difference in between transiting a terminal layer or transiting a dodging layer, but maybe more important is the precise way in which successive layers are connected to each other at the breaking mechanism. The essential data can be described by the connection diagram. Attention will be given to the so-called tunnelling behaviour and the opportunity it provides to create many limit cycles. We will show a number of applications of this technique. Based on the connection diagram we will provide some general results concerning these equations, especially related to totally balanced canard cycles.

We prove that all possible abstract connection diagrams can be realized by a multi-layer canard cycle of a planar polynomial vector field. We also precisely state the conditions under which a connection diagram can be realized by a polynomial Liénard equation.

1.5 Heteroclinic and homoclinic orbits for some delay differential equations (Tibor Krisztin¹)

(1) *University of Szeged, Hungary.*

Abstract

We consider delay differential equations of the form

$$y'(t) = -ay(t) + bf(y(t-1)) \quad (E_f)$$

with parameters $b > a > 0$ and Mackey–Glass-type nonlinear feedback functions $f(\xi) = \frac{\xi^k}{1+\xi^n}$, where $k > 0$ and $n > 0$.

The case $k = 1$ corresponds to the classical Mackey–Glass equation, while the case $k > 1$ appears in population models with an Allee effect. The case $k \in (0, 1)$ arises in certain economic growth models. The pointwise limit of f , as n tends to infinity, is the discontinuous $g : [0, \infty) \rightarrow [0, 1]$ with $g(\xi) = \xi^k$ for $\xi \in [0, 1)$, $g(1) = \frac{1}{2}$, and $g(\xi) = 0$ for $\xi > 1$. The special structure of g makes it possible to analyze the dynamics of the discontinuous equation

$$x'(t) = -ax(t) + bg(x(t-1)) \quad (E_g)$$

It is then shown that several dynamical properties of (E_g) persist for the original equation (E_f) when n is large.

For certain parameter values, stable periodic orbits with complicated structures are constructed. Heteroclinic connections between periodic orbits, as well as between equilibria and periodic orbits, are described. A threshold value $b^* = b^*(a) > a$ is obtained such that, for $b < b^*$, the dynamics is simple, whereas for $b > b^*$, the equation exhibits bistability. For the critical parameter value $b = b^*$, there exists a homoclinic orbit to a saddle point. In this case, a Shilnikov-type chaos is expected in a small neighborhood of the homoclinic orbit.

1.6 Computer-assisted proofs of existence of KAM tori in a planetary dynamical model of the orbital motion of *v*-And **b** (Ugo Locatelli¹)

(1) *Università di Roma Tor Vergata, Italy.*

Abstract

We study the problem of the orbital dynamics of the innermost exoplanet of the *v*-Andromedæ system (i.e., *v*-And **b**) into the framework of a Secular Quasi-Periodic Restricted Hamiltonian model. This means that we preassign the orbits of the planets that are expected to be the biggest ones in that extrasolar system (namely, *v*-And **c** and *v*-And **d**). The Fourier decompositions of their secular motions are injected in the equations describing the orbital dynamics of *v*-And **b** under the gravitational effects exerted by those two exoplanets. By a computer-assisted procedure, we prove the existence of KAM tori corresponding to orbital motions that we consider to be very robust configurations, according to the analysis and the numerical explorations that will be (briefly) discussed and summarized.

Based on a joint work with Rita Mastroianni.

1.7 No infinite spin for total collisions in the spatial N-body problem (Gabriella Pinzari¹, Piotr Zgliczynski)

(1) *University of Padua, Italy.*

Abstract

In the n -body problem, when bodies tend to a total collision, then its normalized shape curve converges to the set of normalized central configurations, which has $SO(3)$ symmetry in the planar case. This leaves a possibility that the normalized shape curve tends to the set obtained by rotations of some central configuration instead of a particular point on it. This is the *infinite spin problem* which concerns the rotational behavior of total collision orbits in the n -body problem. We show that the infinite spin is not possible if the limiting shape is isolated from other connected components of the set of normalized central configurations. Our approach extends the method from recent work for total collision for the planar case by Moeckel and Montgomery. The main tool is a full reduction $SO(3)$ -symmetry in a context of vanishing angular momentum. Include the acknowledgments if necessary

Bibliography

- [1] Gabriella Pinzari and Piotr Zgliczynski. No infinite spin for total collisions in the spatial N-body problem. <https://arxiv.org/abs/2604.22172>
- [2] Richard Moeckel and Richard Montgomery. No infinite spin for planar total collision. *J. Am. Math. Soc.*, 38(1):225–241, 2025.

1.8 Localised structures in spatially extended systems (Jens Rademacher¹)

(1) *University of Hamburg, Germany.*

Abstract

Spatially localised solutions are a characteristic phenomenon of nonlinear spatially extended systems. Classical examples in one spatial dimension are travelling waves such as interfaces in phase field models or pulses in driven-dissipative systems. Generally, a major theme is the existence and stability of such solutions on the real line or lattices. Beyond this, in the past decades also the interaction dynamics of sufficiently separated multiple localised structures has been broadly studied. The talk gives an introduction and then focusses on two directions in which much less is known, reporting results with various collaborators. The first concerns infinite time interaction that involves annihilation of localised states. Here, at the cost of losing spatial dispersion, more can be said for a class of cellular automata and their wave-related non-wandering sets and entropy. The second concerns self-organised motion of a localised state, which has been observed essentially numerically with evidence of chaotic dynamics. On a rigorous level such type of dynamics has recently been verified for a class of multi-component reaction diffusion systems through a singularity unfolding.

2 Communications

2.1 Qualitative Planar Dynamics with a Star Nodes and Homogeneous Non-linearities: Beyond Hilbert's 16th Problem (Begoña Alarcón Cotillas¹)

(1) *Fluminense Federal University, Rio de Janeiro, Brazil.*

Abstract

This is a full study of the dynamics of polynomial planar vector fields whose linear part is a multiple of the identity and whose nonlinear part is a homogeneous polynomial of arbitrary degree $n > 1$. It extends previous work by other authors that was mainly concerned with the existence and number of limit cycles. The general results are also applied to two classes of examples where the nonlinearities have degrees 2 and 3, for which we provide a set of phase portraits.

This is joint work with Isabel Labouriau and Sofia Castro, University of Porto (Portugal). Partially supported by the Spanish Research Project PID2020-113052GB-I00

Bibliography

- [1] B. Alarcón, S.B.S.D. Castro and I.S. Labouriau, Qualitative Planar Dynamics with a Star Nodes and Homogeneous Nonlinearities: Beyond Hilbert's 16th Problem, *To appear in Matemática Contemporânea*, 2026. <https://arxiv.org/abs/2106.07516>
- [2] B. Alarcón, S.B.S.D. Castro and I.S. Labouriau, Global planar dynamics with a star node and contracting nonlinearity, *Research in the Mathematical Sciences* 11-24 (2024).

2.2 The topology behind the affine symmetries of the Julia set induced by a polynomial (Víctor Álvarez-Aparicio¹)

(1) *University of La Rioja, Logroño, Spain.*

Abstract

The affine symmetries of the Julia set of a polynomial (and, more generally, of a rational map) provide relevant information about the dynamical system induced by the iteration of such polynomial, and play a crucial role in answering the following question: When does two polynomials have the same Julia set? In 1990, Alan Beardon provides a complete characterization of such case and describes the group of affine symmetries of a Julia set induced by a polynomial by means of Green functions, showing how to compute it.

In this work, we shall further explore the group of affine symmetries of a Julia set induced by a polynomial and its connection with the topological properties of such polynomial, regarded as a branched covering of the Riemann sphere. In particular, we shall prove that, except for the particular case of power maps, such group of symmetries is precisely the group of automorphisms of the induced polynomial covering map, mostly by topological and algebraic arguments. Moreover, we also present different characterization results that can be useful to compute the group of affine symmetries of the Julia set, and discuss some possible generalizations to the much less understood case of rational maps.

Bibliography

- [1] A. F. Beardon. Symmetries of Julia Sets. *Bulletin of the London Mathematical Society*, 22(6):576–582, 1990.
- [2] A. F. Beardon. Polynomials with identical Julia sets. *Complex Variables, Theory and Application: An International Journal*, 17(3-4):195–200, 1992.
- [3] R. H. Fox. *Covering spaces with singularities*, pages 243–257. Princeton University Press, Princeton, 1957.
- [4] V. Álvarez-Aparicio. *Kaleidoscopic Decomposition, Classification of Finite Branched Coverings of the Sphere and Path-Lifting Algorithms*. Phd thesis, Universidad de la Rioja, 2026.

2.3 Poincaré's stratification for vector fields in $\mathbb{R}^3$ (Clementa Alonso-González¹⁾)

(1) *University of Alicante, Spain.*

Abstract

Let ξ be an analytic vector field in $\mathbb{R}^3$ with isolated singularity and just hyperbolic singular points after reduction of singularities. In this talk we establish, under certain generic conditions, a stratification result that generalizes to three dimensions the classic one, from Poincaré, on the decomposition of the dynamics of an analytic planar vector field into *parabolic*, *elliptic* or *hyperbolic* invariant sectors.

This is a joint work with Fernando Sanz Sánchez (University of Valladolid). Partially supported by the Spanish Research Project PID2020–113052GB–I00

Bibliography

- [1] Alonso-González, C.; Sanz Sánchez, F. *Stratification of three-dimensional real flows I: Fitting domains* Journal of Differential Equations, **361** (2023), 40-96.
- [2] Alonso-González, C.; Sanz Sánchez, F. *Stratification of three-dimensional real flows II: A generalization of Poincaré's planar sectorial decomposition.* Journal of Differential Equations, **436** (2025), 113292.
- [3] Poincaré, H.; Mémoire sur les courbes définies par une équation différentielle. *Journal de Mathématiques Pures et Appliquées*, **7** (1881), 375-422.

2.4 Dynamical Systems in Sobolev Spaces (Assis Azevedo, Davide Azevedo¹, Joana Torres, Mário Bessa)

(1) *University of Minho, Braga, Portugal.*

Abstract

In recent years, we have been working on taking well known dynamical systems theorems and obtaining versions in Sobolev spaces. I will briefly present that work, including versions of the general density theorem and the Oxtoby-Ulam theorem for certain Sobolev spaces. Partially financed by Portuguese Funds through FCT - ‘Fundação para a Ciência e a Tecnologia’ within the research Project UID/00013: Centro de Matemática da Universidade do Minho (CMAT/UM).

Bibliography

- [1] A. Azevedo, D. Azevedo, M. Bessa and M. J. Torres; The closing lemma and the planar general density theorem for Sobolev maps; Proc. Amer. Math. Soc., No. 4, 1687-1696 (2021).
- [2] Genericity of ergodicity for Sobolev homeomorphisms; arXiv:2504.18993.

2.5 Geometry of bifurcation sets: exploring the parameter space

(Roberto Barrio¹, Santiago Ibáñez², Lucía Pérez²)

(1) *University of Zaragoza, Spain.*

(2) *University of Oviedo, Spain.*

Abstract

Inspecting a p -dimensional parameter space by means of $p - 1$ -dimensional slices, changes can be detected that are only determined by the geometry of the manifolds that compose the bifurcation set. We refer to these changes as geometric bifurcations. They can be understood within the framework of the theory of singularities for differentiable mappings and, particularly, the Morse Theory. Working with a three-dimensional parameter space, geometric bifurcations are discussed in the context of three models of neuron activity: the Hindmarsh-Rose, the FitzHugh-Nagumo systems and two coupled neural populations. The first two examples are fast-slow systems with a small parameter that controls the time scale of a slow variable. Geometric bifurcations are observed on slices corresponding to fixed values of this distinguished small parameter, but also in the third example. RB and LP have been supported by the Spanish Research project PID2024-156032NB-I0.

Bibliography

- [1] R. Barrio, S. Ibáñez, L. Pérez: Homoclinic organization in the Hindmarsh–Rose model: A three parameter study, *Chaos: An Interdisciplinary Journal of Nonlinear Science* 30 (5), 053132, 2020
- [2] R. Barrio, S. Ibáñez, L. Pérez: Exploring the geometry of the bifurcation sets in parameter space *Scientific Reports* 14 (1), 10900, 2024.
- [3] A. Mayora-Cebollero, R. Barrio, L. Li, C. Mayora-Cebollero, L. Pérez: Dynamics of coupled neural populations: The role of synaptic dynamics, *Chaos: An Interdisciplinary Journal of Nonlinear Science* 35 (6), 2025.

2.6 When does a nonlinear wave survive a perturbation? A spectral approach for integrable systems (Marcos Caso-Huerta¹, Antonio Degasperis², Sara Lombardo³, Matteo Sommacal⁴)

(1) *University of Oviedo, Spain.*

(2) *Sapienza University of Rome, Italy.*

(3) *Heriot-Watt University, Edinburgh, United Kingdom.*

(4) *Northumbria University, Newcastle upon Tyne, United Kingdom.*

Abstract

Determining whether a solution of a differential equation is stable under small perturbations is one of the central problems in any dynamical setting. For integrable systems (nonlinear PDEs that admit a Lax pair representation), a recently developed algebraic-geometric method transforms this question into a problem of classifying curves in the complex spectral plane. The resulting stability spectrum encodes, in its topology, the complete instability portrait of a given wave solution: whether the background is stable or modulationally unstable, and whether it lies in a regime where extreme localised structures (known as rogue waves) can emerge spontaneously from noise. One of the strenghts of this method is that, unlike most stability methods of its kind, it can be applied even for multicomponent systems.

In this work I will present this framework and illustrate it through a family of integrable systems for long wave–short wave interaction represented by a new two-parameter model, the Yajima-Oikawa-Newell (YON) system, that unifies two classical integrable models discovered in the 1970s. The stability spectrum of the YON system undergoes topological transitions as the background wave parameters vary, in direct analogy with bifurcations in finite-dimensional dynamical systems. When a class of topological objects that we term branches are featured in the spectrum, the system can show a characteristic degeneracy that appears to be both necessary and sufficient for the existence of rogue wave solutions, thus establishing a precise algebraic criterion for the onset of extreme events. This prediction is confirmed by exact rogue wave, breather, and soliton solutions derived via Hirota bilinearisation, and by recent progress on the inverse scattering transform for the system.

Bibliography

- [1] A. Degasperis, S. Lombardo, M. Sommacal (2018). Integrability and linear stability of nonlinear waves, *J. Nonlinear Sci.* **28**.
- [2] M. Caso-Huerta, A. Degasperis, S. Lombardo, M. Sommacal (2021). A new integrable model of long wave-short wave interaction and linear stability spectra, *Proc. R. Soc. A*, **477**: 20210408.
- [3] M. Caso-Huerta, B.-F. Feng, S. Lombardo, K. Maruno, M. Sommacal (2025). Solitons,

breathers and rogue waves of the Yajima-Oikawa-Newell long wave-short wave system,
Wave Motion, **134**: 103511.

2.7 Asymptotic density of limit cycles (Bartomeu Coll¹, Armengol Gasull, Rafael Prohens)

(1) *University of the Balearic Islands, Palma de Mallorca, Spain.*

Abstract

In this work, in the family of polynomial planar differential systems with four monomials, we address the problem of establishing lower bounds for the asymptotic density of systems with a prescribed number of limit cycles. We prove that the asymptotic density of systems possessing at least one, two, or four limit cycles is greater than or equal to $1/4$, $1/16$ or $1/64$, respectively.

To introduce the concept of asymptotic density of systems in the family of polynomial planar differential systems with four monomials having at least p limit cycles, we define the $\mathcal{K}$ function as

$$\mathcal{K}(m, p) = \text{card}\{n = (n_1, n_2, \dots, n_8) \in \mathbb{N}_0^8 : n_i \leq m, \ell(n) = p\},$$

for each $m \in \mathbb{N}$, and we observe that the quotient $\frac{\mathcal{K}(m, p)}{(m+1)^8}$ stands for the relative frequency of systems of degree at most $2m$ having at least p limit cycles, over all systems of this degree in the family, modulus the coefficients of the system. The asymptotic density of systems having at least p limit cycles is defined as

$$\limsup_{m \rightarrow \infty} \frac{\mathcal{K}(m, p)}{(m+1)^8}.$$

It is known that in four monomials differential systems there are many limit cycles, see [1] for instance. We are concerned with the problem of what the abundance of differential systems having a prescribed number of limit cycles will be in such family. We want to thank Prof. Maria Jolis for fruitful discussions and interesting suggestions during the preparation of this work.

The first and third authors are partially supported by the project PID2023-151974NB-I00 funded by the Spanish Ministerio de Ciencia, Innovación y Univ. MICIU/AEI/10.13039/501100011033/FEDER, UE. The second author is partially supported by the Ministerio de Ciencia e Innovación (PID2019-104658GB-I00 grant), by the grant Severo Ochoa and María de Maeztu Program for Centers and Units of Excellence in R&D (CEX2020-001084-M) and also by the Agència de Gestió d'Ajuts Universitaris i de Recerca (2021 SGR 00113 grant).

Bibliography

- [1] A. Gasull and P. Santana. *On a variant of Hilbert's 16th problem*. Nonlinearity 37, 125012 (15pp) (2024).

2.8 Dynamics of the Nonautonomous Bonifacio–Lugiato Model: A Concave–Convex Equation Framework (Jesús Dueñas¹, Carmen Núñez, Rafael Obaya)

(1) *University of Valladolid, Spain.*

Abstract

In this talk, we present tools for studying the global dynamics of nonautonomous scalar ordinary differential equations featuring concave-convex nonlinearities, that is, nonlinearities that exhibit a transition from concavity to convexity in the state variable. These tools are applied to a simple model of superfluorescence in optics. In particular, we investigate the occurrence of uniform stability or bistability in the Bonifacio-Lugiato model by imposing constraints on the nonautonomous input used to excite the system. More generally, models governed by nonautonomous scalar equations may exhibit several transitions between concave and convex regimes, both in the state variable and over time. The theory developed here, which allows for one such change in the state variable, represents a first step toward understanding the full range of dynamical consequences arising from shape constraints on the nonautonomous vector field.

Bibliography

- [1] R. Bonifacio, L.A. Lugiato. Optical bistability and cooperative effects in resonance fluorescence. *Phys. Rev. A* **18**, 1129-1144 (1978).
- [2] J. Dueñas, C. Núñez, R. Obaya. Nonautonomous scalar concave-convex differential equations: conditions for uniform stability or bistability in a model of optical fluorescence. *Chaos* **35** (11), 113105 (2025).
- [3] J. Dueñas, C. Núñez, R. Obaya. Concave-convex nonautonomous scalar ordinary differential equations: from bifurcation theory to critical transitions. *Nonlinearity* **38** (9), 095030 (2025).

2.9 On the dynamics of planar continuous piecewise affine maps (Armengol Gasull¹)

(1) *Autonomous University of Barcelona, Spain.*

Abstract

It is well known that one-dimensional continuous piecewise linear maps can exhibit complex behavior, particularly chaotic dynamics. In a previous work [1] (see also [2]), we introduced a one-parameter family of continuous planar piecewise linear maps possessing one-dimensional invariant sets (graphs) that capture the global dynamics in the plane. Within these graphs, both regular and chaotic dynamics appear for certain parameter values. To the best of our knowledge, this phenomenon in the plane was not been reported before in the literature.

The main goal of this talk is to show that this phenomenon appears to extend to other families that the one studied in the mentioned works. To this end, we focus on the study the continuous dynamical systems given by piecewise-linear (in fact, affine) maps of the form

$$F(x, y) = \begin{cases} F^+(x, y) & \text{if } x \in \mathbb{R}^+, \\ F^-(x, y) & \text{if } x \in \mathbb{R}^-, \end{cases}$$

with $F^\pm(x, y) = A^\pm \cdot (x, y)^T + b^\pm$, where $A^\pm$ are matrices in $\mathbb{R}^{2 \times 2}$, and $b^\pm \in \mathbb{R}^2$. The two regions of definition are determined by the half-planes $\mathbb{R}^+ = \{(x, y); x > 0\}$ and $\mathbb{R}^- = \{(x, y); x \leq 0\}$ and we impose conditions on the matrices and vectors to ensure the global continuity of the map F .

A necessary condition for the existence of an invariant graph (perhaps non compact) that captures the dynamics of the map is that in one of the half-planes, say $\mathbb{R}^-$, the image of the half-plane collapses onto a line (or a half-line). Therefore, in this region, the matrix A^- must necessarily have rank one.

In order for the invariant graph to capture the global dynamics of the map, it is also required that every orbit, except possibly those corresponding to a set of initial conditions located in a finite graph, which will eventually be incorporated into the final invariant graph, enters the half-plane $\mathbb{R}^-$ after finitely many iterations.

In this work, firstly we analyze the conditions under which this occurs and, assuming them, when there appears a graph capturing the dynamics of the whole plane.

As a second step we study whether the dynamics on the obtained graph is simple or complicated. In particular, in some cases we prove that the entropy of this one-dimensional map is positive or we show that a Bernoulli shift does appear.

This talk is based on a joint work with Anna Cima, Victor Mañosa and Francesc Mañosas. The author is supported by the Agencia Estatal de Investigación, grant PID2022-136613NB-I00.

Bibliography

- [1] A. Cima, A. Gasull, V. Mañosa, F. Mañosas. Invariant graphs and dynamics of a family of continuous piecewise linear planar maps. *Qualitative Theory of Dynamical Systems* 24:70 (2025), 103 pp.
- [2] A. Cima, A. Gasull, V. Mañosa, F. Mañosas. Further results for a family of continuous piecewise linear planar maps. [arXiv:2503.02411\[math.DS\]](#)

2.10 Formal Expansions beyond ODE Perturbation Theory (Joan Gimeno¹)

(1) *University of Barcelona, Spain.*

Abstract

The study of normally hyperbolic invariant manifolds (NHIMs) is central in dynamical systems because these objects organize the behavior of nearby trajectories and provide effective reduced descriptions of the dynamics. Their perturbation theory is well developed for ordinary differential equations, but the situation is much less understood when small perturbations change the nature of the equation itself. This happens, for example, when delays, advances, higher derivatives, or fast-slow effects are introduced.

In this talk, I will present a method to construct formal power series expansions for families of solutions emerging from an unperturbed NHIM of an ODE under very general perturbations. The method applies to ordinary perturbations, singular perturbations, delay and advance terms, state-dependent delays, and non-autonomous perturbations. The main result gives a finite recursive procedure to compute, order by order in the perturbation parameter, both the embedding of the perturbed manifold and its internal dynamics, under standard spectral gap assumptions and mild expandability hypotheses on the perturbation.

A key feature of the approach is that it still works when the perturbed equation is no longer an evolution equation. In that setting, the correct object is not an invariant set of initial conditions, but rather a smooth family of approximate solutions parameterized by the reference manifold. I will also explain how the construction fits naturally with a-posteriori results and discuss examples including fast-slow systems, small-delay models, and equations arising in relativistic charged-particle dynamics, where delays are produced by finite propagation speed and the perturbation parameter is $1/c$.

2.11 A dynamical explanation for the elusive nature of Kordylewski clouds (Joan Gimeno, Marc Jorba-Cuscó¹, Begoña Nicolás, Àngel Jorba)

(1) *Polytechnic University of Catalonia, Barcelona, Spain.*

Abstract In this talk we explore dynamical aspects of Kordylewski clouds (dust accumulations located near the triangular libration points of the Earth–Moon system). While the existence of these clouds has been a subject of astronomical debate for decades, this study employs a "scaffolding" approach of increasingly complex models to clarify their stability. We introduce the Augmented Bicircular Problem (ABCP), a planar model that incorporates the gravitational pull of the Earth, Moon, and Sun, along with the Solar Radiation Pressure (SRP).

Our analysis reveals that SRP plays a crucial role in determining the location and stability of the dynamical equivalents of L_4 and L_5 , triggering various bifurcations and resonances as the radiation parameter β varies.

To validate these findings, we conducted numerical simulations using a realistic N -body model. The results demonstrate that while particles can persist in these regions for short times, the vast majority eventually escape the Earth–Moon system i.e. Kordylewski clouds do not exist because there are stable regions in the classical sense, but rather regions of slow diffusion where interplanetary particles are temporarily trapped in a state of continuous renewal.

2.12 Singular perturbation theory in a cardiomyocyte model with three time scales (Jorge A. Jover Galtier¹, Roberto Barrio¹, Lucía Pérez²)

(1) *University of Zaragoza, Spain.*

(2) *University of Oviedo, Spain.*

Abstract

The reduced Luo-Rudy model is a simple 3-variable model of a mammalian ventricle cell that reproduces some interesting phenomena, such as the appearance of early afterdepolarizations (EADs), defective action potentials that when accumulated can lead to arrhythmias and other cardiac problems. The variables of the model present different time scales, which motivates the analysis of the model by singular perturbation theory in two different ways: firstly, by considering the fast evolution of the membrane potential and, secondly, by describing how the difference in time scales of the remaining variables affects the dynamics. We will show how this analysis of the three different time scales of the model is key to understand the emergence of EADs. JAJG, RB and LP have been supported by the Spanish Research project PID2024-156032NB-I0.

Bibliography

- [1] Barrio, R., Jover-Galtier, J. A., Martínez, M. Á., Pérez, L., Serrano, S. (2023). “Mathematical birth of Early Afterdepolarizations in a cardiomyocyte model”. *Mathematical Biosciences*, 366, 109088.

2.13 Stable periodic motions in a bouncing ball model (Stefano Marò¹)

(1) *University of Oviedo, Spain.*

Abstract

We consider the vertical motion of a free falling ball bouncing elastically on a racket moving in the vertical direction according to a regular 1-periodic function f . For fixed coprime p, q we focus on existence, Lyapunov stability and multiplicity of (p, q) -periodic motions. These are p -periodic motions making q bounces in a period. We prove that if $f \in C^3$ and p/q is large, there always exist at least two (p, q) -periodic motions.

We are interested in the stability, in the sense of Lyapunov, of these motions. Under quantitative conditions on f we prove that there exists an infinite sequence of periodic stable motions with $q = 1$. As a consequence, we get an open and unbounded set of initial conditions giving rise to bounded motions. This result relies on the Birkhoff normal form adapted to exact symplectic twist maps. JAJG, RB and LP have been supported by the Spanish Research project PID2024-156032NB-I0.

Bibliography

- [1] S. Marò. *Some remarks on the periodic motions of a bouncing ball*. Bollettino dell'Unione Matematica Italiana 16: 397-409 (2023)
- [2] S. Marò, R. Ortega. *Stability of some periodic configurations of discrete Lagrangian equations*. Nonlinear Analysis 264: 114002 (2026)

2.14 Dynamics of the quadratic family: the origin of the two-dimensional strange attractors (Alejandro Marqués-Lobeiras¹, Antonio Pumariño², José Ángel Rodríguez², Enrique Vigil²)

(1) *Autonomous University of Madrid, Spain.*

(2) *University of Oviedo, Spain.*

Abstract

Homoclinic tangencies for generic families of dissipative 3-D diffeomorphisms were classified by Tatjer [5]. For a certain type of generalized tangency involving a two-dimensional unstable manifold, the return maps have as their limit family the two-dimensional quadratic family

$$T_{a,b}(x, y) = (a + y^2, x + by) \quad (1)$$

As the study of the classical quadratic family was crucial for the proof of the persistence of 1-D strange attractors, the study of family (1) is likely to be the first step towards the proof of the persistence of 2-D strange attractors.

A first analytical approach to the study of the dynamics of family (1) was made by searching for parameter values for which there exist invariant domains [3]. Attractors contained in such invariant domains were then numerically simulated [4]. Furthermore, a numerical analysis based on the Lyapunov exponents allowed to classify the attractors numerically detected for a certain region of parameter values into four types: periodic orbits, invariant closed curves, one-dimensional strange attractors, and two-dimensional strange attractors. See Figure 1 [4]. An analytical proof for the existence and persistence of the two-dimensional strange attractors or even an explanation for the occurrence of the bifurcations between the different types of attractors has not been given yet.

To that end, we carry out a study on the periodic orbits of low period of family (1) and show the coexistence of persistent strange attractors [2]. Also, we construct invariant domains with non-empty interior for sharp regions of parameter values and numerically simulate attractors contained in such domains. I wish to acknowledge the financial support from the "Programa de ayudas Severo Ochoa para la formación en investigación y docencia del Principado de Asturias (Año 2020)" (BP20-063) and the research projects "Génesis de complejidad: dinámica discreta y continua" (PID2020-113052GB-I00) and "Dinámicas locales y globales: bifurcaciones y caos" (PID2023-14761NB-I00). I would also wish to dedicate this work to the memory of professor Antonio Pumariño.

Bibliography

- [1] A. Marqués-Lobeiras, A. Pumariño, J. Á . Rodríguez, and E. Vigil, Splitting and coexistence of 2-D strange attractors in a general family of Expanding Baker Maps, *Nonlinearity* 36 (2023), 4247-4282.

- [2] A. Marqués-Lobeiras, A. Pumariño, J. Á . Rodríguez, and E. Vigil, Dynamics of the 2-D quadratic family: Coexistence of strange attractors, *Commun Nonlinear Sci Numer Simulat* 149 (2025) 108877
- [3] A. Pumariño and J. C. Tatjer, Dynamics near homoclinic bifurcations of three-dimensional dissipative diffeomorphisms, *Nonlinearity* 19 (2006), 2833-2858.
- [4] A. Pumariño and J. C. Tatjer, Attractors for return maps near homoclinic tangencies of three-dimensional dissipative diffeomorphisms, *Discrete Continuous Dyn Syst Ser B* 8 (2007), 971–1005.
- [5] J. C. Tatjer, Three-dimensional dissipative diffeomorphisms with homoclinic tangencies, *Ergod Theory Dyn Syst* 21 (2001), 249-302.

2.15 From statistical properties to young structures in partially hyperbolic systems (João S. Matias¹)

(1) *Centro de Matemática da Universidade do Porto (Portugal)*

Abstract

Classical results due to L.-S. Young [5, 6] show that for dynamical systems admitting suitable inducing schemes, quantitative statistical properties such as decay of correlations and large deviations can be derived from the recurrence behaviour of the inducing structure. Conversely, for non-invertible systems — where no contracting directions are present — it was shown in [2] that decay of correlations can be used to obtain quantitative large deviation estimates, which in turn yield precise recurrence bounds and the existence of Gibbs–Markov inducing schemes. These results were later sharpened in [1] in the exponential and stretched exponential cases, and in [3] in the polynomial case. In the partially hyperbolic setting, where contracting directions are present, the situation is considerably more delicate. Assumptions involving decay of correlations against observables that are measurable with respect to the stable foliation were previously introduced in [4]. However, it remained unclear whether such hypotheses could be used to derive quantitative large deviation estimates. We show that these conditions in fact imply maximal large deviation bounds of stretched exponential or polynomial type. Using these estimates, we prove that the system admits a Young structure whose return-time tails inherit the same asymptotic behaviour, extending the converse results of [2] beyond the non-invertible setting and revealing a new link between decay of correlations, maximal large deviations and Young structures in the presence of contracting directions. These results are part of a joint work with José F. Alves and Ian Melbourne.

Bibliography

- [1] Aimino, R., and Freitas, J. M. Large deviations for dynamical systems with stretched exponential decay of correlations. *Port. Math.* 76, 2 (2019), 143–152.
- [2] Alves, J. F., Freitas, J. M., Luzzatto, S., and Vaienti, S. From rates of mixing to recurrence times via large deviations. *Adv. Math.* 228, 2 (2011), 1203–1236.
- [3] Bunimovich, L. A., and Su, Y. Maximal large deviations and slow recurrences in weakly chaotic systems. *Adv. Math.* 432 (2023), Paper No. 109267, 58.
- [4] Demers, M., Melbourne, I., and Nicol, M. Martingale approximations and anisotropic Banach spaces with an application to the time-one map of a Lorentz gas. *Nonlinearity* 33, 8 (2020), 4095–4113.
- [5] Young, L.-S. Statistical properties of dynamical systems with some hyperbolicity. *Ann. of Math.* (2) 147, 3 (1998), 585–650.
- [6] Young, L.-S. Recurrence times and rates of mixing. *Israel J. Math.* 110 (1999), 153–188.

2.16 Unfolding of a double resonance at a normally elliptic invariant curve in 3D volume-preserving maps (A. Murillo¹, A. Vieiro¹)

(1) *University of Barcelona, Spain.*

Abstract

We investigate the dynamics near a normally elliptic invariant curve of a three-dimensional volume-preserving map, focusing on the unfolding of double resonances, in which the tangent and normal frequencies satisfy two independent resonance relations. Since certain single resonances may destroy the invariant curve or prevent its reducibility, we distinguish several cases within the double-resonant regime. We describe the different bifurcations and classify the three-dimensional resonant structures arising in its neighbourhood. In particular, some of these structures have no analogue in the planar setting and therefore cannot be captured by a two-dimensional reduction. For example, we illustrate mechanisms leading to the creation and destruction of chains of stability bubbles with different topologies.

2.17 Lorenz dynamics prevail under compressibility (Juan Pello¹, Antonio Pumariño¹, Santiago Ibáñez¹)

(1) *University of Oviedo, Spain.*

Abstract

In the present talk we will make the route that goes from the fundamental equations of Fluid Mechanics (continuity equation, Navier-Stokes equations and the conservation of energy law) to the Lorenz equations. Furthermore, knowing the process that led Edward N. Lorenz, in his studies on convection phenomena in the upper layers of the atmosphere, to the discovery in 1963 of the chaotic dynamics of the three-dimensional system it is possible to propose new and consistent problems, on the one hand, in the modification of the approximations made, and on the other hand, in the mathematical study of other more complex possibilities, such as the obtaining of new decoupled differential equation systems, with a larger number of state variables. Hereafter, we set out to reconstruct the process that led to Lorenz system also setting the basis for the development of these other approaches. Moreover, in relation to the modification of the approximations made, we have advanced in the process of obtaining the Lorenz-type equations in the case in which the coefficient of isothermal compressibility is not neglected. This phenomenon suggests that the chaotic, nonlinear behaviors characteristic of the Lorenz model (such as sensitivity to initial conditions and the formation of a strange attractor) continue to exist or dominate even when the fluid or system being modeled is compressible.

2.18 Refining the Dynamic Modeling of Rocking Motion (Set Pérez-González¹, Jesús Suárez Pérez del Río¹, Francisco Prieto¹)

(1) *University of Oviedo, Spain.*

Abstract

Housner’s pioneering study [1] established the dynamics of a rigid block subjected to an external impulse, revealing that Rocking Motion (RM)—despite its seemingly simple setup—can exhibit remarkably complex behavior. This complexity stems from the discontinuous transitions between rest and oscillation about different pivot points, as well as from the intermittent impacts with the supporting surface. More recently, [2] introduced a framework that reformulated the classical piecewise RM equations as a continuous system of differential equations and reinterpreted impact forces—originally modeled through a restitution coefficient—as a coupling mechanism generating instantaneous Dirac-type impulses. This regularized formulation also enables the study of broader classes of dynamical systems.

Starting from the system of central forces that describes Rocking Motion, where angular momentum is a conserved quantity, in our work we managed to generalize it into a singular system that admits a certain polynomial regularization. This allows to analyse the complete dynamics of the system and its corresponding 2-parameter bifurcation diagram. I wish to acknowledge the financial support from the “Programa de ayudas Severo Ochoa para la formación en investigación y docencia del Principado de Asturias (Año 2020)” (BP20-063) and the research projects “Génesis de complejidad: dinámica discreta y continua” (PID2020-113052GB-I00) and “Dinámicas locales y globales: bifurcaciones y caos” (PID2023-14761NB-I00). I would also wish to dedicate this work to the memory of professor Antonio Pumariño.

Bibliography

- [1] Housner G. Housner The behavior of inverted pendulum structures during earthquakes, *Bull Seismol Soc Am* **53** (1963), 403–417.
- [2] Prieto F. Prieto and P.B. Lourenço, On the Rocking Behavior of Rigid Objects, *Meccanica* **40** (2005), 121–133.

2.19 On the stability chart for reversible Hill's equations (Emilio Freire, Manuel Ordóñez, Enrique Ponce¹)

(1) *University of Seville, Spain.*

Abstract

Time-reversal symmetries in periodic linear systems impose a specific structure in their monodromy matrix. Exploiting such structure, after some general results, we study the local geometry of the stability boundaries for reversible Hill's equations of the form $\ddot{x} + q(t)x = 0$, being $q(t)$ an even T -periodic function.

We explain how each curve of the Arnold's tongues is associated to the vanishing of certain entry in the monodromy matrix and how some qualitative features of critical points depend on the coefficients for the Fourier expansion of the stiffness function $q(t)$. Authors are partially supported by Spanish *Ministerio de Ciencia, Innovación y Universidades*, under grant PID2024-158381NB-I00

Bibliography

- [1] Freire E., Ordóñez M., Ponce E. (2022). Hidden regularity of stability boundaries in two-step Hill's equations, *W. Lacarbonara et al. (eds.), Advances in Nonlinear Dynamics, NODYCON Conference Proceedings Series* : 723–733.
- [2] Gan, S., Zhang, M. (2000). Resonance pockets of Hill's equations with two-step potentials. *SIAM J. Math. Anal.* **32**: 651–664.
- [3] Kovacic, I., Rand, R., Sah, S. M. (2018). Mathieu's equation and its generalizations: overview of stability charts and their features. *Appl. Mech. Rev.* **70**: 020802.
- [4] Magnus, W., Winkler, S. (1966). *Hill's Equation*. Wiley, New York.

2.20 The Bosch and Simó conjecture on the Shilnikov-Hopf bifurcation (Alexandre Rodrigues¹)

(1) *University of Lisbon, Portugal.*

Abstract

In this talk, I will investigate the emergence of strange attractors in a neighbourhood of a Shilnikov-Hopf bifurcation. By identifying a broad class of dissipative return maps that capture the essential structure of this codimension-two global bifurcation, we provide a rigorous proof of the Bosch-Simó conjecture (*M. Bosch, C. Simó (1993), Physica D, 217–229*).

We show that, for a set of parameters with positive Lebesgue measure, the dynamics exhibit strange attractors winding around an annular region of phase space. These attractors are “large” in the sense of Broer-Simó-Tatjer. The analysis relies on reducing the system to a one-dimensional model with strong transverse contraction, thereby placing the dynamics within the framework of rank-one strange attractors. In addition, we establish the existence of sequences of parameters for which the family admits superstable periodic orbits, leading to a more complete description of the unfolding of the bifurcation.

2.21 Dimension Reduction and Global Attraction in Delayed Wilson-Cowan Neural Networks (Alfonso Ruiz Herrera¹)

(1) *University de Oviedo, Spain.*

Abstract

Networks of Wilson–Cowan oscillators provide a powerful mathematical framework for modeling neural dynamics. However, despite significant advances, the global behavior of these networks remains poorly understood. In this paper, we develop a systematic methodology to establish criteria for global attraction in networks of Wilson–Cowan oscillators with both intra- and inter-node delays, as well as time-dependent external inputs. Our approach reduces this problem to verifying the uniqueness of solutions to an associated system of inequalities in $\mathbb{R}^2$. This methodology applies to arbitrary network architectures, accommodates both excitatory and inhibitory interactions, and allows general time dependencies in external inputs beyond the T -periodic case. Furthermore, we characterize the global attraction in some classes of networks. From an applied perspective, we identify several stabilizing mechanisms in neural systems: appropriate external inputs or a balance between excitatory and inhibitory connections can effectively stabilize the network dynamics. This is a joint work with Rodrigo Fernández-Martínez.

2.22 Trajectories of vector fields asymptotic to formal invariant curves (F. Sanz Sánchez¹)

(1) *University of Valladolid, Spain.*

Abstract

In this talk we generalize to any dimension a result by F. Dumortier and P. Bonckaert in 1986 concerning the realizability of formal invariant curves of three-dimensional vector fields: Let ξ be a $\mathcal{C}^\infty$ vector field at $(\mathbb{R}^n, 0)$ and suppose that it has a formal invariant curve Γ . Then, as soon as the Taylor expansion of ξ is not identically zero along Γ (a necessary condition), there is a trajectory $\gamma \subset \mathbb{R}^n$ of ξ which has Γ as an asymptotic expansion at the origin.

In fact, we realize the family of all trajectories which are asymptotic to Γ : we construct an invariant $\mathcal{C}^0$ manifold in some open horn around Γ , entirely composed of asymptotic trajectories, and containing the germ of any such trajectory.

Furthermore, if ξ is analytic, we prove that there exists a trajectory γ asymptotic to Γ which is, moreover, non-oscillating with respect to subanalytic sets.

This work made in collaboration with O. Le Gal.

2.23 Nonsmooth bifurcations and fractalization for families of quasiperiodically forced one-dimensional piecewise linear maps (Joan Carles Tatjer¹)

(1) University of Barcelona, Spain.

Abstract

TBA

2.24 Non-uniform expansion and diffusive noise imply dense random horseshoes and existence of a random Gibbs-Markov map
(Giuseppe Tenaglia¹, Dmitry Turaev, Jeroen Lamb)

(1) Imperial College London, United Kingdom.

Abstract

We propose a notion of random horseshoe and prove density of random horseshoes and existence of a random Gibbs-Markov map with exponential tail for non uniformly expanding random dynamical systems with bounded diffusive noise.

Bibliography

- [1] Lamb, Jeroen SW, Giuseppe Tenaglia, and Dmitry Turaev. "Non uniform expansion and additive noise imply random horseshoe." arXiv preprint arXiv:2501.11656 (2025).

2.25 Discrete averaging for maps (A. Viero¹)

(1) *University of Barcelona, Spain.*

Abstract

The discrete averaging theory approximates the dynamics of a map using an autonomous vector field obtained from weighted averages over a segment of trajectory. The methodology provides explicit expressions for the approximating vector fields and yields a direct mechanism for controlling the approximation error. Hence, the discrete averaging provides a simple and effective methodology for finding adiabatic invariants, both numerically and analytically.

We will show how discrete averaging can be applied to investigate the dynamics of a near-identity map as well as the dynamics of a map in a neighbourhood of a resonant fixed point.

This work is a collaboration with Prof. V. Gelfreich from the University of Warwick, UK.

Bibliography

- [1] V. Gelfreich and A. Viero, *Discrete averaging for discrete time dynamical systems*, 2026, arXiv:2603.10737,

3 Poster sessions

3.1 Pattern formation in an active-passive lattice gas (Jesús Ait Idir¹, Maria Bruna, Jose Carrillo)

(1) *University of Oxford, United Kingdom.*

Abstract

We study pattern formation in an active–passive lattice gas through a continuum PDE model obtained as the hydrodynamic limit of an underlying microscopic interacting particle system. The model describes a mixture of active and passive particles and exhibits a rich variety of spatial structures emerging from instabilities of homogeneous states.

While linear stability analysis provides a first characterization of the onset of pattern formation, it does not fully capture the nonlinear selection and evolution of patterns. To address this, we employ weakly nonlinear analysis near the instability threshold and derive amplitude equations governing the slow dynamics of the emerging modes. These reduced equations, typically in the form of coupled ordinary differential equations, offer a systematic framework to classify bifurcations and determine the stability and type of patterns beyond the linear regime.

Our results reveal how the interplay between activity and passive interactions shapes the bifurcation structure of the system and leads to novel pattern selection mechanisms. Moreover, the amplitude equations exhibit a degree of universality, highlighting common features shared across a broad class of pattern-forming PDEs. This provides both qualitative and quantitative insight into the dynamics of active matter systems beyond linear theory.

Bibliography

- [1] J. Mason et al. Dynamical patterns and nonreciprocal effective interactions in an active-passive mixture through exact hydrodynamic analysis. *Nature Communications*, 2025.
- [2] M. Cross, H. Greenside. Pattern Formation and Dynamics in Nonequilibrium Systems. Cambridge University Press, Cambridge (2009).

3.2 Tatjer tangencies in periodic perturbations of generic unfoldings of the three-dimensional nilpotent singularity. (P. Álvarez-Lombardero¹, A. Pumariño¹, J. A. Rodríguez¹)

(1) University of Oviedo, Spain.

Abstract

We study T -periodic perturbations of generic unfoldings of the nilpotent singularity of codimension three in $\mathbb{R}^3$. Specifically, we prove that arbitrarily close to the diffeomorphisms defined by the flow at time $t = T$, there exists dissipative Tatjer tangencies. As a consequence, rich and complex dynamics can be found.

3.3 Spectral Measures for Multivariate Extremes in Dynamical Systems (Alessio Corveddu¹)

(1) *University of Porto, Portugal.*

Abstract

We study the asymptotic behavior of extreme events in multivariate stochastic processes generated by dynamical systems. Extending previous univariate results, we establish the convergence of decorated point processes that record not only the occurrence and intensity of exceedances, but also the multivariate structure of clusters of extremes. A spectral measure for stationary sequences is introduced to characterize extremal profiles and to describe the dependence between components. We demonstrate how this measure can be recovered as a projection of the tail process, and we prove that the multivariate rare event point process converges to a multivariate compound Poisson process. The author was partially supported by CMUP, member of LASI, which is financed by national funds through FCT, IP, under the projects with reference UID/00144/2025 and associated DOI given by <https://doi.org/10.54499/UID/00144/2025>, and by the FCT doctorla scholarship UI/BD/154808/2023

Bibliography

- [1] R. Aimino, A.C.M. Freitas, J.M. Freitas, M. Todd, Multivariate extreme values for dynamical systems, Preprint version Sept 2024, arXiv:2406.14807.
- [2] A.C.M. Freitas, J.M. Freitas, M. Todd, Enriched functional limit theorems for dynamical systems, Ann. Scuola Norm. Sup. Pisa Cl. Sci., to appear (DOI: 10.2422/2036-2145.202401 016).
- [3] Basrak, Bojan, Hrvoje Planinić, and Philippe Soulier. "An invariance principle for sums and record times of regularly varying stationary sequences." Probability Theory and Related Fields 172.3 (2018): 869-914.
- [4] Segers, J. (2012) "Max-stable models for extremal dependence", RevStat Statistical Journal 10, 61–82.
- [5] Nandagopalan, S. Multivariate extremes and estimation of the extremal index. The University of North Carolina at Chapel Hill, 1990.

3.4 Stochastic Properties of Non-Uniformly Hyperbolic Dynamics (Paolo Flavoni¹)

(1) *University of Porto, Portugal.*

Abstract

Chaotic dynamical systems are widely used to model physical phenomena, and the investigation of their statistical properties—often formulated through limit theorems—has played a central role in the development of the field.

In realistic settings, however, such systems are typically affected by small measurement errors, which naturally leads to the consideration of random perturbations of the underlying deterministic dynamics. While one might expect that the presence of noise simplifies the statistical analysis, the opposite is in fact true: establishing analogous results in random dynamical systems is generally much more challenging (see [4] for a general overview).

A key statistical feature of interest is *quantitative recurrence*, described by Hitting Time Statistics (HTS) and Return Time Statistics (RTS), which measure the waiting time before an orbit enters a small target set in phase space. A major breakthrough in this direction was achieved in [3], where an equivalence between the existence of extreme value laws and hitting time statistics was established.

The first work addressing limiting laws for rare events in randomly perturbed systems is [1]. There, the authors consider uniformly expanding systems and obtain results in the *annealed* setting, where one averages over the realizations of the noise to produce a stationary measure with respect to which the statistical properties are studied.

Subsequently, [5] established exponential hitting time statistics for random subshifts of finite type and random expanding maps.

The goal of this project is to extend these results by proving limit theorems for the occurrence of rare events in randomly perturbed non-uniformly hyperbolic systems. Although significant progress has been made in recent years when the unperturbed system exhibits strong hyperbolic properties, moving beyond this framework remains highly challenging. Addressing this problem requires the use of advanced techniques, including Benedicks–Carleson theory (see [2]), point processes, and large deviation methods. The author was partially supported by CMUP, member of LASI, which is financed by national funds through FCT, I.P., under the projects with reference UID/00144/2025 and associated DOI given by <https://doi.org/10.54499/UID/00144/2025>

Bibliography

- [1] H. Ayta, J. M. Freitas, and S. Vaienti, Laws of rare events for deterministic and random dynamical systems, *Trans. Amer. Math. Soc.*, 367(11):8229–8278, 2015.
- [2] M. Benedicks and L. Carleson, On iterations of 1×2 on $(-1, 1)$, *Ann. of Math.* (2), 122(1):1-25, 1985.

- [3] A. C. M. Freitas, J. M. Freitas, and M. Todd, Hitting time statistics and extreme value theory, *Probab. Theory Related Fields*, 147(3-4):675-710, 2010.
- [4] Y. Kifer, Ergodic theory of random transformations, *Progress in Probability and Statistics*, Vol. 10, Birkhauser, Boston, 1986.
- [5] J. Rousseau, B. Saussol, and P. Varandas, Exponential law for random subshifts of finite type, *Stochastic Process. Appl.*, 124(10):3260-3276, 2014.

3.5 Local behavior of Taylor and Padé methods for stiff ODEs

(Philip P. Forrier¹, Joan Gimeno¹)

(1) *University of Barcelona, Spain.*

Abstract

Stiffness is a numerical phenomenon well recognized in practice but which still lacks an accepted mathematical formulation. Since first observed, several ideas have been proposed for its characterization, each refining previous ones. For initial value problems (IVPs), one of the most recent definitions is due to Söderlind, Jay, and Calvo, who characterize stiff regions of the IVP as those exhibiting large time asymmetry; that is, short term reverse time growth is much larger than maximal growth in forward time.

To study the local behavior of numerical integrators for stiff ODEs, we observe that the use of a floating-point arithmetic introduces a perturbation in the initial value that propagates through the computation of its coefficients into the numerical approximation of the step.

At each step of the integration, the Taylor method is constructed by computing recursively derivatives of the vector field at the initial value, so these perturbations introduce terms given by the derivative of the Taylor method w.r.t. the initial value. Being the Taylor method jet-compatible (compatible with jet transport), these terms correspond exactly to a step of the Taylor method on the variational equations and, for stiff ODEs, they are large enough to dominate the integration step, producing stability issues in the method. This is reflected in the numerical integration by a strong reduction in the integration stepsize, which is in practice the observed characteristic behavior of explicit methods when integrating stiff ODEs.

Classically, these stability issues are understood via the stability regions of the methods, given by the application of the method to a scalar linear system. However, we observe that for this to hold, jet-compatibility seems to be fundamental. To show this, we implement Padé methods, defined by constructing a Padé approximant of the solution through its Taylor coefficients. These can be constructed to have good linear stability properties (i.e., A-stability). Nevertheless, the lack of jet-compatibility of these methods makes their error propagation not reduce to a question of linear stability, as the local behavior of a method that is not jet-compatible seems to be determined by its derivative along the orbit rather than by the method itself applied to the variational equations —notions that do coincide for jet-compatible methods—.

Bibliography

- [1] Gustaf Söderlind, Laurent Jay, and Manuel Calvo. Stiffness 1952–2012: Sixty years in search of a definition. *BIT*, 55(2):531–558, June 2015.
- [2] Joan Gimeno, Àngel Jorba, Marc Jorba-Cuscó, Narcís Miguel, and Maorong Zou. Numerical integration of high-order variational equations of ODEs. *Appl. Math. Comput.*, 442:127743, April 2023.

- [3] Philip P. Forrier, Joan Gimeno, and Àngel Jorba. A note on the local behavior of the Taylor method for stiff ODEs. *Appl. Math. Comput.*, 496:129344, July 2025.

3.6 Dynamic methods in non-autonomous differential equations. Stability, bifurcation and critical transitions (Paula Martínez Morais¹)

(1) *University of Valladolid, Spain.*

Abstract This work studies the dynamics of non-autonomous equations from a topological and ergodic perspective. Fundamental classical notions such as exponential dichotomies and their persistence are compiled, as well as concepts of ergodic dynamics applied to non-autonomous systems. Special attention is given to concave flows in finite dimensions, addressing recent results related to stability phenomena, bifurcation, and critical transitions. These models, of particular interest in applied contexts such as mathematical biology or climatology, allow for the capture of relevant nonlinear behaviors in real systems. Additionally, numerical simulations are presented that illustrate the analyzed phenomena and underscore their utility for modeling and qualitative analysis in various branches of applied sciences.

Bibliography

- [1] W. A. Coppel (1978). *Dichotomies in Stability Theory*, Springer, Lecture Notes in Mathematics. ISBN: 9780387085364. DOI: <https://doi.org/10.1007/BFb0067780>
- [2] Peter E. Kloeden and Martin Rasmussen (2011). *Nonautonomous Dynamical Systems*, American Mathematical Society, No. 176.
- [3] Isaac P. Cornfeld, Sergei V. Fomin, and Yakov G. Sinai (2012). *Ergodic Theory*, Springer Science & Business Media, Vol. 245.
- [4] Jesús Dueñas Pamplona et al. (2021). *Métodos dinámicos para el estudio de transiciones críticas en sistemas complejos de la ciencia y la tecnología*.

3.7 Extreme synchronization for product Dynamical Systems (Duarte Sá Pinho¹)

(1) *University of Porto, Portugal.*

Abstract

We establish Extreme Value Distributions for the closest encounter between trajectories generated by different maps defined in the same reference phase space. For a class of strongly mixing maps, we show that the limit distribution depends on the length of the different trajectories. It is also modulated by an Extremal Index, that informs on the tendency of nearby points to diverge along with the evolution of their respective dynamics, serving as an indicator of their compatibility. We give a formula for this quantity for a class of chaotic maps of the interval. We present diverse examples of systems satisfying these assumptions and compute the different parameters modulating the limit distribution. Joint work with Jorge Freitas, Romain Aimino and Théophile Caby. The author was partially supported by CMUP, member of LASI, which is financed by national funds through FCT, I.P., under the projects with reference 2025.04961.BD and associated DOI given by <https://doi.org/10.54499/UID/00144/2025>

Bibliography

- [1] T. Caby, D. Faranda, G. Mantica, S. Vaienti, and P. Yiou, “Generalized dimensions, large deviations and the distribution of rare events,” *Physica D: Nonlinear Phenomena*, vol. 400, p. 132143, 2019. [Online]. Available: <https://arxiv.org/abs/1812.00036>
- [2] V. Lucarini, D. Faranda, A. C. M. Freitas, M. Holland, T. Kuna, M. Nicol, M. Todd, and S. Vaienti, “Extremes and recurrence in dynamical systems,” *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, vol. 374, no. 2065, p. 20150154, 2016.
- [3] B. Saussol, “Absolutely continuous invariant measures for multidimensional expanding maps,” *Israel Journal of Mathematics*, vol. 116, pp. 223–248, 2000.
- [4] G. Keller and C. Liverani, “Rare events, escape rates and quasistationarity: Some exact formulae,” *Journal of Statistical Physics*, vol. 135, no. 3, pp. 519–534, 2009.

3.8 The averaging principle for ordinary differential equations depending on the slow time (Carmen Núñez¹, Rafael Obaya¹, Jorge Rodríguez¹)

(1) *University of Valladolid, Spain.*

Abstract

The work presents a “doubly” nonautonomous version of the averaging principle, applicable to equations that depend on a small parameter ε and on (fast) time τ , but also on slow time $t = \varepsilon\tau$. The objectives are to establish optimal conditions on the dependence of the coefficients of the equations on t under which the averaging principle can be extended and to provide good estimates of the distance between the solutions of the initial equation and those of the averaged equation, always with τ varying in intervals of length proportional to $1/\varepsilon$. The applicability of these results is based on the fact that the estimates obtained are uniform with respect to the initial time, at which the solutions of both equations coincide.

Bibliography

- [1] Z. Artstein (2007) *Averaging of time-varying differential equations revisited*, J. Differ. Equ. 243.2, pgs. 146 – 167.
- [2] Z. Artstein (2010) *Averaging of ordinary differential equations with slowly varying averages*, Disc. Cont. Dyn. B 14.2, pgs. 353 – 365.
- [3] I. Longo, R. Obaya, A. Sanz (2026) *Nonautonomous modelling in energy balance models of climate. Limitations of averaging and climate sensitivity*, Physica D 485, 135038.
- [4] C. Núñez, R. Obaya, J. Rodríguez (2026) *A note on the averaging principle for ordinary differential equations depending on the slow time*, Appl. Math. Letters 178, 109910.
- [5] J. A. Sanders and F. Verhulst (1985) *Averaging methods in nonlinear dynamical systems*, Springer.

3.9 Finitude of Limit Cycles of Linear Piecewise ODEs in the Cylinder (J.L. Bravo¹, R. Trinidad Forte¹)

(1) *University of Extremadura, Spain.*

Abstract

Let $x' = S(t, x)$ be a differential equation in the cylinder, linear piecewise in x and with trigonometric coefficients in t . In this talk, we provide an upper bound on the number of limit cycles in terms of the number of zones of the piecewise equation and the degree of the coefficients, that is, an analog of Hilbert's 16th problem in this context.